Output Feedback Control for Stabilizable and Incompletely Observable Nonlinear Systems: Jet Engine Stall and Surge Control[1]

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Abstract

The problem of controlling surge and stall in jet engine compressors is of fundamental importance in preventing damage and lengthening the life of these components. In this paper, we use the mathematical model developed in [1] to control these two instabilities by output feedback (only one of the three states is measurable). This problem is particularly challenging since the system is not completely observable and, hence, none of the output feedback control techniques found in the literature can be applied to recover the performance of a full state feedback controller. However, we show how to successfully solve it by using a novel output feedback approach for the stabilization of general stabilizable and incompletely observable systems [2]. Simulation results confirm the success of this approach in solving this control problem.

1 Introduction

In this paper we consider the problem of controlling two instabilities which occur in jet engine compressors, namely rotating stall and surge. In [1], Moore and Greitzer developed a three state finite dimensional Galerkin approximation of the nonlinear PDE model describing the compression system. Ever since its development, several researchers have used the Moore-Greitzer three state model (MG3) to design stabilizing controllers for stall and surge. The available control approaches may be divided into three main categories. 1) Linearization and linear perturbation models (e.g., [3, 4, 5] among others). 2) Bifurcation analysis (e.g., [6, 7, 8, 9, 10]). 3) Lyapunov based methods (e.g., [11, 12, 13]). Most of the existing results focus on the development of state feedback controllers, thus complicating their practical implementation (e.g., in [8], the authors need 2D sensor arrays to implement a state feedback control law depending on the squared amplitude of the first harmonic of asymmetric flow and the derivative of the air flow through the compressor). In [11], a partial state feedback controller simplifies practical implementation by requiring measurements of the mass flow and plenum pressure rise, only (hence 2D sensing is not needed). On the other hand, the limitation of this partial state feedback controller lies in the fact that it cannot globally stabilize a unique equilibrium point. To the best of our knowledge, no attempt has been made to design a stabilizing output feedback controller (using only plenum pressure rise feedback) based on a full state feedback control law. This is probably due to the fact that MG3 becomes unobservable when there is no mass flow through the compressor, i.e., the system is not uniformly completely observable (UCO). In this situation, none of the techniques found in the output feedback control literature (e.g., [14, 15, 16, 17, 18]) can be employed for the solution of this problem. In [2, 19], we developed a stable output feedback controller for incompletely observable nonlinear systems which, in particular, can be applied to MG3. In this paper we introduce a new globally stabilizing full state feedback control law for MG3, and we employ the theory developed in [2, 19] to regulate stall and surge by using only pressure measurements.

2 Problem Description

The approximated model introduced in [1] is described by (see [20] for an analogous exposition)

$$\begin{aligned}\dot\Phi &= -\Psi + \Psi_C(\Phi) - 3\Phi R\\ \dot\Psi &= \frac{1}{\beta^2}(\Phi - \Phi_T)\\ \dot R &= \sigma R(1-\Phi^2 - R),\ R(0)\geq 0\end{aligned} \tag{1}$$

where Φ represents the mass flow, Ψ is the plenum pressure rise, $R \geq 0$ is the normalized stall cell squared amplitude, Φ_T is the mass flow through the throttle, $\sigma = 7$ and $\beta = 1/\sqrt{2}$. The functions $\Psi_c(\Phi)$ and $\Phi_T(\Psi)$ are the compressor and throttle characteristics, respectively, and are defined as $\Psi_C(\Phi) =$

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$\Psi_{C_0} + 1 + 3/2\Phi - 1/2\Phi^3$, $\Psi = \frac{1}{\gamma}(1+\Phi_T(\Psi))^2$, where Ψ_{C0} is a constant and γ is the throttle opening, the control input. Given the static relationship existing between Φ_T and γ, without loss of generality, in what follows we will design a controller assuming that Φ_T is our control input. Our control objective is to stabilize system (1) around the critical equilibrium $R^e = 0, \Phi^e = 1, \Psi^e = \Psi_C(\Phi^e) = \Psi_{C0} + 2$, which achieves the peak operation on the compressor characteristic. We shift the origin to the desired equilibrium with the change of variables $\phi = \Phi - 1, \psi = \Psi - \Psi_{C0} - 2$. System (1) then becomes

$$\begin{aligned} \dot{R} &= -\sigma R^2 - \sigma R(2\phi + \phi^2) \\ \dot{\phi} &= -\psi - 3/2\phi^2 - 1/2\phi^3 - 3R\phi - 3R \\ \dot{\psi} &= -\frac{1}{\beta^2}(\Phi_T - 1 - \phi) \end{aligned} \tag{2}$$

The pressure rise (and hence ψ) is the only measurable state variable. In the next section we will apply the theory developed in [2, 19] to develop an output feedback controller to solve the problem above. The procedure outlined in [2, 19] can be summarized as follows.

(i) Extend system (2) with a chain of n_u integrators at the input side (where n_u is determined from the observability mapping).

(ii) Design a stabilizing state feedback control law $\bar{v} = \varphi(x, z)$ for the extended system.

(iii) Implement the nonlinear observer in [2, 19] and choose an appropriate compact set C_ξ satisfying assumption A3 therein.

(iv) Choose the design constant ρ in the observer small enough to guarantee closed-loop stability, and implement the output feedback control law $\hat{v}$ after projecting $\hat{x}$ using the projection defined in [2, 19].

In the following sections we will apply this approach to the jet engine surge and stall control problem.

3 State Feedback Control Design

Here, we apply the output feedback control design outlined in the previous section to system (2), assuming that $y = \psi$, i.e., only the pressure rise is measured. For convenience, in the remainder of the paper we will redefine the control input to be $u = \Phi_T - 1$. Next, notice that Assumption A2 is satisfied since, for example, a stabilizing control law for (2) is given in [20] by means of backstepping design. However, the control law proposed in [20] turns out to be quite complex. In [11], it is shown that a linear partial state feedback control law of the type $u = d_1\psi - d_2\phi$ achieves either a unique asymptotically stable equilibrium point with domain of attraction $\{(R, \phi, \psi) \in \mathbb{R}^3 | R \geq 0\}$ or two equilibria on the axisymmetric and stall characteristic, with domains of attraction $\{(R, \phi, \psi) \in \mathbb{R}^3 | R = 0\}$ and $\{(R, \phi, \psi) \in \mathbb{R}^3 | R > 0\}$, respectively (see Theorem 3.1 in [11]). Here, this problem is overcome by viewing system (2) as an interconnection of two subsystems, namely the R-subsystem and the (ϕ, ψ)-subsystem, and then building a full state feedback controller which makes the origin of (2) an asymptotically stable equilibrium point with domain of attraction $\{(R, \phi, \psi) \in \mathbb{R}^3 | R \geq 0\}$, as seen in the next theorem.

Theorem 1 *For system (2), with the choice of the control law*

$$\bar{u} = (1 - \beta^2 k_1 k_2)\phi + \beta^2 k_2 \psi + 3\beta^2 k_1 R\phi \tag{3}$$

where k_1 and k_2 are positive scalars satisfying the inequalities,

$$k_1 > \frac{17}{8} + \frac{(2C\sigma + 3)^2}{2} \tag{4}$$

$$\left(C\sigma - \frac{105}{64}\right)k_1^2 + \frac{3}{4}\left(-\frac{1}{2}C\sigma + \frac{21}{4}\right)k_1 - (C\sigma + 3)^2 > 0 \tag{5}$$

$$k_2 > k_1 + \frac{9}{4}k_1^2 + \frac{9k_1}{4k_1 - 9/2} + \frac{(k_1^2 - 1)^2}{4} \tag{6}$$

$$C > \frac{3}{2\sigma} \tag{7}$$

the origin is an asymptotically stable equilibrium point with domain of attraction

$$\mathcal{A} = \{(R, \phi, \psi) \in \mathbb{R}^3 | R \geq 0\}.$$

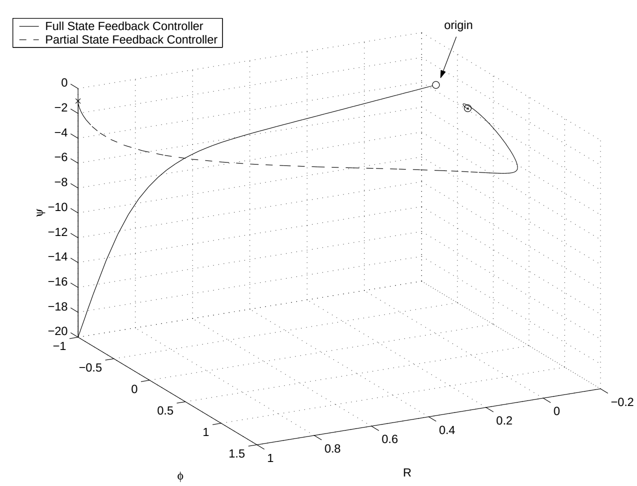

Figure 1: Comparison between the partial state feedback controller developed in [11] and the full state feedback controller (3).

Remark 1: By using inequalities (4)-(7), it is easy to show that the only equilibrium point of the closed-loop systemon the set $\mathcal{A}$ is the origin, as predicted by Theorem 1. Figure 1 shows the evolution of the closed-loop trajectories under the partial state feedback controller developed in [11] and the controller (3) for a particular choice of the coefficients d_1, d_2, k_1, k_2. The partial state feedback controller stabilizes an equilibrium point different from the origin $(R, \phi, \psi) = (0, 0, 0)$.

Remark 2: Inequalities (4)-(7) represent conservative bounds on k_1 and k_2. In practical implementation, these parameters may be chosen significantly smaller after some tuning.

In order to complete the state feedback design, we have to add an appropriate number of integrators at the input side of the system. Following the procedure outlined in [2, 19], we form the observability mapping $\mathcal{H}$:

$$y_e = \begin{bmatrix} y \\ \dot{y} \\ \ddot{y} \end{bmatrix} = \mathcal{H}\left([R, \phi, \psi]^\top, u, \dot{u}\right) =$$
$$\begin{bmatrix} \psi \\ -1/\beta^2(u-\phi) \\ 1/\beta^2\left(-\dot{u} - \psi - 3/2\phi^2 - 1/2\phi^3 - 3R\phi - 3R\right) \end{bmatrix}. \quad (8)$$

Notice that Assumption A1 in [2, 19] is satisfied, for all $\phi \neq -1$, with $n_u = 2$ in that given y_e, u, and $\dot{u}$, one can uniquely find R, ϕ, ψ. The operating point $\phi = -1$ corresponds to $\Phi = 0$, i.e., no mass flow through the compressor which is a condition we would like to avoid during normal engine operation. Since $n_u = 2$, we extend the system with two integrators $\dot{z}_1 = z_2$, $\dot{z}_2 = v$, $u = z_1$. To simplify the notation in the following, define $x = [R, \phi, \psi]^\top$, and rewrite (2) as $\dot{x} = f(x) + g(x)z_1$. Next, we find a stabilizing control law for the extended system by using backstepping: $v = \dot{\alpha} - \tilde{z}_1 - k_4\tilde{z}_2 \triangleq \varphi(x, z)$, where $\tilde{z}_1 = z_1 - \bar{u}$, $\alpha = -k_3\tilde{z}_1 - \dfrac{\partial V}{\partial x}g(x) + \dfrac{\partial \bar{u}}{\partial x}[f(x) + g(x)\, z_1]$, $\tilde{z}_2 = z_2 - \alpha$, and k_3, k_4 are arbitrary positive constants. This completes the design of a stabilizing state feedback for the extended system. The Lyapunov function of the closed-loop extended system is $\bar{V} = V + \frac{1}{2}\tilde{z}_1^2 + \frac{1}{2}\tilde{z}_2^2$, where $V = CR + \frac{1}{2}\phi^2 + \frac{k_1}{8}\phi^4 + \frac{1}{2}\left(\psi - k_1\phi\right)^2$ is the Lyapunov function for the original x-system. Notice that the set $\{[R, \phi, \psi, z_1, z_2]^\top \in \mathbb{R}^5 \,|\, R \geq 0\}$ is invariant, hence by applying the backstepping lemma we guarantee that the origin of the extended system is asymptotically stable with domain of attraction $\mathcal{D} = \mathcal{A} \times \mathbb{R}^2$.

4 Observer Design

The validity of Assumption A1 in [2, 19] allows us to design a stable observer. First, we calculate the Jacobian of the observability mapping (8).

$$\frac{\partial \mathcal{H}}{\partial x} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & \frac{1}{\beta^2} & 0 \\ -\frac{3+3\phi}{\beta^2} & -\frac{3\phi+3R+(3/2)\,\phi^2}{\beta^2} & -\frac{1}{\beta^2} \end{bmatrix} \quad (9)$$

which is an invertible square matrix, as expected. Its inverse is

$$\left[\frac{\partial \mathcal{H}}{\partial x}\right]^{-1} = \begin{bmatrix} -\frac{1}{3(1+\phi)} & -\frac{\beta^2\left(3\phi+3R+(3/2)\phi^2\right)}{3(1+\phi)} & -\frac{\beta^2}{3(1+\phi)} \\ 0 & \beta^2 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad (10)$$

As already pointed out, Assumption A1 in [2, 19] is satisfied on the domain $\mathcal{X} \times \mathcal{U} = \left\{[R, \phi, \psi] \in \mathbb{R}^3 \,|\, \phi > -1\right\} \times \mathbb{R}^2$. Hence, the output feedback approaches found in the literature cannot be used to solve the surge and stall control problem. We first design the nonlinear observer defined in [2, 19]:

$$\begin{aligned}
\dot{\hat{R}} &= -\sigma\hat{R}^2 - \sigma R(2\hat{\phi} + \hat{\phi}^2) - \\
&\quad - \frac{\frac{l_1}{\rho} + \beta^2(3\hat{\phi} + 3\hat{R} + (3/2)\hat{\phi}^2)\frac{l_2}{\rho^2} + \beta^2\frac{l_3}{\rho^3}}{3(1+\hat{\phi})}(\psi - \hat{\psi}) \\
\dot{\hat{\phi}} &= -\hat{\psi} - 3/2\,\hat{\phi}^2 - 1/2\,\hat{\phi}^3 - 3\hat{R}\hat{\phi} - 3\hat{R} + \\
&\qquad + \beta^2(l_2/\rho^2)(\psi - \hat{\psi}) \\
\dot{\hat{\psi}} &= -\frac{z_1 - \hat{\phi}}{\beta^2} + (l_1/\rho)(\psi - \hat{\psi})
\end{aligned}$$

Next, we implement the projection in [2, 19] to confine the observer estimates to within the observable space. First, choose $L = [l_1, l_2, l_3]^\top$ such that $A_c - LC_c$ has eigenvalues placed at -2, then calculate the solution P of the Lyapunov equation associated to $A_c - LC_c$, and its square root $S = S^\top$. Recall that $\hat{\xi} = \mathcal{H}(\hat{x}, z)$, $\dot{\hat{\xi}} = \left\{\dfrac{\partial \mathcal{H}}{\partial \hat{x}}\dot{\hat{x}} + \dfrac{\partial \mathcal{H}}{\partial z}\dot{z}\right\}$, and choose the set $C_\xi(z)$ to be the cube $C_\xi(z) = \{\xi \in \mathbb{R}^3 \,|\, \xi_1 \in [a_1, b_1], \xi_2 \in [-1/\beta^2(z_1 + a_2), -1/\beta^2(z_1 - b_2)], \xi_3 \in [1/\beta^2(-z_2 - a_3), 1/\beta^2(-z_2 + b_3)]\}$, which, when $a_2 < 1$, is contained in $\mathcal{H}(\mathcal{X}, z)$, for all z. Next, the normal vectors $N(\hat{\xi}, z), N_z(\hat{\xi}, z)$ are readily calculated as follows,

$$N(\hat{\xi}, z) = \begin{cases} [1, 0, 0]^\top \text{ if } \hat{\xi}_1 = b_1 \\ [-1, 0, 0]^\top \text{ if } \hat{\xi}_1 = a_1 \\ [0, 1, 0]^\top \text{ if } \hat{\xi}_2 = -\frac{1}{\beta^2}(z_1 - b_2) \\ [0, -1, 0]^\top \text{ if } \hat{\xi}_2 = -\frac{1}{\beta^2}(z_1 + a_2) \\ [0, 0, 1]^\top \text{ if } \hat{\xi}_3 = \frac{1}{\beta^2}(-z_2 + b_3) \\ [0, 0, -1]^\top \text{ if } \hat{\xi}_3 = \frac{1}{\beta^2}(-z_2 - a_3) \end{cases}$$

$$N_z(\hat{\xi}, z) = \begin{cases} [0,0]^\top \text{ if } \hat{\xi}_1 = b_1 \text{ or } \hat{\xi}_1 = a_1 \\ \left[\frac{1}{\beta^2}, 0\right]^\top \text{ if } \hat{\xi}_2 = -\frac{1}{\beta^2}(z_1 - b_2) \\ \left[-\frac{1}{\beta^2}, 0\right]^\top \text{ if } \hat{\xi}_2 = -\frac{1}{\beta^2}(z_1 + a_2) \\ \left[0, \frac{1}{\beta^2}\right]^\top \text{ if } \hat{\xi}_3 = \frac{1}{\beta^2}(-z_2 + b_3) \\ \left[0, -\frac{1}{\beta^2}\right]^\top \text{ if } \hat{\xi}_3 = \frac{1}{\beta^2}(-z_2 - a_3) \end{cases}$$

Thus, the output feedback controller design is completed by using these expressions in the projection defined in [2, 19], and letting $\hat{v} = \varphi(\hat{x}^P, z)$, where $\hat{x}^P$ denotes the projected observer estimate.

5 Simulation Results

Here we present the simulation results when the output feedback controller developed in the previous section is applied to system (2). We choose $k_1 = 25$ and $k_2 = 1.1 \cdot 10^5$ to fulfill inequalities (4)-(7) in Theorem 2. In order to choose the size of the compact set $C_\xi(z)$ so that Assumption A3 is satisfied, we may use the Lyapunov function $\bar{V}$ to calculate $\Omega^x_{c_2}$, choose c_2 small enough to guarantee that $\Omega^x_{c_2} \subset \mathcal{X}$, and use $\mathcal{H}$ to calculate bounds on ξ when $x \in \Omega^x_{c_2}$. However, a more practical way to address the design of $C_\xi(z)$ consists of running a number of simulations for the closed-loop systemunder state feedback corresponding to several initial conditions $[R(0), \phi(0), \psi(0)]^\top$, and calculating upper and lower bounds for ψ, ϕ, and $-\psi - 3/2\phi^2 - 1/2\phi^3 - 3R\phi - 3R$: these will provide the values of a_i, b_i, $i = 1, 2, 3$, respectively. By doing that, we found that whenever $[R(0), \phi(0), \psi(0)]^\top \in \Omega_0 \triangleq \{[R, \phi, \psi]^\top \in \mathbb{R}^3 \,|\, R \in [0, 0.1], \phi \in [-0.1, 0.1], \psi \in [-0.5, 0.5]\}$, we have that $a_1 = -2$, $b_1 = 1$, $a_2 = -0.5$, $b_2 = 1$, $a_3 = -0.5$, $b_3 = 0.3$ satisfy Assumption A3 in [2, 19]. We must point out that our choice of Ω_0 is rather conservative and is made primarily for the sake of illustration. The actual domain of attraction $\mathcal{D}'$ under output feedback control is larger that Ω_0, and can be made arbitrarily close to $\Omega^x_{c_2}$, where c_2 is the largest scalar guaranteeing that $\Omega^x_{c_2} \subset \mathcal{X}$. In Figures 2 to 4 system and controller states, together with the control input, are plotted for three decreasing values of ρ confirming the theoretical predictions about the arbitrary fast rate of convergence of the observer found in Theorem 1 in [2]. Furthermore, the figures also show the operation of the projection which prevents the observer from peaking and guarantees that $\hat{\phi} > -0.5$. Finally, note that the output feedback trajectories tend to the state feedback ones, as showed in Figure 5.

These simulations, besides confirming the theoretical results, illustrate the simplicity of this approach, which has the advantage of being modular, in that the state feedback control design is separated from the observer design, and of working with systems that are not completely observable.

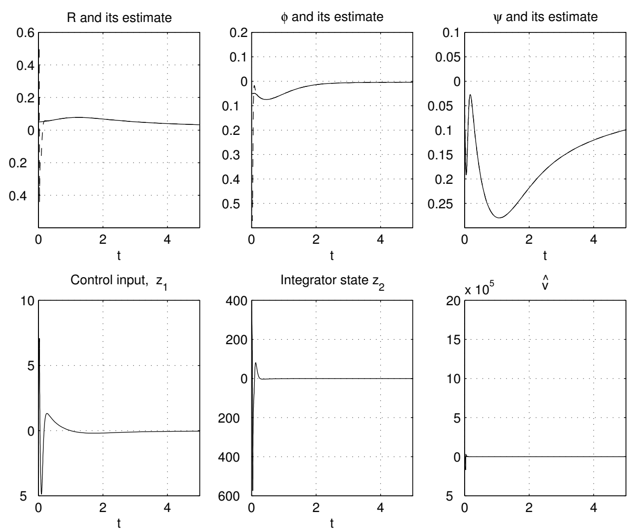

Figure 2: Output feedback control: $\rho = 0.05$.

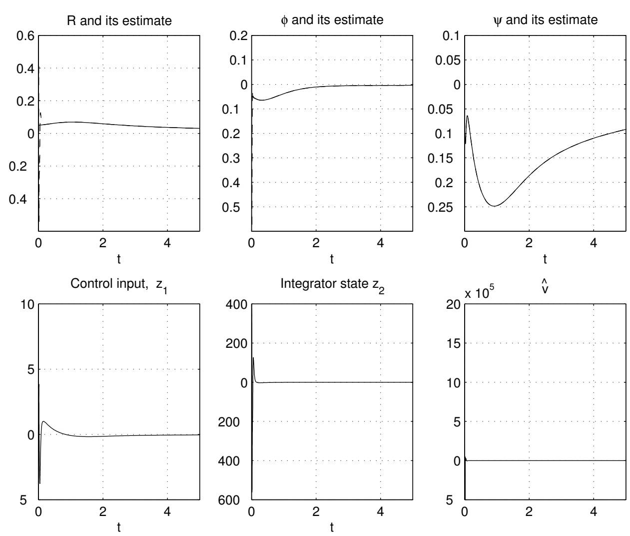

Figure 3: Output feedback control: $\rho = 0.02$.

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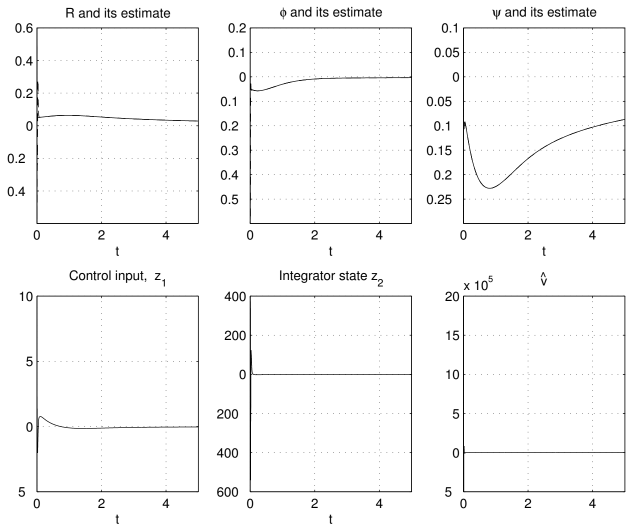

Figure 4: Output feedback control: $\rho = 0.01$.

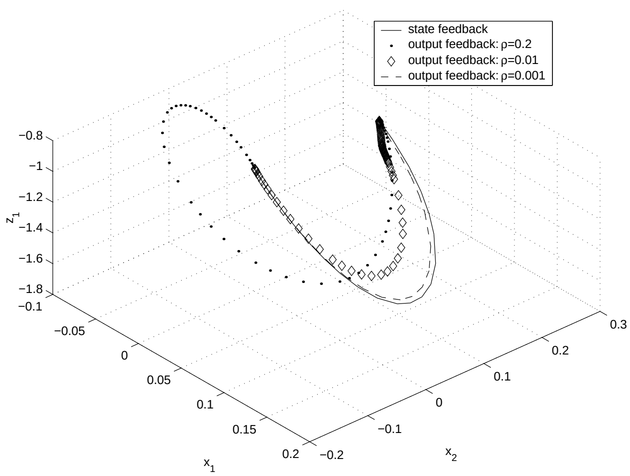

Figure 5: State feedback trajectories and output feedback trajectories for $\rho = 0.05$, $\rho = 0.02$, and $\rho = 0.005$.

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