IMPLEMENTATION OF HIGH PRECISION POSITIONING SYSTEMS USING CONTACTLESS MAGNETIC LEVITATION

by

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A thesis submitted in conformity with the requirements for the degree of Masters of Applied Science Graduate Department of Electrical and Computer Engineering University of Toronto

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Abstract

Implementation of High Precision Positioning Systems Using Contactless Magnetic Levitation

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This thesis presents the implementation and control of two high precision positioning systems based on contactless magnetic levitation, actuated by means of permanent magnet linear synchronous motors. Such positioning technology has applications in the manufacturing of semiconductors and other industrial processes. The first system presented in this thesis employs one permanent magnet linear synchronous motor to actuate two translational degrees-of-freedom and serves as a proof of concept testbed. The second system employs four motors to actuate three translational degrees-of-freedom. For each implementation, a procedure is outlined through which a theoretical state-space model is enhanced using system identification. The resulting mathematical model is then validated using experimental data generated from each apparatus. It is shown that the models of both systems accurately describe the real dynamics over a wide operating range. The models are then used to design nonlinear and linear controllers for set point stabilization and sinusoidal tracking. Comprehensive experimental results demonstrate the superior performance of the nonlinear controllers as compared to their linear counterparts.
Dedication

It is with great pleasure that I dedicate this work to my parents, my siblings, and all the rest of my family and friends. Throughout the many years of my academic pursuits, you have all provided me with unconditional patience and support.

I thank you all.
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Chapter 1

Introduction

1.1 Applying Contactless Positioning to Manufacturing and Research

The manufacturing of semiconductor components is a complex process in which a base substrate material\(^1\) is implanted with constituent microelectronic elements and the corresponding sub-connections. The process requires multiple stages which can include, but is not limited to, lithography, oxidation, ion implantation, rapid thermal processing, as well as chemical etching and deposition [4].

Many of the aforementioned process stages in semiconductor manufacturing require positioning systems, referred to as microsteppers, capable of several degrees of freedom (DOF) with significant speed and precision. To achieve these positioning requirements, the current industry practice is to employ dual-stage microsteppers. The first stage consists of a coarse mechanical actuation that produces high speed movement over a large range, while the second stage makes use of piezoelectric actuators to obtain the necessary degree of precision [26].

\(^1\)Substrate is most often composed of silicon. Other common materials that can be used include germanium and gallium arsenide
As the pace of technology causes the overall dimensions of semiconductors to further decrease, there is an increasing interest in industry to replace traditional dual-stage mechanical microsteppers by contactless positioning devices. One of the main challenges of using mechanical positioning in these applications arises from the fact that mechanical contacts can introduce dust particles and other unacceptable impurities into extremely sensitive processes, such as the lithographic stage, thus decreasing production throughput. Furthermore, mechanical positioning devices wear down over the course of time and eventually require costly maintenance on their components.

Contactless positioning can solve these problems by eliminating the dependence on moving parts. In fact, the ability for contactless positioning systems to accommodate the accuracy demands for future reductions in component size would only be limited by sensor resolution and controller design. Finally, by eliminating any mechanical coupling, contactless positioning devices can potentially reject (if properly controlled) vibrations induced by the surrounding environment which would normally limit accuracy.

While semiconductor manufacturing is the primary motivation for undertaking this research, contactless positioning can be employed in any manufacturing process where significant speed and precision are required at multiple degrees of freedom, and where the aforementioned limitations of mechanical positioning are a serious concern. Some specific examples include transportation systems for factory material [22], suspension systems for vibration isolation [25], and the emerging field of nanotechnology [12] [21]. Other fields that could potentially benefit from contactless positioning include robotics, vehicle design, and space technology.

Beyond industrial applications, contactless positioning is a challenging control problem with interesting nonlinear dynamics. The physical realization of contactless positioning would therefore serve as an excellent testbed through which a variety of control theories and designs could be implemented and verified.
1.2 Thesis Objectives

The overall objectives of this work are as follows:

• Implement a first-generation positioning device employing one permanent magnet linear synchronous motor (PMLSM) to actuate two translational DOFs. Though not fully contactless, this apparatus serves as a proof of concept testbed to validate the mathematical models and controller designs developed in [16].

• Implement a second-generation positioning device employing four PMLSMs to actuate three translational DOFs.

• Use a system identification technique to identify unknown parameters in the 2-DOF and 3-DOF system models.

• Verify the accuracy of the mathematical models resulting from system identification over a suitable operating range. This should be done using experimental measurements of the forces and currents that are generated during testing.

• Employ the validated models in the design of 2-DOF and 3-DOF nonlinear controllers capable of set-point stabilization and sinusoidal tracking. The desired accuracy of the 2-DOF set-point stabilization is specified to be 0.1 mm with a settling time of less than 5s, while the desired 3-DOF set-point accuracy is specified to approach the sensor resolution of\(^2 10 \mu m\) with a settling time of 5s. For sinusoidal tracking, it is only mandated that each system minimize tracking error to as great an extent as possible.

• Compare nonlinear controller performance to a series of linear counterparts.

\(^2\)While the accuracy specifications that have been given are a good starting point, they are nowhere near the resolution required for semiconductors, which is approaching 0.1 \(\mu m\). As future generations of the positioning apparatus are constructed, the required accuracy will increase.
1.3 Existing Contactless Positioning Systems Based on Magnetic Levitation

The implementation of a high precision contactless positioning system based on magnetic levitation is not an original concept and is currently being investigated and developed by other sources. What follow are some examples of related work, including a discussion of the advantages/drawbacks of each implementation.

In [15], Kim and Trumper propose a contactless microstepper for use in semiconductor manufacturing which employs single sided *air cored* permanent magnet linear synchronous motors (PMLSM) to actuate a platform with six degrees-of-freedom. The custom built air cored PMLSMs allow the platform to move above the stators through a repulsive force interaction with no overhead structure or target assembly. The apparatus achieves positioning accuracy with a resolution on the order of 10nm using a linear controller based on decoupled lead-lag compensation, over a planer range of (50 mm × 50 mm), an air-gap range of 400 µm normal to the plane, and rotations on the order of a milli-radian. Since air-cored PMLSMs depend completely on the available 3-phase current supply to generate levitation, it is crucial that the platform weight and the air-gap range be restricted to guarantee power efficiency. Another drawback of the air cored PMLSMs is that they are custom motors not readily available in industry. Finally, the linear controller from [15] suffers from large overshoot and a lack of robustness.

Another positioning implementation based on magnetic levitation with air-cored PMLSMs is developed in [27] specifically for transport applications. A prototype maglev vehicle is proposed with the capability of both horizontal propulsion and repulsive vertical lift, using a direct torque controller. Experimental results demonstrate satisfactory control of the propulsion position and speed over a travel range of 1.5 m. However, the implementation suffers from drawbacks similar to those in [15].

The work in [24] relies on standard *electromagnets*, rather than PMLSMs to imple-
ment contactless positioning. The prototype in [24] employs a set of 10 electromagnets placed within the interior of a cage-structure that houses a movable platen. The electromagnets are positioned in order to allow for up to 6 degrees-of-freedom. Using a nonlinear controller based on feedback linearization and a robust $\mathcal{H}_\infty$ design, the platform is able to perform set-point stabilization and contour tracking to a resolution on the order of 10nm over a planar range of 8mm, an air-gap range of 4mm, and rotations on the order of 1 degree. The results from [24] demonstrate that while the electromagnets are low cost and easy to construct, they are only capable of generating a small range of operation.

Other non-PMLSM solutions can be found in [6] and [13], where multiple degree-of-freedom levitation apparatus are constructed from salient pole permanent magnets and current-controlled hybrid magnets. Using adaptive control strategies, the authors in [6] and [13] achieve set-point positioning with resolution and travel ranges similar to [24].

### 1.4 Magnetic Levitation Using Iron-Cored PMLSMs

In [16], modelling and nonlinear control designs are presented for an idealized 3-DOF contactless positioning device which employs iron cored PMLSMs.

Like the air cored devices in [15], iron cored PMLSMs can produce both a normal and translational force when a variable air-gap is permitted and the device is subjected to appropriate control. Several iron-core PMLSMs can also be combined to produce multiple degrees-of-freedom movement over a large operating range. However, iron-core PMLSMs offer a significant advantage over their air core counterparts. If the mover is placed below the stator surface, an attractive normal force is generated by the interaction between permanent magnet fields on the mover and the iron core of the stator. Such attractive force makes it possible to control the air-gap between stator and mover using significantly less power than in air-cored motors, or lift a greater weight. Additionally, iron-cored PMLSMs are widely used in industry and commonly found on the market.
1.5 Organization of Thesis Material

The Thesis is divided into the following set of chapters:

**Chapter 2** introduces some background material on iron core permanent magnet linear synchronous motors. The chapter begins with a description of the structure of the type of iron core PMLSM used in the construction of each apparatus and its principles of operation. This is followed by a brief summary of the techniques used in [16] to derive the translational and normal forces generated by a single iron core PMLSM. The chapter concludes with a discussion of the practical issues that arise in modelling and implementing an actual PMLSM.

**Chapter 3** provides an overview of the system identification procedure that is applied to the state-space models of each apparatus.

**Chapter 4** details the complete implementation of the actual first-generation 2-DOF contactless position apparatus. A complete physical description of the apparatus is provided, including the details of the constituent elements such as the sensors, linear guides, bearings, current amplifiers, and the real-time controller interface. It is then shown how the material from chapter 2 is applied to the creation of a state-space model of the system dynamics. With this preliminary material in place, chapter 4 proceeds to demonstrate how the system identification technique developed in Chapter 3 is applied to the estimation of the model parameters. The resulting model is then verified by comparing the forces and currents predicted by the model to the actual forces and currents produced by the system. Once the final model and validation procedure are presented, the chapter summarizes the design, implementation and testing process of a nonlinear controller based on feedback linearization and output regulation. Several experimental control results are subsequently included.
The results presented in chapter 4 show that the state-space system model is accurate over a wide range of operation (100 mm × 10 mm). The experimental results also demonstrate that when the nonlinear controller is applied to the actuation of the system, set-point stabilization and sinusoidal tracking can be achieved with performance that meets the accuracy requirements as outlined in the thesis objectives. It is also shown how the nonlinear controller performs significantly better when compared to some PID and linear counterparts.

Chapter 5 details the complete implementation of the actual second-generation 3-DOF contactless position apparatus. In a manner similar to Chapter 4, the 3-DOF hardware setup, model derivation, system identification application, model verification, nonlinear control design, and experimental results are detailed. As in Chapter 4, the results again demonstrate the accuracy of the model over a wide range of operation in addition to the nonlinear controller performance in comparison to PID and linear counterparts. Additional material not contained in the previous chapter demonstrates how antiwindup compensation can be incorporated into both the 3-DOF nonlinear and linear controllers in order to improve performance in the presence of actuator saturation.

Chapter 6 summarizes the final conclusions of the thesis. This chapter includes a description of the major contributions of this research, as well as an outline of how future work will progress in this subject area.
Chapter 2

Review of PMLSMs and The Derivation of The Forces

2.1 The Structure of a PMLSM

In general, a linear synchronous motor has a structure equivalent to a rotary synchronous machine in which the stator and rotor have been cut along their cross-sections and unrolled to produce a flattened configuration [23]. The result is that the flattened rotor, now referred to as a mover, is subject to a linear propulsion rather than a rotary torque. Despite these alterations, many of the same physical models and characteristics from standard rotary machines still apply.

There are numerous types of linear synchronous motors that have been implemented in both research and industry, each having specific advantages depending upon the application. For the solution to the magnetic levitation based contactless positioning problem developed in [16] and investigated in this thesis, the type chosen was a flat, single-sided, iron-cored permanent magnet linear synchronous motor [3]. The physical structure of this type of motor is illustrated in Figure 2.1. For a detailed listing of various types of other linear synchronous motors, the reader is referred to [16].
A flat, single-sided PMLSM configuration was chosen because of its ability to generate two primary forces: a translational force which generates a thrust in the mover parallel to the $x$-axis of Figure 2.1, and a normal force which generates a thrust in the mover along the $y$-axis of Figure 2.1 directly perpendicular to the translational motion.

When the 3-phase stator coils of a PMLSM are excited by an external polyphase AC current source, a magnetic field is generated which can interact with the fields produced within the permanent magnets of the mover. The stator coils are wound such that the field orientations produce a travelling flux wave along the $x$-axis of the PMLSM. Depending upon the alignment of the permanent magnets of the mover relative to the travelling stator field, a translational force is generated that will thrust the mover in either a positive or negative $x$-axis direction. The motion along the $x$-axis can then be controlled through variation of the current amplitude and phase within each of the PMLSM stator coils using some form of feedback mechanism.

The normal force is generated by two different interactions within the air-gap flux:

- The attractive interaction between the permanent magnets of the mover and the iron core of the stator

---

1 Normal motion is restricted in most applications by fixing the air-gap between the stator and mover using bearings or guides. To achieve contactless positioning, the normal motion is not restricted, thereby permitting a variable air-gap.
• The attractive/repulsive interaction between the fields of the permanent magnets and the fields generated by the stator coils

When the $y$-axis air-gap is sufficiently large, the attraction between the permanent magnets of the mover and the iron core of the PMLSM must be supplemented by an attractive field within the stator coils in order to generate the required normal lift. When the air-gap is sufficiently small, the attraction between the permanent magnets and the iron core becomes large enough that it must by counteracted by a repulsive magnetic field within the stator coils of the PMLSM. At a certain air-gap value, the attraction between the permanent magnets and iron core is in perfect equilibrium with the force of gravity. The normal $y$-axis motion of the mover can therefore be regulated through the air-gap flux by making adjustments to the currents within the stator coils.

By choosing a PMLSM configuration with permanent magnets and an iron-cored stator, a larger normal force can be generated and controlled using smaller amounts of stator current than would normally be required in an air-cored PMLSM. As a result, a normal motion can be imparted to a heavier mover over a wider $y$-axis operating range.

In the section that follows, the derivation of the mathematical models that characterize the translational and normal forces will be summarized.

### 2.2 Derivation of the PMLSM Translational and Normal Forces

What follows is a partial summary of the work in [16] leading up to the derivation of the mathematical models describing the translational and normal forces generated by a single PMLSM of the type described in section 2.1. For a more complete description of the derivation process, the reader is referred to [16].

The force models in [16] were obtained by combining the results from [18], [19], and [28]. Specifically, [18, 19] contain force models produced by a PMLSM with a fixed air-
gap, based on the effects of the magnetic fields produced by both the mover and stator. Such models take into account, to some extent, the attenuation of the magnetic field in the air-gap caused by the stator slots. The work in [16] modifies the results in [18, 19] to account for a *variable* air-gap. The force models in [18, 19] do not take into account the spatial modulation effects produced in the magnetic fields by the stator slots. On the other hand, the results from [28] develop modelling to account for this effect in a DC rotary machine using the concept of *relative permeance*. The modelling of the slot effect from [28] was subsequently adapted in [16] for application to PMLSMs.

Consider the inertial frame of a single PMLSM shown in Figure 2.2. Referring to the figure, let $g$ be the air-gap length along the $y$-axis normal range, $d$ the translational displacement of the mover relative to the stator along the $x$-axis, $L_A$ the depth of each permanent magnet (PM) along the $z$-axis, $h_m$ the height of the PM’s, $p_m$ the number of PM’s, $t_i$ the slot pitch, $b_0$ the slot aperture, $\tau$ the PM pole pitch, $\tau_p$ the PM pole arc, $\mu_{rec}$ the relative PM recoil permeability, and $\sigma_m$ the surface magnetic charge. To account for the effects of the stator slots, replace the air-gap $g$ by the effective air-gap $g_e$, with

![Figure 2.2: Inertial frame of a single PMLSM](image-url)
Chapter 2. Review of PMLSMs and The Derivation of The Forces

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g_e = gK_c, where \( K_c \) denotes Carter’s coefficient and has the form

\[ K_c = \frac{t_1}{t_1 - g\gamma_1}, \]

\[ \gamma_1 = \frac{4}{\pi} \left[ \frac{b_0}{2g} \arctan \left( \frac{b_0}{2g} \right) - \ln \sqrt{1 + \left( \frac{b_0}{2g} \right)^2} \right]. \]

In addition, let \( I_a, I_b, \) and \( I_c \) be the phasors of the 3-phase currents and \( I_a, I_b, \) and \( I_c \) their magnitudes. Define the instantaneous currents to be \( i_a, i_b, \) and \( i_c, \) such that \( i_a(t) \leq I_a, i_b(t) \leq I_b, \) and \( i_c(t) \leq I_c. \) Define \( W \) to be the number of turns of wire on each phase, \( p \) the number of pole pairs in the stator, \( w_c \) the coil pitch, and \( k_{wn} \) the winding factor. Finally, let \( i_d \) and \( i_q \) define the direct and quadrature currents, which are related to the phase currents \( i_a, i_b, \) and \( i_c \) by the relationship

\[
\begin{bmatrix}
i_d \\
i_q
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
\cos(\frac{\tau d}{2}) & \cos(\frac{\tau d - \frac{2\pi}{3}}{2}) & \cos(\frac{\tau d + \frac{2\pi}{3}}{2}) \\
-\sin(\frac{\tau d}{2}) & -\sin(\frac{\tau d - \frac{2\pi}{3}}{2}) & -\sin(\frac{\tau d + \frac{2\pi}{3}}{2})
\end{bmatrix} \begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix}.
\] (2.1)

With all of the necessary definitions in place, the mathematical models of the PMLSM translational and normal forces that were derived in [16] can be presented.

2.2.1 Model of PMLSM Translational Force

The translational force of a single PMLSM is derived in [16] by combining the effects of the magnetic fields produced by the permanent magnets, the magnetic fields produced by the stator and the reluctance difference produced by the slots. The final result was the following mathematical model describing the translational force

\[
F_x(g, i_q) = -\frac{12\sqrt{2}Wk_{wn}p_mL_A\sigma_m\mu_0\tilde{\lambda}(g)}{\pi pK_c(g)} \sinh(\frac{\pi}{2\tau}(h_m + g)) \sin(\frac{\pi\tau}{2\tau})i_q,
\] (2.2)

where \( \tilde{\lambda}(g) \) is the relative permeance of the stator slots. This is modelled with an acceptable accuracy using the fundamental component of its Fourier approximation

\[
\tilde{\lambda}(g) = 1 - \frac{b_0^2}{4t_1(g + \frac{b_0}{2} + \frac{h_m}{\mu_{rec}})}.
\] (2.3)
Equation (2.2) indicates that the translational thrust of the PMLSM is a nonlinear function of the air-gap\(^2\). In particular, it shows that the normal motion affects the translational one.

### 2.2.2 Model of PMLSM Normal Force

The normal force is also derived in [16] through combination of the various magnetic fields with the effects of the stator slots. The result is the following mathematical model for the normal force

\[
F_y(g, i_q, i_d) = -K_2(g) + K_3(g)i_d - K_4(g)(i_d^2 + i_q^2),
\]

(2.4)

where

\[
K_2(g) = \frac{\tilde{\lambda}(g)L_{Ap}m\tau B_{pmy1}(g)^2}{4\mu_0},
\]

\[
K_3(g) = \frac{\tilde{\lambda}(g)3\sqrt{2}L_{Ap}mWk_{w1}B_{pmy1}(g)\coth(\frac{\pi}{\tau}(h_m + g))}{\pi^2K_c(g)},
\]

\[
K_4(g) = \frac{\tilde{\lambda}(g)18L_{Ap}mW^2k_{w1}^2\mu_0\coth^2(\frac{\pi}{\tau}(h_m + g))}{\tau\pi^2K_c(g)^2}.
\]

In the model of the normal force, \(B_{pmy1}(g)\) represents the magnetic field produced by the permanent magnets of the mover\(^3\), while \(\tilde{\lambda}(g)\) is defined in (2.3). Similarly to the translational force, the normal force depends nonlinearly on the air-gap position, but it does not depend on the translational position of the mover. It is also clear from (2.4) that the normal force is subject to a nonlinear coupling that occurs between the direct and quadrature current inputs of the PMLSM.

\(^2\)Note that if we had fixed the mover of a PMLSM at a constant air-gap, the translational force becomes a linear function of the quadratic current \(i_q\).

\(^3\)Throughout the experimental procedures, this function was approximated by a 12\(^{th}\) degree polynomial designed according to a procedure detailed in [16].
2.3 Practical Modelling and Implementation Issues

The expressions for the translational and normal forces that were given in (2.2) and (2.4) are utilized in Chapters 4 and 5 to construct the required state-space models and controllers for both of the 2-DOF and 3-DOF contactless positioning systems.

While ideal simulations of state-space models and controllers based on these forces demonstrates the feasibility of the theoretical contactless positioning system design in [16], it is not immediately clear how accurately the above models describe the physical behavior of a PMLSM. Two issues need to be addressed before implementing any controller on the experimental apparatus.

First, several parameters in the model are either not accurately known or they are completely unknown. This is the case for all parameters dealing with magnetic characteristics of the mover and stator. Additionally, the model appears to be sensitive to small measurement errors in certain dimensional parameters such as the height of the permanent magnets.

Second, the mathematical model does not characterize certain practical implementation effects: friction, end-effects, actuator saturation and cogging forces\(^4\). It must be determined whether such effects should be accounted for in the model.

The two issues described above motivate the need for a system identification procedure to identify unknown or not accurately known parameters in the model, as well as a model validation procedure to determine a range of operation within which the model accurately represents the forces generated by a PMLSM. To this end, Chapter 3 describes a system identification algorithm to estimate uncertain parameters. Chapters 4 and 5 contain, among other things, procedures that rely on the system identification algorithm to validate the model of the 2-DOF and 3-DOF system, respectively.

\(^4\)The cogging force will be described in more detail in Chapter 4.
Chapter 3

Overview of System Identification Procedure

As discussed in Section 2.3, several physical parameters of the PMLSM are not accurately known or are completely unknown. This chapter presents a system identification procedure for parameter estimation developed by Didinsky, Pan, and Başar in [9] (see also [1, 20]). This technique is applied in Chapters 4 and 5 to the 2-DOF and 3-DOF system, respectively.

3.1 Optimal parameter identification

The work in [9] considers the class of systems modelled by the differential equation

\[ \dot{x} = A(x, u)\theta + b(x, u) + \omega(t), \quad x(0) = x_0, \]  

where \( x \in \mathbb{R}^n \) and \( u \in \mathbb{R}^p \). The functions \( A(x, u) : \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}^{n \times r} \), and \( b(x, u) : \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}^n \) are assumed to be smooth. The vector \( \theta \in \mathbb{R}^r \) contains unknown parameters in the system, while \( \omega(t) \in \mathbb{R}^n \) is a an unknown time-varying disturbance. The goal is to find an estimate \( \hat{\theta}(t) \) such that \( \lim_{t \to \infty} \hat{\theta}(t) = \theta \) irrespective of \( \omega(t) \) (under a suitable persistence of excitation condition).
Let $\delta(t, \eta_t)$ denote the estimator function at time $t$, where $\eta_t = \{(x(s), u(s)), \forall s \leq t\}$ is the vector containing the available information at time $t$. Let $\Delta$ denote the class of admissible estimator functions. Define the performance functional

$$L(\delta) = \sup_{x_0, \theta_0, \omega_{[0, \infty)}} \left\{ \frac{\|\theta - \hat{\theta}\|_{Q(t,x,u)}^2}{\|\omega\|^2 + |\theta - \theta_0|_{Q_0}^2} : \hat{\theta}(t) = \delta(t, \eta_t) \right\},$$

(3.2)

where $\omega_{[0, \infty)} = \{\omega(t) : t \in [0, \infty)\}$, $\| \cdot \|_Q$ is the weighted $L_2$ norm, and $| \cdot |_{Q}$ is the weighted matrix two-norm. The matrix function $Q(\cdot)$ is positive semidefinite and the matrix $Q_0$ is positive definite. Finally, the vector $\bar{\theta}_0$ represents an initial guess for $\theta$.

The identification goal can be rephrased as finding an estimator function $\delta^* \in \Delta$ minimizing the cost functional (3.2). Let $\gamma^{*2} := \inf_{\delta \in \Delta} L(\delta)$.

As pointed out in [9, 1] this variational problem can be interpreted as a differential game in which $\delta$ is viewed as a minimizer and the initial conditions and noise $(x_0, \theta_0, \omega_{[0, \infty)})$ are viewed as the maximizers. In this case, the square of the attenuation level $\gamma^*$ is the upper value of the game. A cost function associated with this differential game is

$$J_\gamma(\delta; x_0, \theta_0, \omega_{[0, \infty)}) = \int_0^\infty \left[ |\theta - \delta(t, \eta_t)|_{Q(t,x,u)}^2 - \gamma^2 |\omega(t)|^2 \right] dt - \gamma^2 |\theta - \bar{\theta}_0|_{Q_0}^2.$$

An estimator $\delta(t, \eta_t) \in \Delta$ is said to be cost bounding for a fixed $\gamma > 0$ if

$$\sup_{x_0, \theta_0, \omega_{[0, \infty)}} J_\gamma(\delta; x_0, \theta_0, \omega_{[0, \infty)}) < \infty.$$

An estimator $\bar{\delta}(t, \eta_t) \in \Delta$ is said to be minimax for a given $\gamma$ if

$$\inf_{\delta \in \Delta} \sup_{x_0, \theta_0, \omega_{[0, \infty)}} J_\gamma(\delta; x_0, \theta_0, \omega_{[0, \infty)}) = \sup_{x_0, \theta_0, \omega_{[0, \infty)}} J_\gamma(\bar{\delta}; x_0, \theta_0, \omega_{[0, \infty)}).$$

It can be shown [1] that $\gamma^*$ is the smallest value of $\gamma$ for which the upper value of the game $J_\gamma$ is zero. We are now ready to define the optimal parameter identification problem.

**Problem 1:** Given an information set $\eta_t$, find the least value of $\gamma > 0$, $\gamma^*$, such that $J_\gamma$ admits a cost-bounding minimizer. For each $\gamma > \gamma^*$, find a minimax estimator policy $\delta_\gamma$ such that, letting $\hat{\theta}_\gamma(t) := \delta_\gamma(t, \eta_t)$, $\hat{\theta}_\gamma(t) \to \theta$ as $t \to \infty$. 
The work in [9] provides a complete solution to Problem 1. In the next two sections we present the results in [9] which are most relevant to this thesis.

### 3.1.1 Full State Derivative Information (FSDI) Case

We begin by discussing the solution to Problem 1 for the case where

\[ \eta_t = \{(x(s), \dot{x}(s), u(s), \forall s \leq t)\}, \]

that is, when the time derivative \( \dot{x}(t) \) is available for feedback along with the state and control inputs (the more reasonable situation when \( \dot{x}(t) \) is not available for feedback is discussed in the next section). In this case the estimator \( \delta(\gamma(t), \eta_t) \) solving Problem 1 is given in differential form within [9] by

\[
\dot{\hat{\theta}}_\gamma = \Sigma_\gamma^{-1} A(x, u)^T [\dot{x} - A(x, u)\hat{\theta}_\gamma - b(x, u)] , \quad \hat{\theta}_\gamma(0) = \bar{\theta}_0 ,
\]

(3.3)

where

\[
\dot{\Sigma}_\gamma = A(x, u)^T A(x, u) - \gamma^{-2} Q(x, u) , \quad \Sigma(0) = Q_0 .
\]

The above estimator is referred to as a full state derivative information parameter estimator (FSDI). The parameter \( \gamma \) can be interpreted as an attenuation factor tuned to improve estimator convergence. Note that a corollary in [9] suggests the following form for \( Q(x, u) \) when a proper choice of \( \gamma \) is made

\[
Q(x, u) = A(x, u)^T A(x, u).
\]

(3.4)

The following result gives conditions for the state of the estimator (3.3) to asymptotically converge to the true system parameters \( \theta \).

**Theorem 3.1.1.** Consider the dynamic system in (3.1) along with the identification scheme proposed in (3.3). Let all disturbances be in \( L_2 \) and assume that the following persistence of excitation condition is satisfied

\[
\lim_{T \to \infty} \lambda_{\text{min}} \left( \int_0^T Q(x, u) dt \right) = \infty .
\]

(3.5)

Then, if \( \gamma^* \) is finite, for all \( \gamma > \gamma^* \), \( \hat{\theta}_\gamma(t) \to \theta \) as \( t \to \infty \).
3.1.2 Noise Perturbed Full State Information (NPFSI) Case

In most practical situations, it is difficult to obtain accurate measurements of the state derivative. In addition, the measurement of the state vector may be perturbed by sensor noise. As a result, the authors in [9] derive a parameter estimator that is based solely on a noisy measurement of the state vector. Such an estimator is referred to as a noise perturbed full state information (NPFSI) parameter estimator.

The class of systems considered here is described by

\[
\dot{x} = A(y, u)\theta + b(y, u) + \omega, \quad x(0) = x_0
\]

\[
y = x + \varepsilon \nu,
\]

where \( y \) is a noisy measurement of the state, \( \nu \) is the measurement disturbance, and \( \varepsilon \) is the confidence level in the state measurement. Here the information set is \( \eta_t = \{(y(s), u(s)), \forall s \leq t\} \). The NPFSI estimator solving Problem 1 (under suitable conditions) is given by

\[
\begin{bmatrix}
\dot{x} \\
\dot{\theta}_\gamma
\end{bmatrix} =
\begin{bmatrix}
0 & A \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{x} \\
\dot{\theta}_\gamma
\end{bmatrix} +
\begin{bmatrix}
b \\
0
\end{bmatrix} + \frac{1}{\varepsilon^2} \Sigma^{-1}
\begin{bmatrix}
I \\
0
\end{bmatrix}
(y - \hat{x}),
\]

(3.6)

where

\[
\dot{\Sigma} = -\Sigma
\begin{bmatrix}
0 & A \\
0 & 0
\end{bmatrix} - \begin{bmatrix}
0 & 0 \\
A^T & 0
\end{bmatrix} \Sigma + \begin{bmatrix}
\varepsilon^{-2}I & 0 \\
0 & -\gamma^{-2}Q(y, u)
\end{bmatrix} - \Sigma
\begin{bmatrix}
I & 0 \\
0 & 0
\end{bmatrix} \Sigma,
\]

(3.7)

\[
\Sigma(0) =
\begin{bmatrix}
P_0 & 0 \\
0 & Q_0
\end{bmatrix},
\begin{bmatrix}
\dot{x}(0) \\
\dot{\theta}_\gamma(0)
\end{bmatrix} =
\begin{bmatrix}
\bar{x}_0 \\
\bar{\theta}_0
\end{bmatrix}.
\]

It is noted in [9] that, in addition to assuming persistency of excitation and \( L_2 \) disturbances, the existence of a solution to Problem 1 requires that certain restrictions be placed on the form of the output measurement noise. The paper provides a detailed discussion concerning the properties that the output noise should satisfy to guarantee convergence of the estimator state to the true system parameters. While the results are
too extensive to present in this overview, it is noted that by assuming boundedness of the measurement disturbances it is always guaranteed that the output noise belongs to the class of acceptable functions. The convergence theorem for the NPFSI estimator can therefore be summarized as follows.

Theorem 3.1.2. Under the assumptions of Theorem 3.1.1, and provided $\nu(t)$ is bounded, for all $\gamma > \gamma^*$, $\hat{\theta}_\gamma(t) \to \theta$ as $t \to \infty$.

3.1.3 Reduced Order NPFSI Parameter Estimator

The NPFSI parameter estimator in (3.6)-(3.7) has high order. It is desirable, from an implementation viewpoint, to reduce its complexity. To this end, the authors in [9] developed the following approximation to (3.6)-(3.7) which is valid for small values of $\varepsilon$.

$$\dot{\hat{\theta}}_\gamma = \varepsilon^{-1} \Sigma_\gamma^{F^{-1}} A(y, u)^T (y - \hat{x}), \quad \hat{\theta}_\gamma(0) = \bar{\theta}_0 ,$$

(3.8)

where

$$\dot{\Sigma}_\gamma^{F} = A(y, u)^T A(y, u) - \gamma^{-2} Q(y, u), \quad \Sigma_\gamma^{F}(0) = Q_0 ,$$

$$\dot{x} = A(y, u) \hat{\theta}_\gamma + b(y, u) + \varepsilon^{-1} (y - \hat{x}), \quad \hat{x}(0) = \bar{x}_0 .$$

Here $Q(y, u)$ has the same form as in (3.4).

3.2 System ID Implementation Issues

Throughout the remainder of this thesis we employ the estimator in (3.8) to perform the required system identification because of its simplified structure and the fact that it does not require measurement of the time derivative of the system state. It is shown in the following chapters that the sensors employed in each apparatus are highly accurate with a minimal amount of noise disturbance. The level of confidence in the output measurement
is therefore high enough to allow for a small $\varepsilon$, which allows for the utilization of the reduced order NPFSI estimator.

There are practical issues that need to be addressed in implementing the system identification procedure. For example, some basic control is needed in order to actuate each positioning system and guarantee the necessary persistency of excitation. Even with this basic control system in place, the restricted range of travel in both 2-DOF and 3 DOF positioning systems limits the inputs that can be imparted to the systems, thus impacting the amount of excitation that can be generated. Another issue is that the sensors in each contactless positioning system measure the position states but not the velocity states. This means that, technically, the full state of the systems is not available for measurement.

The chapters that follow demonstrate how the reduced order NPFSI estimator is applied to the estimation of the model parameters, and how the aforementioned implementation issues can be resolved.
Chapter 4

The 2-DOF Contactless Positioning System

This chapter details the implementation and control of the 2-DOF positioning system, a fundamental first step towards the more advanced 3-DOF implementation. It is crucial to verify the basic theoretical predictions in [16] concerning the accuracy of the mathematical model of a PMLSM and the effectiveness of a nonlinear controller in achieving a large 2-DOF range of operation. It turns out that modifications are needed in the nonlinear controller developed in [16]. These modifications are described later.

We begin this chapter by discussing the hardware setup. We then derive the state-space model using the expression for the forces generated by a PMLSM derived in Sections 2.2.1 and 2.2.2. The parameter identification algorithm described in Section 3.1.3 is then used to estimate various unknown parameters in the system model and forms the basis for a model verification procedure. Once the accuracy of the model is verified, we design a nonlinear controller based on feedback linearization and output regulation to achieve set-point stabilization and sinusoidal tracking. Detailed experimental results are provided which compare the performance of our nonlinear controller to PID compensators and linearization-based controllers.
4.1 Hardware Setup

A photo of the complete hardware implementation is shown in Figure 4.1, while the hardware specifications are summarized in Table 4.1. This experiment was constructed by Quanser Consulting based on specifications in [2]. It employs a single iron-cored PMLSM. The physical characteristics of the single motor allow for a horizontal range of motion of approximately ±50 mm and a vertical range of motion of approximately ±10 mm. The individual components of the system are detailed as follows.

![Figure 4.1: 2-DOF contactless positioning hardware implementation](image)

4.1.1 PMLSM Mover and Stator

The *stator* of the PMLSM, which is fixed in place to a heavy aluminium frame, is longitudinally laminated and transversally slotted in order to accommodate a single layer of 3-phase winding. The *mover*, which is attached to a movable platform, is composed of a set of four type N35 permanent magnets (PM) attached to a ferromagnetic backing.
Table 4.1: Specifications for 2-DOF positioning hardware

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>units</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator slot width</td>
<td>$b_0$</td>
<td>mm</td>
<td>12.7</td>
</tr>
<tr>
<td>Stator slot pitch</td>
<td>$t_1$</td>
<td>mm</td>
<td>19.05</td>
</tr>
<tr>
<td>Turns per phase</td>
<td>$W$</td>
<td>–</td>
<td>900</td>
</tr>
<tr>
<td>Coil pitch</td>
<td>$\omega_c$</td>
<td>mm</td>
<td>57.15</td>
</tr>
<tr>
<td>Stator pole pairs</td>
<td>$p$</td>
<td>–</td>
<td>3</td>
</tr>
<tr>
<td>Number of stator slots</td>
<td>$z_1$</td>
<td>–</td>
<td>18</td>
</tr>
<tr>
<td>PM height</td>
<td>$h_m$</td>
<td>mm</td>
<td>5</td>
</tr>
<tr>
<td>PM length</td>
<td>$L_A$</td>
<td>mm</td>
<td>50</td>
</tr>
<tr>
<td>Number of PM’s</td>
<td>$p_m$</td>
<td>–</td>
<td>4</td>
</tr>
<tr>
<td>Pole pitch</td>
<td>$\tau$</td>
<td>mm</td>
<td>57.15</td>
</tr>
<tr>
<td>PM width</td>
<td>$\tau_p$</td>
<td>mm</td>
<td>28.58</td>
</tr>
<tr>
<td>PM coercivity</td>
<td>$H_c$</td>
<td>A/m</td>
<td>875400</td>
</tr>
<tr>
<td>Back iron height</td>
<td>$h_b$</td>
<td>mm</td>
<td>4.7</td>
</tr>
<tr>
<td>Back iron width</td>
<td>–</td>
<td>mm</td>
<td>50</td>
</tr>
<tr>
<td>Back iron length</td>
<td>–</td>
<td>mm</td>
<td>200.0</td>
</tr>
<tr>
<td>Horizontal Mover Mass</td>
<td>$M_h$</td>
<td>Kg</td>
<td>1.594</td>
</tr>
<tr>
<td>Vertical Mover Mass</td>
<td>$M_v$</td>
<td>Kg</td>
<td>4.350</td>
</tr>
</tbody>
</table>
4.1.2 Linear Guides, Stoppers and Sensors

The PMLSM mover is attached to a platform using two orthogonally mounted linear guides that allow for horizontal motion. The platform is in turn connected to a set of 4 vertically mounted linear guides which permit the vertical motion.

As a result of the incorporation of linear guides, the 2-DOF prototype is clearly not a fully contactless positioning system. However, linear guides are necessary in order to prevent the platform from pitching and rolling, which cannot be compensated for using a single PMLSM. Despite this incorporation, the power transfer itself is still contactless. In future generations of the apparatus the dependence on linear guides will be greatly reduced and eventually eliminated. An advantage of using linear guides is that they facilitate incorporation of sensors within the apparatus. The 2-DOF hardware employs two linear optical encoders to measure the horizontal and vertical position of the platform. Each encoder had a resolution of 10 µm and, being digital, are noise-free.

A set of vertical and horizontal stoppers allow for the adjustment of the operating range. The stoppers also allow one to reduce the degrees-of-freedom by constraining either the horizontal or the vertical motion. This is useful for parameter identification, as shown in the sections that follow.

Figures 4.2 and 4.3 illustrate the setup of the linear guides, stoppers and position sensors incorporated into the apparatus.

4.1.3 Power Delivery

The 3-phase AC current required to actuate the stator coils is provided by a set of three linear current amplifier modules (LCAM) supplied by Quanser. Current commands are sent to the LCAM’s through a PC interface. Each LCAM is capable of supplying 7A continuous and 9A peak. To account for the inductance of the stator coils, each LCAM was factory tuned for optimum power delivery.
Figure 4.2: Close-up of the 2-DOF horizontal linear guides

Figure 4.3: Close-up of the 2-DOF vertical linear guides
4.1.4 Computer Interface and Real-Time Control Environment

The magnetic levitation apparatus is connected to a standard PC running Windows XP using a Quanser Multi-Q PCI I/O Board. The Quanser board provides all of the necessary analog and digital I/O, and is operated at a sampling frequency of 2 KHz.

Control of the contactless positioning hardware is implemented through the Multi-Q interface using the Quanser WINCON real-time control environment. The software is fully integrated into MATLAB and allows for the construction of controllers through Simulink diagrams.

A block diagram detailing the complete interconnection between the contactless positioning system and the PC is provided in figure 4.4.

![Block diagram of interconnection between the system hardware and PC.](image-url)
4.2 Derivation of System State-Space Model

To develop a mathematical model for the 2-DOF system we simply write Newton’s equations using the expressions for the translational and normal forces presented in chapter 2, based on the orientations from figure 4.5.

\[ \begin{align*}
M_v \ddot{g} &= M_v G + F_y(g, i_q, i_d), \\
M_h \ddot{d} &= F_x(g, i_q),
\end{align*} \]

(4.1)

where \( G \) is the gravitational constant, \( F_x(g, i_q) \) the translational force from (2.2), \( F_y(g, i_q, i_d) \) the normal force from (2.4), and \( M_h, M_v \) are the horizontal and vertical masses of the movable platform, respectively. Using (2.2) and (2.4) we obtain

\[ \begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= G - L_4(x_1)[u_1^2 + u_2^2] + L_3(x_1)u_2 - L_2(x_1), \\
\dot{x}_3 &= x_4, \\
\dot{x}_4 &= -L_1(x_1)u_1,
\end{align*} \]

(4.2)

Figure 4.5: Forces in the 2-DOF contactless positioning system
where

\[ x = [g, \dot{g}, d, \dot{d}]^T, \quad u = [i_q, i_d]^T, \]

\[ L_1(x_1) = \frac{K_1(x_1)}{M_h}, \]

\[ L_i(x_1) = \frac{K_i(x_1)}{M_v}, \quad i = 2, \ldots, 4, \]

\[ K_1(x_1) = \frac{12\sqrt{2}W k_{w1} p_m L_A \sigma_m \mu_0 \tilde{\lambda}(x_1) \sinh\left(\frac{\pi}{\tau} h_m\right) \sin\left(\frac{\pi \tau_p}{2}\right)}{\pi p K_c(x_1) \sinh\left(\frac{\pi}{\tau}(h_m + x_1)\right)}, \]

\[ K_2(x_1) = \frac{\tilde{\lambda}(x_1) L_A p_m \tau B_{pmy1}(x_1)^2}{4\mu_0}, \]

\[ K_3(x_1) = \frac{\tilde{\lambda}(x_1) 3\sqrt{2} L_A p_m W k_{w1} B_{pmy1}(x_1) \coth\left(\frac{\pi}{\tau}(h_m + x_1)\right)}{p^2 K_c(x_1)}, \]

\[ K_4(x_1) = \frac{\tilde{\lambda}(x_1) 18 L_A p_m W^2 k_{w1}^2 \mu_0 \coth^2\left(\frac{\pi}{\tau}(h_m + x_1)\right)}{\tau p^2 K_c(x_1)^2}, \]

\[ \tilde{\lambda}(x_1) = 1 - \frac{b_0^2}{4t_1(x_1 + \frac{b_0}{2} + \frac{h_m}{\mu_{rec}})}. \]

As mentioned in chapter 2, the function \( B_{pmy1}(x_1) \) represents the magnetic field produced by the permanent magnets and is approximated using a 12\textsuperscript{th} degree polynomial, while \( i_d \) and \( i_q \) represent the direct and quadrature PMLSM current inputs from (2.1).

The model (4.2) does not take into account friction, cogging force, end effects, and any other physical uncertainty that may affect the system. In what follows we verify to what extent such effects can be neglected within a reasonable range of operation.

In preparation for the application of the system identification technique to estimate the unknown model parameters, we rewrite (4.2) lumping together all unknown (or not
perfectly known) parameters into four constants \( C_1, C_2, C_3 \) and \( C_4 \) as follows

\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= G - C_4 \frac{\tilde{\lambda}(x_1) \coth^2(\frac{\pi}{2}(h_m + x_1))}{K_2^2(x_1)} [u_1^2 + u_2^2] \\
&\quad + C_3 \frac{\tilde{\lambda}(x_1) B_{pmy1}(x_1) \coth(\frac{\pi}{2}(h_m + x_1))}{K_c(x_1)} u_2, \\
&\quad - C_2 \tilde{\lambda}(x_1) B_{pmy1}(x_1)^2 \\
\dot{x}_3 &= x_4, \\
\dot{x}_4 &= -C_1 \frac{\tilde{\lambda}(x_1)}{K_c(x_1) \sinh(\frac{\pi}{2}(h_m + x_1))} u_1,
\end{align*}
\]

(4.3)

where

\[
\begin{align*}
C_1 &= \frac{12\sqrt{2}Wk_{w1}p_m L_A \sigma_m \mu_0 \sinh(\frac{\pi}{2}h_m) \sin(\frac{\pi p}{2})}{M_h \pi p}, \\
C_2 &= \frac{L_A p_m \tau}{M_v \mu_0}, \\
C_3 &= \frac{3\sqrt{2}L_A p_m Wk_{w1}}{M_c p^2}, \\
C_4 &= \frac{18L_A p_m W^2 k_{w1}^3 \mu_0}{M_v \pi p^2}.
\end{align*}
\]

4.3 Implementation of System Identification Technique

The model (4.3) fits the form required by the system identification algorithm summarized in chapter 3 because the unknown constants \( C_1, \ldots, C_4 \) enter the differential equation linearly. The next few sections discuss various implementation issues that must be resolved before implementing the algorithm.

4.3.1 Persistency of Excitation

In order to guarantee the necessary amount of persistency of excitation required by the reduced-order NPFSI technique, two PID regulators are employed in the control of the current inputs \( i_d \) and \( i_q \), as illustrated in figure 4.6.

The chosen controller structure has poor control performance. This is due to the high nonlinearity of the system and the coupling between the control inputs \( i_d \) and \( i_q \). However, since the goal here is merely the generation of sufficient persistency of excitation
for system identification, high accuracy is not required. As a result, a manual tuning of each PID is sufficient for the task at hand.

The \( i_d \) and \( i_q \) control currents are converted to 3-phase current using the transformation (2.1). It was discovered through the course of experimentation that, because of sensor reset\(^1\) and the displacement of the permanent magnets relative to the stator coils, an offset of \( \tau / 2 = 28.575 \text{ mm} \) had to be incorporated into (2.1) to accommodate the desired horizontal operating range. The offset is illustrated below and discussed in detail in appendix C

\[
\begin{align*}
    i_d &= \frac{2}{3} \cos\left(\frac{\pi}{\tau}(x_3 - \frac{\tau}{2})\right)i_a + \frac{2}{3} \cos\left(\frac{\pi}{\tau}(x_3 - \frac{\tau}{2}) - \frac{2\pi}{3}\right)i_b + \frac{2}{3} \cos\left(\frac{\pi}{\tau}(x_3 - \frac{\tau}{2}) + \frac{2\pi}{3}\right)i_c, \\
    i_q &= -\frac{2}{3} \sin\left(\frac{\pi}{\tau}(x_3 - \frac{\tau}{2})\right)i_a - \frac{2}{3} \sin\left(\frac{\pi}{\tau}(x_3 - \frac{\tau}{2}) - \frac{2\pi}{3}\right)i_b - \frac{2}{3} \sin\left(\frac{\pi}{\tau}(x_3 - \frac{\tau}{2}) + \frac{2\pi}{3}\right)i_c. 
\end{align*}
\]

Other additions to the PID controller setup include a safety mechanism to shut-down the controller when instability is detected, and a pre-filtering of the reference commands to account for the performance limits of the PID regulators.

The final PID controller implementation is used to generate the required data for system identification. Rather then depending on condition (3.5), it was found through successive experimentation that the following set of sinusoidal summations generate a

\(^{1}\)The linear optical encoders’ count is reset every time the controller is run.
sufficient amount of persistency of excitation

\[
H_{ref}(t) = 0.06 \left( \frac{1}{4} \sin(\pi t + \frac{\pi}{2}) + \frac{1}{2} \sin(2\pi t + \frac{4\pi}{3}) + \frac{1}{2} \sin(3\pi t + \frac{\pi}{4}) + \frac{1}{4} \sin(4\pi t + \frac{\pi}{6}) \right),
\]

\[
V_{ref}(t) = 0.02 - 0.005 \left( \frac{1}{4} \sin(\pi t - 0.8\pi) + \frac{1}{4} \sin(2\pi t - 1.6\pi) + \frac{1}{4} \sin(3\pi t - 1.8\pi) + \frac{1}{4} \sin(4\pi t - 3.2\pi) \right).
\]

Using the above reference commands, the system is actuated over the approximate horizontal range of \([-50\text{mm}, 50\text{mm}]\) and the approximate vertical range of \([15\text{mm}, 25\text{mm}]\).

### 4.3.2 Velocity Estimation

Although the NPFSI technique does not require state derivative information, it does depend on measurement of the full state vector. While the position states \(x_1\) and \(x_3\) are measured using optical encoders, the velocity states \(x_2\) and \(x_4\) are not available. Instead of incorporating partial observation into the system ID procedure, we use high-gain observers to derive the velocity states \(x_2\) and \(x_4\) from the position measurements \(x_1\) and \(x_3\). The Simulink function blocks implementing the high-gain observer used in each case is shown in figure 4.7.

![High-gain observer used to derive each velocity state from each position state](image)

The first block is an approximate derivative function and the second block is a low-pass filter that removes distortion. The parameters in the two blocks are tuned to guarantee good derivative estimation performance for some representative sample signals.
4.3.3 Remaining Implementation Issues

The final implementation issue to be addressed before the system identification procedure can be applied is the choice of the NPFSI parameters $\gamma$ and $\varepsilon$. Recall that $\gamma$ is an attenuation factor that can be tuned by the user to improve estimator convergence, while $\varepsilon$ reflects the confidence level in the state measurement. It was found through successive experimentation and help from [9] that choosing $\gamma \simeq 1.1$ and $\varepsilon \simeq 0.01$ produce good parameter convergence.

4.4 Model Verification of 2-DOF System

The system identification procedure is applied to a set of 3 separate parameter estimation experiments:

- The vertical position of the platform is held in place at a series of fixed air-gaps, while the system identification procedure is used to estimate $C_1$ and verify the model of the horizontal dynamics.

- In the second experiment, the horizontal position of the platform is held in place at a series of fixed displacements, while the system identification procedure is used to estimate $C_2$, $C_3$, and $C_4$ from the vertical dynamics for each position.

- In the third and final experiment, the parameters $C_1, \ldots C_4$ are simultaneously estimated, while the system is actuated over the complete horizontal and vertical operating range.

The results from each parameter estimation experiment are used to verify the accuracy of our mathematical model.
4.4.1 Verification of Horizontal Dynamics

The air-gap of the 2-DOF positioning system is fixed to a constant value $\bar{x}_1$. Thus, the horizontal dynamics from (4.2) become

$$
\dot{x}_3 = x_4, \\
\dot{x}_4 = -L_1(\bar{x}_1)u_1.
$$

This is an LTI system that can be solved to get $x_3(t)$ as

$$
x_3(t) = -\frac{1}{2}L_1(\bar{x}_1)u_1 t^2 + x_3(0).
$$

Note that we assume zero initial horizontal velocity and constant $u_1$. Equation (4.6) indicates that if a constant $u_1$ is applied when the air-gap is fixed, the horizontal displacement of the mover exhibits a parabolic response. By recording the horizontal position response, it is possible to curve-fit a parabola to the data points and obtain an estimate of $L_1(\bar{x}_1)$ at the air-gap in question. In turn, once $L_1(\bar{x}_1)$ is known, one can easily infer an estimate of $C_1$,

$$
C_1 = \frac{K_c(\bar{x}_1) \sinh(\frac{\bar{z}}{\tau}(h_m + x_1))}{\tilde{\lambda}(x_1)} L_1(\bar{x}_1).
$$

The estimate of $L_1(\bar{x}_1)$ is compared to the value of $L_1(\bar{x}_1)$ obtained through the application of the NPFSI estimator from (3.8) to (4.5), viewing $L_1(\bar{x}_1)$ as an unknown parameter that enters the horizontal dynamics linearly. This is summarized in the procedure that follows.
Procedure 1: Model verification procedure for horizontal dynamics

1. The motion of the system is constrained (by hardware) to lie on the horizontal axis at a fixed air-gap

2. Position data corresponding to different air-gap values \{\bar{x}_1^1, \ldots, \bar{x}_1^k\} are collected

3. A set of parabolas are fitted to horizontal position data at various air-gaps \{\bar{x}_1^1, \ldots, \bar{x}_1^k\} in order to obtain an estimate of \(L_1(\bar{x}_i^1), i = 1, \ldots, k\) by means of (4.6)

4. For each air-gap \(\bar{x}_1^i\), the NPFSI estimator (3.8) is applied to (4.5) to estimate \(L_1(\bar{x}_1^i)\)

5. The results produced by the two methods are subsequently compared.

Figure 4.8 shows a few examples of the data points obtained and the parabolas that were fitted. Note how the curves closely approximate the position data, demonstrating the correctness of the horizontal dynamics model in (4.5). Air-gap values range between 10 and 27.5 mm at 2.5 mm intervals. In each case, the mover is initialized near \(x_3 = -50\) mm and accelerated to about \(x_3 = 50\) mm using a current of \(u_1 = -0.5\) A.

A comparison of the \(L_1(\bar{x}_1^i)\) estimates obtained using parabolic curve-fitting and the NPFSI estimator with different values of \(\varepsilon\) is found in Figure 4.9. The plot demonstrates that two different estimation techniques produce very similar results, confirming the effectiveness of the NPFSI estimation technique and indicating that the horizontal portion of the model accurately describes the physical behavior of the system. Note that the NPFSI estimates obtained for \(\varepsilon = 0.05\) exhibit increased divergence. This is not surprising, since as the value of \(\varepsilon\) is increased the estimator puts less emphasis on the estimation error and, as a result, the convergence performance of the estimator worsens.
Figure 4.8: Curve-fitting of parabolas to actual horizontal position data

Figure 4.9: Comparison of various estimations of $L_1(x_1)$
4.4.2 Verification of Vertical Dynamics

In order to validate the vertical dynamics, the mover is fixed at \( x_3 = 0 \) mm, which constrains the motion to lie on the vertical axis. At the same time, \( u_1 = 0 \) is imposed. As a result, from (4.2) we obtain the vertical dynamics

\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= G - L_4(x_1)u_2^2 + L_3(x_1)u_2 - L_2(x_1),
\end{align*}
\]

(4.7)

where as in (4.3), \( L_2, \ldots L_4 \) are defined in terms of the unknown parameters \( C_2, \ldots C_4 \) as follows

\[
\begin{align*}
L_2(x_1) &= C_2 \frac{\tilde{\lambda}(x_1)B_{pmy1}(x_1)^2}{K_c(x_1)}, \\
L_3(x_1) &= C_3 \frac{\tilde{\lambda}(x_1)B_{pmy1}(x_1) \coth\left(\frac{\tau}{2} h_m + x_1\right)}{K_c(x_1)}, \\
L_4(x_1) &= C_4 \frac{\tilde{\lambda}(x_1) \coth^2\left(\frac{\tau}{2} (h_m + x_1)\right)}{K_c(x_1)}.
\end{align*}
\]

(4.8)

From (4.7), the current \( \bar{u}_2 \) needed to maintain the air-gap at a desired equilibrium \( \bar{x}_1 \) is found by solving the equation

\[
G - L_4(\bar{x}_1)u_2^2 + L_3(\bar{x}_1)u_2 - L_2(\bar{x}_1) = 0.
\]

(4.9)

If the model (4.7) associated with the vertical dynamics is correct, the equilibrium current \( \bar{u}_2 \) predicted by (4.9),

\[
\bar{u}_2 = \frac{L_3(\bar{x}_1) - \sqrt{L_3^2(\bar{x}_1) - 4L_4(\bar{x}_1)(L_2(\bar{x}_1) - G)}}{2L_4(\bar{x}_1)},
\]

(4.10)

should be close to the equilibrium current measured experimentally. The validation procedure for the vertical dynamics is now clear.
Procedure 2: Model verification procedure for vertical dynamics

1. The motion of the system is constrained (by hardware) to lie on the vertical axis with the mover positioned at $x_3 = 0$ mm

2. $u_1 = 0$ is set

3. For each of the air-gaps in the set $\{\bar{x}_1^1, \ldots, \bar{x}_1^k\}$, the corresponding equilibrium currents $\{\bar{u}_2^1, \ldots, \bar{u}_2^k\}$ are measured

4. The NPFSI estimator is applied to (4.7) to estimate the unknown constants $C_2$, $C_3$, and $C_4$

5. The estimates of $C_2$, $C_3$, and $C_4$ are used in (4.8) to derive $L_2$, $L_3$, and $L_4$

6. The values $L_2(\bar{x}_1^i)$, $L_3(\bar{x}_1^i)$, and $L_4(\bar{x}_1^i)$ are used to predict the theoretical equilibrium current at each air-gap $x_1^i$, by means of (4.10)

7. The theoretical equilibrium current predictions are compared to the measured currents across the entire air-gap range

To apply procedure 2, air-gap values are chosen to range between 10 and 25 mm with 1 mm increments. The reduced-order NPFSI system identification procedure, meanwhile, is applied to the positioning hardware for three different values of $\varepsilon$. The estimation results are summarized in table 4.2. In Figure 4.10 the theoretical equilibrium currents

| Table 4.2: Vertical model parameters using NPFSI estimator when $x_3 = 0$ mm |
|-----------------|--------|--------|--------|
| Parameters | $\varepsilon = 0.001$ | $\varepsilon = 0.01$ | $\varepsilon = 0.05$ |
| $C_2$      | 790.97 | 791.00 | 788.98 |
| $C_3$      | 30.47  | 30.90  | 32.44  |
| $C_4$      | 0.11   | 0.12   | 0.18   |
found using procedure 2 (based on the parameters $C_2, \ldots C_4$ estimated with $\varepsilon = 0.01$) are compared to the measured equilibrium currents.

![Graph comparing measured $i_d$ currents and theoretical predictions](image)

Figure 4.10: Comparing measured $i_d$ currents and the theoretical predictions

From the plot it is clear that while the theoretical equilibrium currents begin to diverge from the actual measurements for air-gaps smaller than 15 mm, within the range 15 mm to 25 mm the model accurately described the behavior of the 2-DOF positioning system. The results therefore validate the vertical dynamics of the model within the range of 15 and 25 mm and provide further proof of the effectiveness of the NPFSI technique.

The divergence for smaller air-gaps is most likely the result of physical uncertainties that are not taken into account within the model. In the following section, evidence is provided that suggests the cogging force accounts for the bulk of the uncertainty.

### 4.4.3 Analysis of the cogging force

The goal of this section is to determine the source of the discrepancy, observed in Figure 4.10, between theoretical and measured equilibrium currents at air-gaps smaller than 15 mm. We claim that the *cogging force* is primarily responsible for such discrepancy.
The cogging force of a linear synchronous motor is defined in [7] to be the force produced by the interaction between the teeth of the stator and the edges of the permanent magnets of the mover. It is described by a periodic function of the horizontal position of the mover over the slot pitch of the stator. Equation (4.11) provides a good mathematical representation of this force

\[ F^c_x = \xi(x_1) \sin \left( \frac{\pi}{t_1} x_3 \right). \] (4.11)

The function \( \xi(x_1) \), representing the peak magnitude of the cogging force, is inversely proportional to the air-gap \( x_1 \), meaning that the cogging force becomes stronger as the mover approaches the stator. Notice that the peaks of the cogging force occur at odd integer multiples of \( t_1/2 = 9.525 \) mm. Hence, if the mover is held at a constant position \( x_3 = 28.575 \) mm = \( 3t_1/2 \), then the total normal force exerted by the PMLSM on the surface of the mover is given by

\[ F_n(x_1) \bigg|_{x_3=3t_1/2} = K_2(x_1) + K_3(x_1)u_2 + K_4(x_1)u_2^2 + \xi(x_1), \]

where, as before, we set \( u_1 = 0 \). On the other hand, when \( x_3 = 0 \) mm, the cogging force vanishes and the normal force can be accurately represented by the nominal model

\[ F_n(x_1) \bigg|_{x_3=0} = K_2(x_1) + K_3(x_1)u_2 + K_4(x_1)u_2^2. \]

The unknown constants \( C_2, C_3, \) and \( C_4 \) in the functions \( K_2, K_3, \) and \( K_4 \) have already been estimated in the previous section (recall that \( L_i(x_1) = K_i(x_1)/M_v, i = 2, \ldots, 4 \)). Let \( \hat{K}_2(x_1) = K_2(x_1) + \xi(x_1) \). By estimating \( \hat{K}_2, \xi(x_1) \) is obtained as \( \hat{K}_2(x_1) - K_2(x_1) \). This simple idea was the basis of the next procedure.
Chapter 4. The 2-DOF Contactless Positioning System

Procedure 3: Estimation of peak cogging force in 2-DOF system

1. The motion of the system is constrained (by hardware) to lie on the vertical axis with the mover positioned at $x_3 = 28.575 \text{ mm} = 3t_1/2$

2. The constants $C_3$ and $C_4$ are assumed to be known and equal to the values estimated in Section 4.4.2. The constant $C_2$ is assumed to be unknown and is estimated by applying the NPFSI estimator to (4.7)

3. The value of $C_2$ just found was used to generate an approximation of $\hat{K}_2(x_1)$ as

$$\hat{K}_2(x_1) = C_2 \tilde{\lambda}(x_1)B_{pmy_1}(x_1)^2$$

4. The difference between $\hat{K}_2(x_1) - K_2(x_1)$, where $K_2(x_1)$ was obtained using the $C_2$ from Section 4.4.2, represents a rough estimate of the peak of the cogging force $\xi(x_1)$ as a function of the air-gap.

Using the NPFSI estimator as described in procedure 3 with $\varepsilon = 0.01$, $C_2 = 769.99$ was obtained. Applying the rest of procedure 3 produces the estimate of $\xi(x_1)$ in Figure 4.11. The plot shows that the estimated peak cogging force is appreciable (greater than 2 N) when the air-gap is smaller than 15 mm, but otherwise negligible for larger air-gaps.

The negligible cogging force is confirmed in Figure 4.12, where the theoretical equilibrium currents are compared with the actual measurements of $u_2$ along the same lines as in procedure 2, with and without horizontal offset. While the predictions were identical over most of the desired air-gap range, they diverge at air-gaps smaller than 15 mm.

Since the value of the normal force over the range of operation is of the order of 10 N (it has to be in equilibrium with the gravitational force on average), it is clear from Figure 4.11 that, within the air-gap range between 15 mm and 25 mm, the cogging force at each horizontal position is a relatively small percentage of the total force, so it can be ignored in this range. However, Figure 4.11 indicates that for smaller air-gaps
Figure 4.11: Estimate of the peak cogging force over the entire air-gap range

Figure 4.12: Measured and theoretical equilibrium $i_d$ currents at each air-gap
the discrepancy becomes significant. These results appear to confirm our claim that
the cogging force is primarily responsible for the discrepancy, in the vertical dynamics,
between our theoretical model and the physical behavior of the system at air-gaps smaller
than 15mm.

4.4.4 Verification of Complete Model Dynamics

With the horizontal and vertical dynamics of the magnetic levitation model verified
separately, the final task is to confirm that all of the model parameters can be estimated
simultaneously using the NPFSI estimator and still result in a valid 2-DOF model. To
help confirm this, the previous horizontal and vertical model verification procedures are
combined as described in the following procedure.

<table>
<thead>
<tr>
<th>Procedure 4: Model verification of complete 2-DOF system dynamics</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The mover is allowed to travel freely across the complete horizontal and vertical operating range</td>
</tr>
<tr>
<td>2. The reduced-order NPFSI system identification procedure is applied to the simultaneous estimation of the unknown parameters ( C_1, C_2, C_3 ) and ( C_4 ) using the complete 2-DOF lumped parameter model from (4.3)</td>
</tr>
<tr>
<td>3. The parameter ( C_1 ) is used to derive ( L_1(x_1) ). This resulting function is then compared over the air-gap range with the previous values of ( L_1(x_1) ) obtained in procedure 1</td>
</tr>
<tr>
<td>4. The parameters ( C_2, C_3, ) and ( C_4 ) are used to derive ( L_2, L_3, ) and ( L_4 ). These terms are in turn used to predict the equilibrium current ( u_2 ) as a function of the air-gap. The predicted equilibrium current is then compared to the results from procedure 2.</td>
</tr>
</tbody>
</table>

As indicated in procedure 4, the reduced-order NPFSI estimator is applied across the entire range of operation simultaneously at 3 values of \( \varepsilon \), in order to simultaneously estimate the 4 model parameters. The results are provided in table 4.3. Figure 4.13 com-
Table 4.3: 2-DOF model parameters estimated using NPFSI

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\varepsilon = 0.001$</th>
<th>$\varepsilon = 0.01$</th>
<th>$\varepsilon = 0.05$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>13.97</td>
<td>14.20</td>
<td>16.23</td>
</tr>
<tr>
<td>$C_2$</td>
<td>796.93</td>
<td>796.99</td>
<td>795.01</td>
</tr>
<tr>
<td>$C_3$</td>
<td>30.33</td>
<td>31.04</td>
<td>33.44</td>
</tr>
<tr>
<td>$C_4$</td>
<td>0.06</td>
<td>0.07</td>
<td>0.15</td>
</tr>
</tbody>
</table>

The comparisons show that the complete model generated using the full-span NPFSI estimator describes the physical behavior of the system to a reasonable degree of accuracy over the desired air-gap range. However, it should be noted that the resulting horizontal dynamics produced by the 2-DOF calibration procedure appear to diverge from what
was estimated in the fixed air-gap case. This may be due to a lack of persistency of excitation in the 2-DOF calibration procedure or quite simply to an inherent inaccuracy in one of the procedures.

Figures 4.15 and 4.16 and compare predictions of the system states using our mathematical model and the parameters in table 4.3 to the actual state of the system when the system is made to track the reference signals that were used for system identification.

The complete model verification results have demonstrated the accuracy of our mathematical model. We are now ready for control design.
Figure 4.15: Comparison between the actual vertical states and the model predictions

Figure 4.16: Comparison between the actual vertical states and the model predictions
4.5 Nonlinear Controller Design

In [16], a nonlinear controller is proposed for a PMLSM contactless positioning system. While the design is a good starting point, it was found that applying it directly to the positioning apparatus was not effective. There are two main problems that limit the effectiveness of the controller in [16]. First, the original nonlinear controller is designed for set-point stabilization only and is not meant to achieve sinusoidal tracking. Second, the original nonlinear controller is not capable of rejecting constant disturbances produced by such effects as friction and bearing misalignment in the linear guides of the actual system. Hence, the control design in [16] needs to be modified to achieve set-point stabilization and sinusoidal tracking, while at the same time rejecting constant disturbances.

4.5.1 Nonlinear Controller Specifications

The nonlinear controller should meet the following specifications:

- It should be capable of performing both set-point stabilization and sinusoidal tracking over a horizontal operating range of $[-50 \text{ mm}, 50 \text{ mm}]$ and a vertical operating range of $[15 \text{ mm}, 25 \text{ mm}]$.

- The maximum steady-state error for set-point stabilization should be below 0.1 mm across the entire operating range. The settling time should be below 5s.

- The steady-state error for sinusoidal tracking should be as close to 0.1 mm as possible.

- Constant disturbances should be rejected.
4.5.2 Nonlinear Controller Design and Implementation

Consider again the model in (4.2) and incorporate two unknown parameters $\Delta_1$ and $\Delta_2$ representing constant uncertainties due to friction and bearing misalignment

$$
\dot{x}_1 = x_2,
\dot{x}_2 = G - L_4(x_1)[u_1^2 + u_2^2] + L_3(x_1)u_2 - L_2(x_1) + \Delta_2,
\dot{x}_3 = x_4,
\dot{x}_4 = -L_1(x_1)u_1 + \Delta_1,
$$

Consider next the feedback transformation

$$
\begin{align*}
\dot{u}_1 &= -\frac{v_1}{L_1(x_1)}, \\
\dot{u}_2 &= \frac{L_3(x_1) - \sqrt{R(x_1, v_1, v_2)}}{2L_4(x_1)},
\end{align*}
$$

(4.12)

where

$$
R(x_1, v_1, v_2) = L_3(x_1)^2 + 4L_4(x_1)(-v_2 - L_4(x_1)U(x_1, v_1) - L_2(x_1) + G),
$$

$$
U(x_1, v_1) = \left(\frac{v_1}{L_1(x_1)}\right)^2.
$$

The resulting system reads as

$$
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= v_2 + \Delta_2, \\
\dot{x}_3 &= x_4, \\
\dot{x}_4 &= v_1 + \Delta_1,
\end{align*}
$$

(4.13)

with output $y = (x_1, x_3)$. Before proceeding further with the design of the controller terms $v_1$ and $v_2$, notice that the feedback transformation (4.12) is well defined on the set

$$
\mathcal{A} = \{(x, v) : R(x_1, v_1, v_2) \geq 0, L_1(x_1) \neq 0, L_4(x_1) \neq 0\}.
$$

Thus, in order for the controller to make sense, an estimate of the range of operation of the device is needed, that is, the largest set of feasible initial conditions $x(0)$ that guarantees $(x(t), v(t)) \in \mathcal{A}$ for all positive time. This was precisely what was done in Procedure 2 of [16]. Owing to
the fact that the choice of $v$ here is different from what done in [16], Procedure 2 in [16] requires modification. However, doing this is beyond the scope of this thesis and will be the subject of future investigations. Instead, we focus on showing experimentally that the controller is well-defined over the range of interest.

The control terms $v_1$ and $v_2$ are now chosen to make $x_1$ and $x_3$ track a constant step or sinusoid of fixed frequency $\omega_0$ (or a combination of the two) while rejecting the constant disturbances $\Delta_1$, $\Delta_2$. This control specification is best posed as a linear output regulation problem [8], [11], [10] where the exosystem is

$$\dot{w}_1 = \omega_0 w_2,$$
$$\dot{w}_2 = -\omega_0 w_1,$$
$$\dot{w}_3 = \omega_0 w_4,$$
$$\dot{w}_4 = -\omega_0 w_3,$$
$$\dot{w}_5 = 0,$$
$$\dot{w}_6 = 0,$$
$$\dot{w}_7 = 0,$$
$$\dot{w}_8 = 0,$$

$$(r_v, r_h, \Delta_1, \Delta_2) = (w_1 + w_5, w_3 + w_6, w_7, w_8), \quad (4.14)$$

and the output to be regulated is $e = (e_1, e_2) = (x_1 - r_v, x_3 - r_h)$. The internal model was made of two copies of the same system

$$\dot{\xi}_v = \phi \xi_v + Ne_1, \quad y_2 = \Gamma \xi_v,$$
$$\dot{\xi}_h = \phi \xi_h + Ne_2, \quad y_1 = \Gamma \xi_h, \quad (4.15)$$

where

$$\phi = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -\omega_0^2 & 0 \end{bmatrix}, \quad N = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \Gamma = [1 \ 0 \ 0].$$
It is useful to think of the $\xi_v$ and $\xi_h$ subsystems in (4.15) as being two internal models for the vertical and horizontal dynamics, respectively. For simplicity of implementation, $w_1, \ldots, w_4$ in the exosystem (4.14) and $x_1, \ldots, x_4$ in (4.13) are assumed to be available for feedback and the output regulator design is completed by letting

$$v_1 = K_h \begin{bmatrix} e_2 \\ \dot{e}_2 \\ \xi_h \end{bmatrix} + \Gamma \xi_h,$$

$$v_2 = K_v \begin{bmatrix} e_1 \\ \dot{e}_1 \\ \xi_v \end{bmatrix} + \Gamma \xi_v,$$

where the state feedback controller gains $K_h$ and $K_v$ are chosen according to the internal model principle, using pole placement or LQR, with the goal being to stabilize the augmented system consisting of the cascade connection of the plant and internal models when $\Delta_1 = \Delta_2 = r_v = r_h = 0$. That is, $K_h$ and $K_v$ are chosen so as to stabilize the LTI system

$$\dot{x}_1 = x_2,$$

$$\dot{x}_2 = K_v \begin{bmatrix} x_1 \\ x_2 \\ \xi_v \end{bmatrix} + \Gamma \xi_v,$$

$$\dot{x}_3 = x_4,$$

$$\dot{x}_4 = K_h \begin{bmatrix} x_3 \\ x_4 \\ \xi_h \end{bmatrix} + \Gamma \xi_h,$$

$$\dot{\xi}_v = \phi \xi_v + N x_1,$$

$$\dot{\xi}_h = \phi \xi_h + N x_3.$$
The final controller is given by (4.12), (4.15), and (4.16). Its block diagram is depicted in Figure 4.17.

![Block Diagram of Nonlinear Controller](image)

Figure 4.17: Simplified block diagram of nonlinear controller

With the complete nonlinear controller design in place, the following section presents the experimental results. The results demonstrate the set-point and tracking performance of the nonlinear controller in comparison to other linear counterparts.

### 4.6 Experimental Results of 2-DOF Controller Implementation

The nonlinear controller was implemented using WINCON. Its ability to track step and sinusoidal responses is evaluated and compared to some linear counterparts.

#### 4.6.1 Set-point Stabilization

The goal of the set-point stabilization experiments is to implement both the nonlinear controller and the linear counterparts to stabilize set-points in the horizontal operating range of $[-50 \text{ mm}, 50 \text{ mm}]$ and the vertical operating range of $[15 \text{ mm}, 25 \text{ mm}]$. 
The gains $K_h$ and $K_v$ in the nonlinear controller are chosen by applying pole placement to the cascaded connection of the controller and the feedback transformed system model from (4.13). Specifically, the poles corresponding to the horizontal dynamics are placed at $[-9, -10, -11, -12, -13]$, while the poles corresponding to the vertical dynamics are placed at $[-11, -12, -13, -14, -15]$. The result is

$$K_h = [-1182, -55, -154440, -45128, -11924],$$

$$K_v = [-1662, -65, -360360, -103350, -20332].$$

Figure 4.18 illustrates the response of the vertical system states to a sequence of step commands and the absolute vertical positioning accuracy. Figure 4.19 presents the same response results for the horizontal system states. Finally, Figure 4.20 depicts the amount of current input supplied by the controller to the contactless positioning apparatus in order to achieve these results.

![Figure 4.18: Plots of vertical system state responses to the set-point commands](image)

Both the horizontal and vertical position responses demonstrate that for each of the step commands the nonlinear controller is able to successfully stabilize the contactless positioning system.

![Figure 4.18: Plots of vertical system state responses to the set-point commands](image)
Figure 4.19: Plots of horizontal system state responses to the set-point commands

Figure 4.20: Plots of current inputs required for set-point commands
positioning hardware to the desired position, with the required steady-state accuracy below 0.1 mm and settling time well below 5s (approaching 0.5s), thus meeting our performance specifications. The peak and average commanded currents in Figure 4.20 fall within the performance limits of the linear current amplifier modules.

On the other hand, the overshoot in the vertical dynamics is quite large (at one point reaching 100%). This large overshoot limits the range of possible motion. Even though our performance requirements do not include any specification for overshoot, it is interesting to try to decrease the vertical overshoot. To this end, we adjust the gain $K_v$ to place the poles at $[-57.9, -9.9 \pm 11.3i, -1.6 \pm 1.4i]$. As a result, $K_v$ is now

$$K_v = [-1592 -81 -55736 -11573 -15825].$$

Figure 4.21 illustrates the new vertical response to the same sequence of steps. The results demonstrate how the vertical step response exhibits a significantly smaller overshoot at the price of a longer, but not unreasonable, settling time.

![Vertical Step Response](image)

**Figure 4.21:** Plots of vertical step response when controller gains are adjusted

Depending upon the industrial application in which the contactless positioning system is utilized, it is possible that either the overshoot or settling time may turn out to be
a more important performance factor. The experimental results show that by adjusting
the nonlinear controller gains either specification requirement can be accommodated for
by the nonlinear controller.

The next step is to compare the performance of the nonlinear controller to a linearization-
based controller. Specifically, the linear controller under consideration is the same output
regulator in (4.16), but without the incorporation of the feedback linearization transforma-
tion given in (4.12). To make a fair comparison, we have to ensure that for each
set-point the gains of the linear controller are chosen to produce the closed-loop poles as
in the nonlinear case. More specifically, the original nonlinear model in (4.2) is linearized
about the equilibria corresponding to the desired set-points. For each linear model, an
output regulator with the same structure as (4.16) is designed so that the poles of the
closed-loop system coincide with the poles induced by the nonlinear controller by means
of feedback linearization. This ensures that, in a small neighborhood of each set-point,
the linearization-based controller has the same properties of the nonlinear controller.

The nonlinear and linear controllers are compared by subjecting them to a set of four
separate set-point commands, starting from a constant initial condition. Each set-point
requires a successively larger travel distance.

Figure 4.22 depicts the response of the controllers for the first step command, start-
ing from the initial condition of \( [x_1, x_2, x_3, x_4]^T = [0.026 \text{ m}, 0, 0, 0]^T \) and terminating at
\( [x_1, x_2, x_3, x_4]^T = [0.024 \text{ m}, 0, 0.005 \text{ m}, 0]^T \). The results show that while both the linear
and nonlinear controllers are successful, the linear controller yields a response with a
larger settling time due to the system nonlinearity.

Figure 4.23 depicts the response of the controllers for the second step response com-
mand, starting from the initial condition of \( [x_1, x_2, x_3, x_4]^T = [0.026 \text{ m}, 0, 0, 0]^T \) and ter-
minating at \( [x_1, x_2, x_3, x_4]^T = [0.022 \text{ m}, 0, 0.010 \text{ m}, 0]^T \). The results again show that while
both the linear and nonlinear controllers are successful, the linear controller performance
is degraded by the nonlinearity.
Figure 4.22: Comparison of the nonlinear and linear controllers (first step response)

Figure 4.23: Comparison of the nonlinear and linear controllers (second step response)
Chapter 4. The 2-DOF Contactless Positioning System

Figure 4.24 demonstrates the performance of the nonlinear and linear controllers for the third step command, starting from the initial condition of $[x_1, x_2, x_3, x_4]^T = [0.026 \text{ m}, 0, 0, 0]^T$ and terminating at $[x_1, x_2, x_3, x_4]^T = [0.020 \text{ m}, 0, 0.015 \text{ m}, 0]^T$. Again the linear and nonlinear controllers both handle the step command. However, note that in this case the overshoot of the linear controller is now higher than that of the nonlinear controller. This suggests that the performance limitations of the linear controller may have almost been reached.

Figure 4.24: Comparison of the nonlinear and linear controllers (third step response)

Figure 4.25 demonstrates the performance of the nonlinear and linear controllers for the final step command, starting from the initial condition of $[x_1, x_2, x_3, x_4]^T = [0.026 \text{ m}, 0, 0, 0]^T$ and terminating at $[x_1, x_2, x_3, x_4]^T = [0.018 \text{ m}, 0, 0.020 \text{ m}, 0]^T$. For this final step command, the linear controller yields an unstable response, while the nonlinear controller still exhibits excellent performance. It should be noted that around 5.5 seconds the instability exhibited by the linear controller triggers a safety mechanism within the real-time code resulting in a complete shut-down of the system. This is why the oscillations halt at that point in time within figure 4.25.
Overall, the results of this comparison demonstrate the superior performance of the nonlinear controller over the linear servomechanism controller in two respects. Not only does the nonlinear controller exhibit a better transient behavior, but it also shows the ability to perform set-point stabilization over a wider operating range.

The final set-point stabilization experiments involve comparing the performance of the nonlinear controller with the PID regulators that were used for system identification in Section 4.3. Recall that the PID setup was illustrated in Figure 4.6.

Figure 4.26 demonstrates a successful step response using the PID regulators. It can clearly be seen that the PID step response, even though successful, suffers from extremely poor transient behavior (particularly in the vertical range of motion). It should also be noted that even with extensive tuning to the PID parameters in an attempt to improve performance, it was only possible to obtain a successful step response over the small distance visible within the figure.

Figure 4.27 confirms that for even a slightly larger step command the PID regulators completely fail in performing the desired set-point stabilization.
Figure 4.26: Step response of PID regulators (small step)

Figure 4.27: Step response of PID regulators (large step)
The experiments therefore show that direct comparison of the PID with the nonlinear controller is not necessary. It is clear that the PID regulators cannot achieve the same set-point stabilization performance as the nonlinear controller or even the linear version of it. Although PID control would likely have been effective for pure horizontal operation at a fixed air-gap it is not capable of coping with the highly nonlinear dynamics that are produced by a variable air-gap.

4.6.2 Sinusoidal Tracking

For the experiments that follow, a horizontal sinusoid reference is applied with amplitude 30 mm and frequency $1.5\pi$ rad/sec. At the same time, a vertical sinusoid reference is simultaneously applied with amplitude 5 mm, offset 20 mm, and frequency $1.5\pi$ rad/sec. These amplitudes and frequencies allow tracking actuation over the complete range of operation near the maximum allowable speed permitted by the LCAM current supply.

The nonlinear controller employs the same controller gains, $K_h$ and $K_v$, from (4.17). Figure 4.28 illustrates the horizontal and vertical tracking responses. Figure 4.29 illustrates the horizontal and vertical tracking error between the actual system responses and the desired reference signals.

From the tracking error data shown in figure 4.29, it is found that the horizontal tracking error averages about 0.11 mm with occasional peaks that reach 0.4 mm at random intervals. The vertical tracking, meanwhile, averages 0.24 mm with peaks that occasionally reach 0.8 mm. The tracking error settles to around its average value in less than a second. The resulting nonlinear tracking performance is deemed to be satisfactory, despite the fact that the vertical tracking error is about twice the desired specification of 0.1 mm. Several improvements to the tracking performance were attempted, including the adjustment of the controller and model parameters, as well as the testing of a basic coulomb friction compensator. However, despite these adjustments, no significant improvements were achieved with the tracking error beyond the results presented here.
Figure 4.28: Plots showing the tracking responses of the nonlinear controller

Figure 4.29: Plots showing the tracking errors of the nonlinear controller
The conclusion is that the nonlinear controller in its current form reaches the limitations of the contactless positioning hardware implementation and as a result, cannot further compensate for the uncertainties that are introduced by such effects as friction and bearing misalignment.

Figure 4.30 illustrates the horizontal and vertical tracking responses obtained when PID regulators are used to track the same tracking references as in the nonlinear case. For a consistent comparison, figure 4.31 illustrates the horizontal and vertical tracking errors between the actual responses and the desired PID reference signals.

![Figure 4.30: Plots showing the tracking responses of the PID regulators](image)

From the plots, it is clearly seen that the PID regulators produce a significantly larger tracking error for both horizontal and vertical motions. Specifically, the average horizontal error is now about 0.19 mm, while the vertical error now approaches 0.9 mm. When compared with the PID regulators, the nonlinear controller has been shown to produce an improvement of at least 50% in the tracking error performance.

In addition to the larger tracking error, it should be noted that the comparison of the PID regulators to the nonlinear controller is essentially unnecessary, since the linear...
Figure 4.31: Plots showing the tracking errors of the PID regulators

PID configuration would completely fail at performing the sinusoidal tracking without the incorporate of reference signal pre-filtering. This further illustrates how unsuitable the PID configuration is for the type of sinusoidal tracking that can be achieved using the nonlinear controller.
Chapter 5

The 3-DOF Contactless Positioning System

This chapter provides complete implementation details of the 3-DOF positioning system. With the 2-DOF hardware serving as a successful proof of concept testbed, the goal here is to implement a more advanced second-generation positioning apparatus. In addition to the extra degree-of-freedom, the 3-DOF hardware is the result of intensive design and is constructed with better linear guides. It is therefore possible to produce a more rigorous set of specifications for nonlinear control design. Despite these improvements, it is shown how the 3-DOF hardware introduces new challenges, such as actuator saturation and uncontrollable pitch and roll dynamics. Overall, the 3-DOF implementation results lay the groundwork for future generations of positioning devices to be discussed later.

The chapter is organized as follows. The hardware setup is first described. The state-space model of the system is then derived, and a system identification procedure similar to that in Chapter 4 is outlined. Next, a nonlinear controller based and feedback linearization and output regulation is designed, and experimental results compare the performance of the nonlinear controller to that of linear counterparts.
Chapter 5. The 3-DOF Contactless Positioning System

5.1 Hardware Setup

Like its predecessor, the second-generation 3-DOF contactless positioning system was built by Quanser Consulting based on the specifications in [2] and successive modifications. The apparatus is constructed using a set of four iron-cored PMLSMs. The physical characteristics of the system allow for $x$-axis and $z$-axis (both horizontal) range of motion of approximately $\pm 50$ mm, and a $y$-axis (vertical) range of motion of approximately $\pm 10$ mm. A photo of the complete hardware implementation is shown from various angles in Figure 5.1, while the hardware specifications are summarized in Table 5.1. The individual components of the system are detailed in the following sections.

Figure 5.1: 3-DOF hardware implementation (TOP), Top view showing 4 PMLSMs (BOTTOM-LEFT), Close-up of single PMLSM mover and stator (BOTTOM-RIGHT)
Table 5.1: Specifications for 3-DOF positioning hardware

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall hardware dimensions</td>
<td>–</td>
<td>mm</td>
<td>1200 x 1200 x 300</td>
</tr>
<tr>
<td>Motor pair separation</td>
<td>–</td>
<td>mm</td>
<td>400.0</td>
</tr>
<tr>
<td>Stator slot width</td>
<td>$b_0$</td>
<td>mm</td>
<td>12.7</td>
</tr>
<tr>
<td>Stator slot pitch</td>
<td>$t_1$</td>
<td>mm</td>
<td>19.05</td>
</tr>
<tr>
<td>Turns per phase</td>
<td>$W$</td>
<td>–</td>
<td>900</td>
</tr>
<tr>
<td>Coil pitch</td>
<td>$\omega_c$</td>
<td>mm</td>
<td>57.15</td>
</tr>
<tr>
<td>Stator pole pairs per motor</td>
<td>$p$</td>
<td>–</td>
<td>3</td>
</tr>
<tr>
<td>Number of stator slots per motor</td>
<td>$z_1$</td>
<td>–</td>
<td>18</td>
</tr>
<tr>
<td>PM height</td>
<td>$h_m$</td>
<td>mm</td>
<td>5</td>
</tr>
<tr>
<td>PM length</td>
<td>$L_A$</td>
<td>mm</td>
<td>50</td>
</tr>
<tr>
<td>Number of PM’s per motor</td>
<td>$p_m$</td>
<td>–</td>
<td>4</td>
</tr>
<tr>
<td>Pole pitch</td>
<td>$\tau$</td>
<td>mm</td>
<td>57.15</td>
</tr>
<tr>
<td>PM width</td>
<td>$\tau_p$</td>
<td>mm</td>
<td>28.58</td>
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<tr>
<td>PM coercivity</td>
<td>$H_c$</td>
<td>A/m</td>
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<td>Y-axis mover mass</td>
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<td>Kg</td>
<td>6.9</td>
</tr>
</tbody>
</table>

5.1.1 PMLSM Mover, Stator and Power Interconnections

The four PMLSM stators, fixed in place to a heavy aluminium frame, are longitudinally laminated and transversally slotted to accommodate a single layer of 3-phase winding. The four corresponding movers, attached to the movable platform, are each composed of four type N35 permanent magnets (PM) attached to a ferromagnetic backing.
Consider the diagram in Figure 5.2. The four PMLSMs are combined into 2 pairs by wiring together the 3 phases of each pair in parallel. One pair of PMLSMs is oriented along the $x$-axis (coil-pair alpha), while the other pair is oriented along the $z$-axis (coil-pair beta). The result is that the complete system requires only two 3-phase current inputs, or equivalently, 2 pairs of direct and quadrature current inputs, for a total of four control inputs.

### 5.1.2 Linear Guides, Stoppers and Sensors

The platform of the 3-DOF hardware is designed to move as one complete piece using a series of Del-Tron S2-6AC linear guides. The platform is directly attached to a set of four vertically mounted linear guides permitting the $y$-axis range of operation. These vertical guides are in turn attached to a set of four orthogonally mounted linear guides permitting motion across the $z$-axis. Finally, this setup rests on top of another set of four orthogonally mounted linear guides permitting motion across the $x$-axis.

As before, the linear guides are required to limit the amount of pitching and rolling that would naturally occur in the motion of the platform, meaning that the 3-DOF prototype is not fully contactless. The improved quality of the guides and precision
alignment helps to limit the amount of friction they introduce. The goal is still to eventually eliminate the need for linear guides in future prototypes.

A set of fixed horizontal limits and adjustable vertical stoppers are also incorporated into the 3-DOF apparatus to allow for the restriction of the platform operating range. At the same time, a set of brackets permits each individual degree-of-freedom to be fixed in place to allow for isolated experiments on each axis of motion.

A set of three linear optical encoders are used to measure the position of the platform across each axis of motion. The encoders are the same type employed in the 2-DOF prototype and have a resolution of 10 $\mu$m.

Figures 5.3 and 5.4 illustrate the setup of the linear guides, stoppers and position sensors incorporated into the 3-DOF position apparatus.

Figure 5.3: Close-up of the 3-DOF linear guides and vertical stoppers
5.1.3 Power Delivery

In order to generate the 3-phase AC current for actuation of each pair of PMLSMs, the 3-DOF contactless positioning system requires a pair of custom built power supplies developed by Quanser Consulting. Each power supply consists of three linear current amplifier modules (LCAM). A PMLSM pair is connected to each power supply so that the individual phases within the pair are controlled by an LCAM. As with the original power supply setup from the 2-DOF prototype, current commands are sent to the LCAMs through an interface from a PC. Each LCAM is factory limited to supply around 4A continuous and 5A peak. The LCAM’s are also tuned for optimum power delivery based on the stator coil inductance.

5.1.4 Computer Interface and Real-Time Control Environment

The Quanser Multi-Q PCI data acquisition board cannot be used with the 3-DOF apparatus because it does not possess sufficient analog outputs. As a result, the 3-DOF
positioning hardware is connected to the PC using a Quanser Q8 data acquisition board. The upgraded Quanser board provides all of the necessary analog and digital I/O, and allows the hardware to be operated at a sampling frequency of 5 KHz.

Control of the 3-DOF hardware is still implemented through the Q8 interface using the Quanser WINCON real-time control environment, which allows construction of controllers through Simulink diagrams in MATLAB. The equipment connections are made according to the same block diagram structure shown in figure 4.4 in Chapter 4.

### 5.2 Derivation of System State-Space Model

Consider the diagram in Figure 5.5 displaying the forces acting on the mover of the 3-DOF positioning system.

![Figure 5.5: Forces in the 3-DOF contactless positioning system](image)

The equations of motion are obtained by straightforward application of Newton’s law,

\[
\begin{align*}
M_y \ddot{y} &= M_y G + 2F_{y1}(g, i_{q2}, i_{d2}) + 2F_{y2}(g, i_{q1}, i_{d1}), \\
M_x \ddot{x} &= 2F_x(g, i_{q2}), \\
M_z \ddot{z} &= 2F_z(g, i_{q1}),
\end{align*}
\] (5.1)
where \( G \) denotes the gravitational constant, \( F_x(g, i_{q2}) \) the translational force generated by coil-pair alpha, \( F_z(g, i_{q1}) \) the translational force generated by coil-pair beta, \( F_{y1}(g, i_{q2}, i_{d2}) \) the normal force generated by coil-pair alpha, and \( F_{y2}(g, i_{q1}, i_{d1}) \) the normal force generated by coil-pair beta. \( M_x, M_z \) and \( M_y \) are the masses of the movable platform over each axis of motion. Substituting the expressions in (2.2) and (2.4) we get

\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= G - 2L_5(x_1)[u_1^2 + u_2^2 + u_3^2 + u_4^2] + 2L_4(x_1)[u_3 + u_4] - 4L_3(x_1), \\
\dot{x}_3 &= x_4, \\
\dot{x}_4 &= -2L_1(x_1)u_2, \\
\dot{x}_5 &= x_6, \\
\dot{x}_6 &= -2L_2(x_1)u_1,
\end{align*}
\]

(5.2)

where

\[
x = [g, \dot{g}, \delta x, \dot{\delta x}, \delta z, \dot{\delta z}]^T, \quad u = [i_{q1}, i_{q2}, i_{d1}, i_{d2}]^T,
\]

\[
L_1(x_1) = \frac{K_1(x_1)}{M_x}, \quad L_2(x_1) = \frac{K_1(x_1)}{M_z},
\]

\[
L_i(x_1) = \frac{K_{i-1}(x_1)}{M_y}, \quad i = 3, \ldots, 5,
\]

\[
K_1(x_1) = \frac{12\sqrt{2}Wk_{w1}p_mL_A\sigma_m\mu_0\tilde{\lambda}(x_1) \sinh\left(\frac{\pi}{\tau}p_m\sin\left(\frac{\pi\tau_p}{2\tau}\right)\right)}{\pi p K_c(x_1) \sinh\left(\frac{\pi}{\tau}(h_m + x_1)\right)},
\]

\[
K_2(x_1) = \frac{\tilde{\lambda}(x_1)L_A p_m \tau B_{pmy1}(x_1)^2}{4\mu_0},
\]

\[
K_3(x_1) = \frac{\tilde{\lambda}(x_1)3\sqrt{2}L_A p_m W k_{w1} B_{pmy1}(x_1) \coth\left(\frac{\pi}{\tau}(h_m + x_1)\right)}{p^2 K_c(x_1)},
\]

\[
K_4(x_1) = \frac{\tilde{\lambda}(x_1)18L_A p_m W^2 k_{w1}^2\mu_0 \coth^2\left(\frac{\pi}{\tau}(h_m + x_1)\right)}{\tau p^2 K_c(x_1)^2},
\]

\[
\tilde{\lambda}(x_1) = 1 - \frac{b_0^2}{4t_1(x_1 + \frac{b_0}{2} + \frac{h_m}{\mu_{rec}})}.
\]

As mentioned in Chapter 2, the function \( B_{pmy1}(x_1) \) represents the magnetic field produced by the PM’s and is approximated using a 12th degree polynomial, while \( i_{q2}, i_{d2} \)
and $i_q, i_d$ are the direct and quadrature current inputs to the PMLSM coil-pairs alpha and beta, respectively.

Notice that the model in (5.2) is overactuated because we have four control inputs and three degrees-of-freedom. Future research will exploit such overactuation to control the roll and pitch of the platform. Throughout the reminder of this thesis, we eliminate the overactuation by setting

$$u_3 = u_4 = \bar{u}. \tag{5.3}$$

In what follows, we apply the same system identification technique from Chapter 4 to estimate the unknown 3-DOF model parameters. In preparation for this, the model from (5.2) is rewritten with the parameters lumped together into five unknown constants $C_1, C_2, C_3, C_4$ and $C_5$ as follows

$$
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= G - 2C_5 \frac{\tilde{\lambda}(x_1) \coth^2(\frac{\tau}{2}(h_m + x_1))}{K_c(x_1)} [u_1^2 + u_2^2 + u_3^2 + u_4^2] \\
&\quad + 2C_4 \frac{\tilde{\lambda}(x_1) B_{pnyl}(x_1) \coth(\frac{\tau}{2}(h_m + x_1))}{K_c(x_1)} [u_3 + u_4] \\
&\quad - 4C_3 \tilde{\lambda}(x_1) B_{pnyl}(x_1)^2, \\
\dot{x}_3 &= x_4, \\
\dot{x}_4 &= -C_1 \frac{\tilde{\lambda}(x_1)}{K_c(x_1) \sinh(\frac{\tau}{2}(h_m + x_1))} u_2, \\
\dot{x}_5 &= x_6, \\
\dot{x}_6 &= -C_2 \frac{\tilde{\lambda}(x_1)}{K_c(x_1) \sinh(\frac{\tau}{2}(h_m + x_1))} u_1,
\end{align*}
$$

where

$$
\begin{align*}
C_1 &= \frac{12\sqrt{2}Wk_{\text{ew1}}\mu_0 L_A \sigma_m \mu_0 \sinh(\frac{\pi}{2}h_m) \sin(\frac{\pi \tau}{2})}{M_z \tau p}, \\
C_2 &= \frac{12\sqrt{2}Wk_{\text{ew1}}L_A \sigma_m \mu_0 \sinh(\frac{\pi}{2}h_m) \sin(\frac{\pi \tau}{2})}{M_z \tau p}, \\
C_3 &= \frac{3\sqrt{2}L_A \mu_0}{M_z \tau p^2}, \\
C_4 &= \frac{18L_A \mu_0 W^2 k_{\text{ew1}}^2 \mu_0}{M_z \tau p^2}.
\end{align*}
$$
5.3 Implementation of System Identification Technique

In order to apply the reduced-order NPFSI system identification technique from [9] to the estimation of the unknown constants $C_1, \ldots, C_5$ within the lumped model (5.3), the same implementation issues discussed in Chapter 4 need to be addressed. In this section we briefly review them.

Three PID controllers are used to achieve rough simultaneous tracking of reference motions on the three axes of the system. The associated block diagram is illustrated in Figure 5.6. The controllers were tuned manually. The reference signals used for system identification are

\[
\begin{align*}
X_{\text{ref}}(t) &= 0.045 \left( \frac{1}{2} \sin(0.75 \pi t - \frac{\pi}{3}) + \frac{1}{4} \sin(1.5 \pi t + \frac{\pi}{4}) \\
&\quad + \frac{1}{2} \sin(2.25 \pi t - \frac{\pi}{5}) + \frac{1}{4} \sin(3 \pi t + \frac{\pi}{6}) \right), \\
Y_{\text{ref}}(t) &= 0.005 \left( \frac{1}{4} \sin(\pi t - 0.8 \pi) + \frac{1}{4} \sin(2 \pi t - 1.6 \pi) \\
&\quad + \frac{1}{4} \sin(3 \pi t - 2.4 \pi) + \frac{1}{4} \sin(4 \pi t - 3.2 \pi) \right), \\
Z_{\text{ref}}(t) &= 0.045 \left( \frac{1}{2} \sin(0.75 \pi t - \frac{\pi}{3} - 0.469 \pi) + \frac{1}{4} \sin(1.5 \pi t + \frac{\pi}{4} - 0.938 \pi) \\
&\quad + \frac{1}{2} \sin(2.25 \pi t - \frac{\pi}{5} - 1.406 \pi) + \frac{1}{4} \sin(3 \pi t + \frac{\pi}{6} - 1.875 \pi) \right),
\end{align*}
\]

These reference signals actuate the 3-DOF positioning system over the approximate $x$ and $z$-axis ranges of $[-50 \text{ mm}, 50 \text{ mm}]$ and the approximate $y$-axis range of $[20 \text{ mm}, 30 \text{ mm}]$.

In the conversion between three-phase currents and direct/quadrature components the same offset as in (4.4) is included. Refer to Appendix C for more details.

In addition to the persistency of excitation generation, we make use of the same high-gain observers from chapter 4 to derive the velocity states from the position states. The same $\gamma$ and $\varepsilon$ parameter values are also used.
5.4 Model Verification of 3-DOF System

The system identification procedure is applied to a set of 3 separate parameter estimation experiments:

- The $y$-axis position of the platen is held in place at a series of fixed air-gaps, while the system identification procedure is used to estimate $C_1$ and $C_2$, both individually and simultaneously.

- The platen is held in place at a series of fixed displacements on $x$-$z$ plane, while the system identification procedure is used to estimate $C_3$, $C_4$, and $C_5$ from the $y$-axis dynamics for each position.

- The parameters $C_1, \ldots, C_5$ are simultaneously estimated, while the system is actuated over the complete operating range of the device.
5.4.1 Verification of \( x \) and \( z \)-axis dynamics

The \( x \) and \( z \)-axis dynamics are verified through application of a slightly modified version of Procedure 1 in Chapter 4. Constraining the motion to lie on the \( x-z \) plane, we obtain the horizontal dynamics

\[
\begin{align*}
\dot{x}_3 &= x_4, \\
\dot{x}_4 &= -2L_1(\bar{x}_1)u_2, \\
\dot{x}_5 &= x_6, \\
\dot{x}_6 &= -2L_2(\bar{x}_1)u_1.
\end{align*}
\] (5.4)

For zero initial velocities we obtain the solution

\[
\begin{align*}
x_3(t) &= -L_1(\bar{x}_1)u_2 t^2 + x_3(0), \\
x_5(t) &= -L_2(\bar{x}_1)u_1 t^2 + x_5(0).
\end{align*}
\] (5.5)

Procedure 1 is applied separately to both the \( x \) and \( z \) axes over an air-gap range between 15 and 35 mm at 2.5 mm intervals. For each air-gap position, the platform is first accelerated from \( x_3 = -60 \) mm to \( x_3 = 60 \) mm, with \( u_2 = -3 \) A and \( x_5 = 0 \) mm, in order to estimate \( L_1(\bar{x}_1) \). Next, the platform is acceleration from \( x_5 = -60 \) mm to \( x_5 = 60 \) mm, with \( u_1 = -3 \) A and \( x_3 = 0 \) mm, in order to estimate \( L_2(\bar{x}_1) \) at the same air-gap.

Figures 5.7 and 5.8 provide a few examples of the data points obtained along with the fitted parabolas. It should be noted that data could not be obtained for air-gaps below 15 mm, due to the difficulty of obtaining a leveled air-gap in the presence of the large magnetic attraction between the PM’s and stators which causes a rotation of the platform. The resulting comparisons between the \( L_1(\bar{x}_1) \) and \( L_2(\bar{x}_1) \) estimates obtained using procedure 1 are found in Figure 5.9.

The results confirm that two different estimation techniques predicted a similar response for the \( x \) and \( z \)-axis dynamics of the 3-DOF system, with the estimates obtained for the \( \varepsilon = 0.05 \) case exhibiting the expected increased divergence. The correspondence between the two techniques is not as close as in the original 2-DOF system (but still...
Figure 5.7: Curve-fitting of parabolas to actual $x$-axis position data

Figure 5.8: Curve-fitting of parabolas to actual $z$-axis position data
Figure 5.9: Comparison of various estimations of $L_1(\bar{x}_1^i)$ and $L_2(\bar{x}_1^i)$

sufficient for our purposes). A likely explanation for this is that the platform is subjected to a smaller acceleration than in the 2-DOF case, and thus friction affects the estimation of $L_1$ to a greater extent.

From the $L_1(\bar{x}_1^i)$ and $L_2(\bar{x}_1^i)$ estimates we get estimates of $C_1$ and $C_2$. Their average values are

$$C_1 = 0.6739,$$

$$C_2 = 0.9225. \quad (5.6)$$

As a final confirmation of the accuracy of the model of the $x$ and $z$ dynamics, Figure 5.10 demonstrates that when $L_1(\bar{x}_1^i)$ and $L_2(\bar{x}_1^i)$ are simultaneously estimated using the NPFSI estimator with simultaneous $x$ and $z$ axis excitation, the results are virtually identical to those displayed in Figure 5.9

### 5.4.2 Verification of $y$-axis Dynamics

To validate the $y$-axis dynamics, Procedure 2 from Chapter 4 is applied to the 3-DOF system. Recall that we constrain the control inputs by imposing that $u_3 = u_4 = \bar{u}$. At
Figure 5.10: Comparison of single axis and simultaneous estimations of $L_1(\bar{x}_1^i)$ and $L_2(\bar{x}_1^i)$
the same time, we constrain $x_3 = x_5 = 0$ mm by hardware and set $u_1 = u_2 = 0$ A. From (5.2), using the above constraints, we obtain the $y$-dynamics as follows

$$
\dot{x}_1 = x_2, \\
\dot{x}_2 = G - 4L_5(x_1)\bar{u}^2 + 4L_4(x_1)\bar{u} - 4L_3(x_1).
$$

Procedure 2 in Chapter 4 can now be applied to compare the actual measured 3-DOF equilibrium current with the theoretical current predictions at a series of fixed air-gap values, based on

$$
\bar{u} = \frac{4L_4(\bar{x}_1) - \sqrt{16L_4^2(\bar{x}_1) - 16L_5(\bar{x}_1)(4L_3(\bar{x}_1) - G)}}{8L_5(\bar{x}_1)}.
$$

To apply Procedure 2, air-gap values are chosen to range between 30 and 17 mm with 1 mm increments. Because of power limitations, the platform cannot be levitated at air-gaps larger than 30 mm. Furthermore, air-gap below 20 mm yield an increasing amount of uncontrollable pitching and rolling. This pitching and rolling make current measurements impossible for air-gaps smaller than 17 mm. The results from the application of the NPFSI estimator for three different values of $\varepsilon$ are summarized in table 5.2.
Table 5.2: $y$-axis model parameters using NPFSI estimator when $x_3 = x_5 = 0$ mm

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\varepsilon = 0.001$</th>
<th>$\varepsilon = 0.01$</th>
<th>$\varepsilon = 0.05$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_3$</td>
<td>339.00</td>
<td>334.52</td>
<td>332.01</td>
</tr>
<tr>
<td>$C_4$</td>
<td>8.47</td>
<td>8.71</td>
<td>8.63</td>
</tr>
<tr>
<td>$C_5$</td>
<td>0.0030</td>
<td>0.0045</td>
<td>0.0110</td>
</tr>
</tbody>
</table>

The theoretical equilibrium currents found using Procedure 2 (based on the parameters $C_3, \ldots C_5$ estimated with $\varepsilon = 0.01$) are compared to the measured equilibrium currents in Figure 5.11.

![Figure 5.11: Comparing measured $i_d$ currents and the theoretical 3-DOF predictions](image)

Figure 5.11: Comparing measured $i_d$ currents and the theoretical 3-DOF predictions

The plot demonstrates that the model accurately describes the behavior of the 3-DOF positioning hardware for the chosen air-gap range between 17 and 30 mm. Note that, unlike the results obtained for the 2-DOF prototype, there does not appear to be a systematic increase in prediction divergence for smaller air-gap values. Based on the assumption from Chapter 4 that the cogging force is primarily responsible for model
divergence, it makes sense that the divergence is now less significant because the minimum air-gap is now larger. This is confirmed in the next section.

### 5.4.3 Analysis of the cogging force

In order to obtain an estimate of the intensity of the cogging force over the air-gap range of the device, we employ Procedure 3 in Chapter 4. Using the NPFSI estimator as described in Procedure 3 with $\varepsilon = 0.01$, we obtain $C_3 = 330.01$. The resulting peak of the cogging force as a function of the air-gap is depicted in Figure 5.12.

![Figure 5.12: Estimate of the peak 3-DOF cogging force over the chosen air-gap range](image)

The plot demonstrates that the peak cogging force is below 0.5N in an air-gap range between 20 and 30mm\(^1\). Since the intensity of the normal force typically generated by the device is of the order of 10N (to be in equilibrium with the gravitational force), it is clear from Figure 5.12 that, within the air-gap range between 20mm and 30mm, the cogging force at each $x$ and $z$ displacement is essentially negligible. This result confirms

---

\(^{1}\)This air-gap range constitutes the $y$-axis operating region in the upcoming controller implementations.
the finding, displayed in Figure 5.11, that there is little difference between the theoretical equilibrium currents and their actual measurements.

![Figure 5.13: Measured and theoretical 3-DOF equilibrium currents at each air-gap](image)

5.4.4 Verification of Complete Model Dynamics

We complete model verification by using the NPFSI estimator to simultaneously derive $C_1, \ldots, C_5$ using Procedure 4 in Chapter 4. The results are provided in Table 5.3.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\varepsilon = 0.001$</th>
<th>$\varepsilon = 0.01$</th>
<th>$\varepsilon = 0.05$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>0.6158</td>
<td>0.6135</td>
<td>0.6328</td>
</tr>
<tr>
<td>$C_2$</td>
<td>0.8310</td>
<td>0.8224</td>
<td>0.8566</td>
</tr>
<tr>
<td>$C_3$</td>
<td>327.51</td>
<td>327.50</td>
<td>325.00</td>
</tr>
<tr>
<td>$C_4$</td>
<td>8.27</td>
<td>8.48</td>
<td>8.53</td>
</tr>
<tr>
<td>$C_5$</td>
<td>0.0084</td>
<td>0.0070</td>
<td>0.0090</td>
</tr>
</tbody>
</table>
Figure 5.14 compares the estimate of $L_1$ and $L_2$ generated using the estimates of $C_1, \ldots, C_5$ with the estimates obtained at fixed air-gaps in the previous section. Figure 5.15 compares the theoretical equilibrium currents $\bar{u}$ produced by the complete 3-DOF model with the measured equilibrium currents.

![Graph](image)

Figure 5.14: Comparison between various estimates of $L_1$ and $L_2$

The results confirm the accuracy of the model. They also indicate that the simultaneous estimation of the unknown constants $C_1, \ldots, C_5$ gives good results.

We conclude the verification results by comparing the state trajectories predicted by the mathematical model to the actual system trajectories when the PID controllers in Figure 5.6 are employed to track the reference signals used for system identification. The result, displayed in Figures 5.16, 5.17, and 5.18, is that predictions are almost indistinguishable from the actual measurements.
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Figure 5.15: Comparing predicted $i_d$ current with actual 3-DOF measurement

Figure 5.16: Comparison between the actual $x$-axis states and the NPFSI predictions
Figure 5.17: Comparison between the actual z-axis states and the NPFSI predictions

Figure 5.18: Comparison between the actual y-axis states and the NPFSI predictions
5.5 Nonlinear Controller Design

In this section we present an extension of the nonlinear controller introduced in Chapter 4 to deal with the 3-DOF setup. The controller should meet the following specifications.

- It should be capable of performing both set-point stabilization and sinusoidal tracking over a $x$ and $z$-axis operating range of $[-50 \text{ mm}, 50 \text{ mm}]$ and a $y$-axis operating range of $[20 \text{ mm}, 30 \text{ mm}]$.

- The nonlinear controller should achieve a set-point steady-state error of $0.1 \text{ mm}$ in under 3 seconds, and attain the sensor encoder resolution of $10 \mu\text{m}$ in under 5 seconds.

- For all set-point commands, the overshoot should be ideally less than 30%, but under no circumstances greater than 50%.

- It is desired that the nonlinear controller be capable of performing sinusoidal tracking with a tracking error as close to the encoder resolution of $10 \mu\text{m}$ as possible, but otherwise no additional tracking performance specifications is imposed.

We begin the design process by rewriting the system model and incorporating constant unknown terms $\Delta_1, \ldots, \Delta_3$ representing friction or bearing misalignment.

\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= G - 2L_5(x_1)[u_1^2 + u_2^2 + u_3^2 + u_4^2] + 2L_4(x_1)[u_3 + u_4] - 4L_3(x_1) + \Delta_3, \\
\dot{x}_3 &= x_4, \\
\dot{x}_4 &= -2L_1(x_1)u_2 + \Delta_2, \\
\dot{x}_5 &= x_6, \\
\dot{x}_6 &= -2L_2(x_1)u_1 + \Delta_1,
\end{align*}
\]
Recall that in writing the equation above we let \( u_3 = u_4 = \bar{u} \). Consider the feedback transformation

\[
\begin{align*}
    u_1 &= -\frac{v_1}{2L_2(x_1)}, \\
    u_2 &= -\frac{v_2}{2L_1(x_1)}, \\
    \bar{u} &= \frac{L_4(x_1) - \sqrt{R(x_1, v_1, v_2, v_3)}}{2L_5(x_1)},
\end{align*}
\]

(5.7)

where

\[
R(x_1, v_1, v_2, v_3) = L_4(x_1)^2 + L_5(x_1) (-v_3 - L_5(x_1) U(x_1, v_1, v_2) - 4L_3(x_1) + G),
\]

\[
U(x_1, v_1, v_2) = 2 \left( \frac{v_1}{2L_2(x_1)} \right)^2 + 2 \left( \frac{v_2}{2L_1(x_1)} \right)^2.
\]

The resulting linearized 3-DOF model reads as follows \( y = (x_1, x_3, x_5) \)

\[
\begin{align*}
    \dot{x}_1 &= x_2, \\
    \dot{x}_2 &= v_3 + \Delta_3, \\
    \dot{x}_3 &= x_4, \\
    \dot{x}_4 &= v_2 + \Delta_2, \\
    \dot{x}_5 &= x_6, \\
    \dot{x}_6 &= v_1 + \Delta_1.
\end{align*}
\]

(5.8)

The feedback transformation (5.7) is well defined on the set \( \mathcal{A} = \{(x, v) : R(x_1, v_1, v_2, v_3) \geq 0, L_1(x_1) \neq 0, L_2(x_1) \neq 0, L_5(x_1) \neq 0\} \). As in Chapter 4, we do not attempt to rigorously estimate the domain of attraction of the equilibrium of the closed-loop system, we rather show experimentally that the control input is always well-defined during operation.

The control terms \( v_1, v_2 \) and \( v_3 \) were next chosen to make \( x_1, x_3 \) and \( x_5 \) track a constant step or a sinusoid of fixed frequency \( \omega_0 \) (or a combination of the two) while rejecting the constant disturbances \( \Delta_1, \Delta_2 \) and \( \Delta_3 \). As done in Chapter 4, we pose this
problem as one of output regulation with the exosystem

\[
\begin{align*}
\dot{w}_1 &= \omega_0 w_2, & \dot{w}_7 &= 0, \\
\dot{w}_2 &= -\omega_0 w_1, & \dot{w}_8 &= 0, \\
\dot{w}_3 &= \omega_0 w_4, & \dot{w}_9 &= 0, \\
\dot{w}_4 &= -\omega_0 w_3, & \dot{w}_{10} &= 0, \\
\dot{w}_5 &= \omega_0 w_6, & \dot{w}_{11} &= 0, \\
\dot{w}_6 &= -\omega_0 w_5, & \dot{w}_{12} &= 0,
\end{align*}
\]

\[
(\mathbf{r}_y, \mathbf{r}_x, \mathbf{r}_z, \Delta_1, \Delta_2, \Delta_3) = (w_1 + w_7, w_3 + w_8, w_5 + w_9, w_{10}, w_{11}, w_{12}), \tag{5.9}
\]

where the output to be regulated is \(e = (e_1, e_2, e_3) = (x_1 - r_y, x_3 - r_x, x_5 - r_z)\). The internal model is composed of three copies of the same system

\[
\begin{align*}
\dot{\xi}_y &= \phi \xi_y + Ne_1, & y_3 &= \Gamma \xi_y, \\
\dot{\xi}_x &= \phi \xi_x + Ne_2, & y_2 &= \Gamma \xi_x, \\
\dot{\xi}_z &= \phi \xi_z + Ne_3, & y_1 &= \Gamma \xi_z,
\end{align*}
\]

\[
\tag{5.10}
\]

where as before

\[
\phi = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & -\omega_0^2 & 0
\end{bmatrix}, \quad N = \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}, \quad \Gamma = [1 \ 0 \ 0].
\]

The output regulator design is completed by letting

\[
\begin{align*}
v_1 &= K_z \begin{bmatrix} e_3 \\ \dot{e}_3 \\ \xi_z \end{bmatrix} + \Gamma \xi_z, & v_2 &= K_x \begin{bmatrix} e_2 \\ \dot{e}_2 \\ \xi_x \end{bmatrix} + \Gamma \xi_x, & v_3 &= K_y \begin{bmatrix} e_1 \\ \dot{e}_1 \\ \xi_y \end{bmatrix} + \Gamma \xi_y,
\end{align*}
\]

\[
\tag{5.11}
\]
where the state feedback controller gains $K_x$, $K_z$ and $K_y$ are chosen so as to stabilize the system

\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= K_y \begin{bmatrix} x_1 \\ x_2 \\ \xi_y \end{bmatrix} + \Gamma \xi_y, \\
\dot{x}_3 &= x_4, \\
\dot{x}_4 &= K_x \begin{bmatrix} x_3 \\ x_4 \\ \xi_x \end{bmatrix} + \Gamma \xi_x, \\
\dot{x}_5 &= x_6, \\
\dot{x}_6 &= K_z \begin{bmatrix} x_5 \\ x_6 \\ \xi_z \end{bmatrix} + \Gamma \xi_z,
\end{align*}
\]

The final controller is given by (5.7), (5.10), and (5.11). Its block diagram is depicted in Figure 5.19.

### 5.6 Experimental Results of 3-DOF Controller Implementation

In this section we present the experimental results obtained using the controller designed in the previous section. We also compare the performance of the nonlinear controller to that of a linearization-based output regulator and a gain scheduling controller.
5.6.1 Set-point Stabilization

We choose $K_x$ and $K_Z$ by placing the poles of both the $(x_3, x_4, \xi_x)$ and $(x_5, x_6, \xi_z)$ subsystems at $[-31.6, -2.8 \pm 5.6i, -2.1 \pm 1.7i]$, while the poles of the $(x_1, x_2, \xi_y)$ subsystem are placed at $[-129.1, -3.3 \pm 5.8i, -2.2 \pm 1.9i]$. We obtain

$$K_x = \begin{bmatrix} -354 & -41 & -8945 & -1201 & -1472 \end{bmatrix},$$

$$K_z = \begin{bmatrix} -354 & -41 & -8945 & -1201 & -1472 \end{bmatrix},$$

$$K_y = \begin{bmatrix} -1487 & -140 & -50001 & -560 & -7836 \end{bmatrix}.$$

Figure 5.20 depicts the response of the $y$-axis states to a series of step commands, as well as the absolute $y$-axis positioning accuracy. Figures 5.21 and 5.22 present the same response results for the $x$ and $z$-axis states. Finally, Figure 5.23 displays the amount of current input supplied by the controller to the PMLSM pairs in the 3-DOF contactless positioning apparatus in order to achieve these results.

Notice that the overshoots are consistently below 30% and the tracking error gets below 0.1mm in less than 3 seconds. However, while the $y$-axis tracking error reaches the
Figure 5.20: Plots of $y$-axis state responses to the set-point commands

Figure 5.21: Plots of $x$-axis state responses to the set-point commands
Figure 5.22: Plots of z-axis state responses to the set-point commands

Figure 5.23: 3-DOF currents required in each PMLSM pair for set-point commands
encoder resolution of 0.1µm in less than 5 seconds, the x and z-axis tracking errors fail to meet this particular performance specification. When trying to tune the controller gains in order to speed up the transient (at the expense of increased overshoot), the result is failure of the nonlinear controller to stabilize some of the larger step commands. It is then clear that something in the system prevents one from trading-off transient speed with overshoot (which proved to be possible in Chapter 4). In the next section we investigate the source of this problem.

5.6.2 Improving Nonlinear Step Responses Using Antiwindup

The problem raised in the previous section was traced to the two key performance limitations within the linear current amplifiers used to power the PMLSM pairs:

1. When only one of either the direct or quadrature inputs $i_q, i_d$ are supplied to an individual PMLSM pair with the other at zero, the corresponding LCAM is limited to supply about $4 - 5A$ continuous current before saturating.

2. When the direct and quadrature inputs $i_q, i_d$ are not zero and one of them, say $i_q$, is commanded to be greater than $3 - 4A$, $i_d$ experiences a drop from its commanded value.

It is clear from Figure 5.23 that aggressive controller commands may easily demand currents that produce the aforementioned control signal saturation and drop (especially at larger air-gap values) resulting in instability. This is the reason why the nonlinear controller cannot be tuned to exert more aggressive control action to decrease the time it takes the tracking error to reach encoder resolution. An obvious solution to this problem would be to saturate the output of the nonlinear controller, but this would significantly affect the transient performance of the system. To avoid this problem, we design an antiwindup mechanism around the saturation. The antiwindup methodology we use is
developed in [14] and [5], and is a generalization of the traditional integrator antiwindup methodology which is applicable to general LTI state-space models.

To implement the antiwindup compensation we begin by saturating the control input \( u_1, u_2, \) and \( \bar{u} \) so that they lie within the range \([-4A, 4A]\). In other words, defining

\[
sat(u) = \begin{cases} 
  u & |u| \leq 4 \\
  4 & u > 4 \\
  -4 & u < -4
\end{cases}
\]

we replace \( u_1, u_2, \) and \( \bar{u} \) by \( sat(u_1), sat(u_2), \) and \( sat(\bar{u}) \), respectively. Next, we apply the antiwindup mechanism to the state of the internal models. To do this, we replace (5.10) by

\[
\begin{align*}
\dot{\xi}_y &= \phi \xi_y + Ne_1 + E_{cy}[sat(\bar{u}) - \bar{u}], \quad y_3 = \Gamma \xi_y, \\
\dot{\xi}_x &= \phi \xi_x + Ne_2 + E_{cx}[sat(u_2) - u_2], \quad y_2 = \Gamma \xi_x, \\
\dot{\xi}_z &= \phi \xi_z + Ne_3 + E_{cz}[sat(u_1) - u_1], \quad y_1 = \Gamma \xi_z,
\end{align*}
\]  

(5.13)

where \( E_{cy}, E_{cx}, E_{cz} \in \mathbb{R}^{3 \times 1} \) represent antiwindup compensation gains which have to be tuned. The antiwindup compensators incorporated into (5.13) act as dead-zone nonlinearities within the nonlinear controller, vanishing when the control signals are within the saturation limits and reducing integrator windup when they are not. We tune the antiwindup gains based on the procedure that follows.
Procedure 5: Implementation of Antiwindup Compensation

1. The nonlinear controller gains $K_y$, $K_x$ and $K_z$ are designed for aggressive transient performance under the assumption that no LCAM actuator saturation is present.

2. The antiwindup compensation from (5.13) and the aforementioned saturation limits are incorporated into the nonlinear controller.

3. The antiwindup gains $E_{cy}$, $E_{cx}$ and $E_{cz}$ are manually adjusted through experimentation in order to regain the desired controller performance despite the presence of the controller saturation limits.

We design $K_x$ and $K_z$ to place the poles of both the $(x_3, x_4, \xi_x)$ and $(x_5, x_6, \xi_z)$ subsystems at $[-5, -6, -7, -8, -9]$. We design $K_z$ to place the poles of the $(x_1, x_2, \xi_y)$ subsystem at $[-91.3, -4.2 \pm 6.4i, -2.3 \pm 2.0i]$. This results in the gains

$$K_x = K_z = [-463, -35, -15121, -997, -2548],$$

$$K_y = [-1274, -104, -50001, -3936, -7774].$$

By manual tuning we get the following antiwindup gains

$$E_{cy} = \begin{bmatrix} 1e-5 \\ 1e-5 \\ 1e-5 \end{bmatrix}, \quad E_{cx} = \begin{bmatrix} 1.1e-4 \\ 1.1e-4 \\ 1.1e-4 \end{bmatrix}, \quad E_{cz} = \begin{bmatrix} 1.25e-4 \\ 1.25e-4 \\ 1.25e-4 \end{bmatrix}. \quad (5.15)$$

With the modified controller gains and antiwindup compensation, the same set-points commands from Section 5.6.1 are applied to the system. Figure 5.24 compares the step responses to what was obtained in the previous section.

The results demonstrate that the adjusted controller gains and the incorporation of antiwindup significantly reduce the transient time of the nonlinear controller in the $x$ and $z$-axis responses, with each now reaching encoder resolution in under 5 seconds. The responses exhibit increased overshoot, and depending on the application either overshoot
or settling time may be a more important factor. The results show that either requirement can be accommodated for using the 3-DOF nonlinear controller. The effects of the antiwindup compensation are quite evident when the improved 3-DOF nonlinear controller step responses are measured with and without antiwindup compensation. The comparison is displayed Figure 5.25. When antiwindup compensation is removed, the 3-DOF nonlinear controller exhibits a noticeable increase in overshoot before eventually failing on one of the larger step responses.

It is important to note that [14] and [5] provide procedures to estimate the stability region of the antiwindup compensation based on solving a series of linear matrix inequalities (LMI). Due to time constraints and the lack of a functioning LMI solver, we did not investigate this issue.

5.6.3 Comparing Nonlinear and Linear Step Responses

In this section we compare the performance of our nonlinear controller to a linearization-based output regulator and later to a gain scheduling controller. We begin by comparing
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Figure 5.25: Improved 3-DOF nonlinear controller steps with and without antiwindup

the nonlinear controller without antiwindup to its linearization-based counterpart. In

designing the linear controller we adopt the same “fairness” criteria outlined in Section 4.6.1, page 54. A series of set-points commands of increasing amplitude are applied to the system using the nonlinear and linear controllers.

Figure 5.26 displays the performance of the two controllers for the first step command, starting from \([x_1, x_2, x_3, x_4, x_5, x_6]^T = [0.029 \text{ m}, 0, 0, 0, 0, 0]^T\) and terminating at \([x_1, x_2, x_3, x_4]^T = [0.027 \text{ m}, 0, 0.005 \text{ m}, 0, 0.005 \text{ m}, 0]^T\). The results show that while both controllers are successful, the linear response suffers from a longer transient.

Figure 5.27 displays the performance of the 3-DOF controllers for the second step command, starting from \([x_1, x_2, x_3, x_4, x_5, x_6]^T = [0.029 \text{ m}, 0, 0, 0, 0, 0]^T\) and terminating at \([x_1, x_2, x_3, x_4]^T = [0.025 \text{ m}, 0, 0.010 \text{ m}, 0, 0.010 \text{ m}, 0]^T\).

While the 3-DOF nonlinear controller is successful, the linear controller exhibits excessive instability. Although the linear controller does eventually settle to the desired set-point, the initial instability is so large that the platform hits against the vertical stopper limits. So despite the recovery, the linear controller has failed in a practical sense,
Figure 5.26: Comparison of the nonlinear and linear 3-DOF controllers (first step)

Figure 5.27: Comparison of the nonlinear and linear 3-DOF controllers (second step)
and therefore no further step response comparisons are needed for the present controller configurations. Recalling the original results from Chapter 4, it is evident that the linear controller in its present form performs much worse than it did in the 2-DOF case. This is due to the same LCAM actuation limitations discussed in the previous section.

To overcome this problem, both nonlinear and linearization-based controllers are augmented with a saturation plus antiwindup mechanism, as outlined in Procedure 5. The gains of the nonlinear controller are chosen as in (5.14). The gains of the linearization-based controller are modified accordingly. The results for the first step command are illustrated in Figure 5.28. Figure 5.29 depicts the controller responses to the second step command.

Although both step commands confirm that the nonlinear controller outperforms its linear counterpart, the linear controller performs better and no longer fails on the second step command. Figure 5.30 depicts the response of the two controllers for the third and final step command, starting from $[x_1, x_2, x_3, x_4, x_5, x_6]^T = [0.029 \text{ m}, 0, 0, 0, 0, 0]^T$ and terminating at $[x_1, x_2, x_3, x_4]^T = [0.023 \text{ m}, 0, 0.015 \text{ m}, 0, 0.015 \text{ m}, 0]^T$. For this larger
step command, the nonlinear controller is successful while the linear controller hits the stopper limits, resulting in practical failure.

Overall, the comparisons demonstrate the superior performance of the nonlinear controller over its linear counterpart, even when antiwindup is implemented. Not only does the 3-DOF nonlinear controller exhibit a better transient behavior, but it also displays the ability to perform set-point stabilization over a wider 3-DOF operating range.

We next compare the nonlinear controller performance to that of a linear controller with very basic gain-scheduling. The gain-scheduling is such that the controller switches between the linear gains employed in the previous experiments according to a set of air-gap operating regions illustrated in table 5.4. Figure 5.31 shows the results of the comparison with the nonlinear controller.

Despite the incorporation of gain-scheduling, the linear controller still suffers from instability, failing on several set-points. However, it must be noted that the gain-scheduling design was very crude. It is likely that a more rigorous gain-scheduling design would have produced better results, and this could be pursued in future research.
Figure 5.30: Improved nonlinear and linear 3-DOF controllers (third step)

Table 5.4: Switching characteristics of 3-DOF gain-scheduling controller

<table>
<thead>
<tr>
<th>Region</th>
<th>Air-gap Range (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$x_1 &gt; 0.026$</td>
</tr>
<tr>
<td>2</td>
<td>$0.024 \leq x_1 \leq 0.026$</td>
</tr>
<tr>
<td>3</td>
<td>$0.022 \leq x_1 &lt; 0.024$</td>
</tr>
<tr>
<td>4</td>
<td>$x_1 &lt; 0.022$</td>
</tr>
</tbody>
</table>
We conclude this section by comparing the performance of the nonlinear controller to that of the PID regulators used in the system identification procedure. Figure 5.32 demonstrates a successful step using the PID regulators. The response, although successful, suffers from poor transient behavior (particularly in the $y$-axis range).

Figure 5.33 shows that for a larger step command the PID regulators fail in performing the desired $y$-axis set-point stabilization.

5.6.4 Sinusoidal Tracking

For each of the sinusoidal tracking experiments, the $x$ and $z$-axis positions of the 3-DOF platform are each actuated using a sinusoidal reference with an amplitude of 30 mm and a frequency of $1.5\pi$ rad/sec. At the same time, The $y$-axis position is simultaneously actuated using a sinusoidal reference of amplitude 5 mm, an offset of 25 mm, and a frequency of $1.5\pi$ rad/sec. The nonlinear controller employs the gains $K_x$, $K_z$ and $K_y$, in (5.12). Figures 5.34, 5.35, and 5.36 illustrate the tracking responses and absolute tracking errors obtained for each of the $y$, $x$ and $z$ reference commands, respectively.
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Figure 5.32: Step response of 3-DOF PID regulators (small step)

Figure 5.33: Step response of 3-DOF PID regulators (large step)
Figure 5.34: $y$-axis tracking response and error using 3-DOF nonlinear controller

Figure 5.35: $x$-axis tracking response and error using 3-DOF nonlinear controller
The x and z-axis tracking errors average at about 0.12 mm and 0.13 mm respectively with occasional peaks of 0.4 mm at random intervals. The y-axis tracking error average at 59 µm with occasional peaks of 0.2 mm. Each tracking error settles around the average value in less than 3 seconds. It is worth noticing that the y-axis tracking performance is drastically improved when compared to the vertical tracking error obtained with the 2-DOF prototype. This result is not surprising, as the 3-DOF hardware is the result of a more rigorous design process and is constructed with a better set of linear guides.

When the 3-DOF nonlinear controller tracking responses are compared to the basic PID regulators used in the system identification experiments, the performance differences are more dramatic than in the 2-DOF experiments. The results using PID regulators are shown in Figures 5.37, 5.38 and 5.39.

Using the 3-DOF PID regulators, the average x and z-axis tracking error is now about 1.64 mm, while the y-axis tracking error approaches 1.44 mm. So it is clear then that even with the pre-filtering advantage, the best error values are still over 10 times greater than what was produced with the 3-DOF nonlinear controller.
Figure 5.37: $y$-axis tracking response and error using 3-DOF PID regulators

Figure 5.38: $x$-axis tracking response and error using 3-DOF PID regulators
Figure 5.39: z-axis tracking response and error using 3-DOF PID regulators
Chapter 6

Conclusions

6.1 Thesis Summary

This thesis undertook the implementation and evaluation of two high precision positioning systems based on contactless magnetic levitation generated with iron-core PMLSMs.

The first apparatus was constructed by Quanser Consulting and based on a single PMLSM which allowed for 2-DOF motion. The 2-DOF apparatus was implemented as a proof of concept testbed for the theoretical results from [16]. The second apparatus was also constructed by Quanser Consulting and based on a set of four PMLSMs which allowed for 3-DOF motion. The 3-DOF apparatus was implemented as a more advanced second-generation successor to the original 2-DOF prototype.

Contactless positioning can be applied to any application requiring high-speed precision movement over multiple DOF. An example would be the manufacturing of semiconductors. As these components undergo future size reductions, contactless positioning has the potential to meet accuracy requirements while reducing maintenance costs and the introduction of impurities. And while contactless positioning is currently being developed by other sources, the use of industry standard iron-core PMLSMs makes the technology much more suitable for eventual incorporation into the manufacturing environment.
The main results of this thesis are as follows:

- The theoretical model and controller derivations from [16] provided a good starting point for this research, but could not be implemented directly onto the actual 2-DOF and 3-DOF positioning devices because of their idealized natures that did not take into account uncertain model parameters and unmodelled forces.

- A system identification procedure was implemented and applied to the estimation of the unknown or not perfectly known parameters within the 2-DOF and 3-DOF models. The procedure was taken from [9] and was capable of estimating time invariant parameters that entered linearly into nonlinear systems. Implementation required PID regulators to generate sufficient persistency of excitation, as well as high-gain observers to derive the full state vector.

- A series of model verification experiments were applied to the 2-DOF positioning hardware in order to determine the accuracy of the 2-DOF state-space model once it had been enhanced with estimated parameters. It was found that the enhanced 2-DOF model could accurately predict the behavior of the physical system in an air-gap range between 15 and 25 mm and a horizontal range between $-50$ and 50 mm. For air-gaps smaller than 15 mm, the effects of the cogging force became too significant to ignore.

- The verified 2-DOF model was used to implement a nonlinear controller based on feedback linearization and output regulation. Experimental testing demonstrated that the nonlinear controller was capable of performing set-point stabilization to an accuracy of 0.1 mm, as well as sinusoidal tracking with a horizontal tracking error of about 0.1 mm and a vertical tracking error of approximately 0.2 mm. The nonlinear controller was also found to outperform both its linear counterpart and the original PID regulators.
Chapter 6. Conclusions

- A series of similar model verification experiments were applied to determine the accuracy of the 3-DOF hardware state-space model enhanced with estimated parameters. It was found that the enhanced 3-DOF model could predict the behavior of the physical system in a $y$-axis range between 20 and 30 mm and a $x$ & $z$-axis range between $-50$ and 50 mm. Despite the insignificant effects of the cogging force, power supply limitations combined with uncontrollable pitching and rolling at smaller air-gaps prevented a larger $y$-axis operating region.

- The verified 3-DOF model was used to implement the same type of nonlinear controller on the 3-DOF hardware as in the original 2-DOF case. Experimental testing demonstrated that, with the help of some antiwindup compensation, the 3-DOF nonlinear controller was capable of performing set-point stabilization to an accuracy of $10 \, \mu m$. Sinusoidal tracking was also achieved using the 3-DOF nonlinear controller with exceptional $y$, $x$ and $z$ tracking error. It was also demonstrated that the nonlinear controller could outperform a 3-DOF linear counterpart (with and without gain-scheduling), as well as the 3-DOF PID regulators.

6.2 Contributions of Thesis Work

The main contribution of this thesis has been to demonstrate the feasibility of implementing a multiple degree-of-freedom high-precision contactless positioning system based on iron-core PMLSMs. The 2-DOF hardware was able of prove the concepts laid out in the original theoretical work from [16], while the 3-DOF hardware demonstrated that future prototypes have the potential to achieve continued performance improvements.

Other important contributions included the implementation of effective system ID and model verification procedures that can be used in future contactless positioning experiments. The antiwindup compensation implemented within the 3-DOF nonlinear controller should also prove useful if actuator saturation remains a problem.
6.3 Future Work

The following are suggested areas that could be pursued in future research:

- Suppose the existing 3-DOF hardware were modified to allow for the independent control of each PMLSM. If some additional sensors were incorporated in conjunction with sufficient LCAM-based power capability, it would be possible to perform limited 5-DOF control experiments without building a new positioning apparatus. The result would be the elimination of the uncontrollable pitching and rolling, allowing for an increase in the feasible $y$-axis range of motion.

- At some point, a third generation 5-DOF positioning system will be constructed with the linear guides completely eliminated, resulting in true contactless operation. Such an apparatus will present some challenging design issues. The most significant of which will be the development of a radically different sensing system, since the lack of linear guides will mean that the existing encoders can no longer be used.

- The existing system identification technique can only be applied to estimate time invariant parameters. As a result, all estimation experiments had to be carried out offline before any control experiments could be undertaken. It would be extremely useful if an online system identification technique could be employed directly within future controller designs. The advantage would be that, in the event of any sudden change in the model parameters, the controller would have the capability of compensating without failure or shutdown.

- The focus in both the 2-DOF and 3-DOF implementations was the nonlinear controller based on feedback linearization and output regulation. Future work could investigate other nonlinear controller implementations. Other possibilities include adaptive control, linear controllers based on more sophisticated gain-scheduling, or even some form of hybrid control.
# Appendix A

## Symbol Description

- $b_0$: Slot width
- $B_{pmyl}$: Permanent magnet flux density
- $d$: Mover displacement relative to stator
- $F_x, F_z$: Translational Force
- $F_y$: Normal Force
- $G$: Gravitational constant
- $g$: Length of the air gap
- $g_e$: Effective length of the air gap
- $h_b$: Height of the back iron on the movers
- $H_c$: Permanent magnet coercivity
- $h_m$: Permanent magnets height
- $i_a, i_b, i_c$: 3-Phase current
- $i_d, i_q$: Direct & quadrature current
- $K_c$: Carter’s coefficient
- $k_{wn}$: Winding factor
- $L_A$: Length of the poles
- $M_h$: Horizontal Mass of 2-DOF platen
- $M_v$: Vertical Mass of 2-DOF platen
- $M_x$: X-axis Mass of 3-DOF platen
- $M_y$: Y-axis Mass of 3-DOF platen
- $M_z$: Z-axis Mass of 3-DOF platen
- $p$: Number of pole pairs in each stator
- $p_m$: Number of permanent magnets
- $t_1$: Slot pitch
- $W$: Number of turns of wire per phase
- $w_c$: Coil pitch
- $z_1$: Number of stator slots per motor
- $\gamma$: NPFSI attenuation factor
- $\varepsilon$: NPFSI confidence level
- $\lambda$: Relative permeance
- $\mu_0$: Permeability of the free space
- $\mu_r$: Relative permeability
- $\mu_{rec}$: Relative PM recoil permeability
- $\sigma_m$: Surface magnetic charge
- $\tau$: Permanent magnet pole pitch
- $\tau_p$: Permanent magnet pole arc
Appendix B

Acronyms

AC  Alternating current
CSA  Canadian Space Agency
DC  Direct current
DOF  Degree(s) of freedom
FSDI  Full state derivative information
LCAM  Linear current amplifier module
LIM  Linear induction motor
LMI  Linear matrix inequality
LQR  Linear quadratic regulator
LSM  Linear synchronous motor
NPFSI  Noise perturbed full state information
NSERC  National Science and Engineering Research Council of Canada
PCI  Peripheral component interface
PID  Proportional-integral-derivative
PM  Permanent magnet
PMLSM  Permanent magnet linear synchronous motor
Appendix C

Setup of PMLSM Commutation

Before any of the original 2-DOF and 3-DOF controllers can be implemented, it is crucial that each of the PMLSMs be subjected to proper 3-phase commutation. Without this commutation, it is not possible to fully actuate the 2-DOF and 3-DOF platforms over their respective ranges of operation.

Recall that a PMLSM produces a translational and normal force through interactions between the magnetic fields produced by the permanent magnets of the mover and the 3-phase coils of the stator. When a constant positive or negative current is applied to each phase of the stator, the 3-phase windings generate the magnetic orientations described in table C.1. Note that these orientations are relative to the mover.

Table C.1: Magnetic orientations of 3-phase stator windings

<table>
<thead>
<tr>
<th>Current Type</th>
<th>Phase-u</th>
<th>Phase-v</th>
<th>Phase-w</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive Current</td>
<td>North</td>
<td>North</td>
<td>South</td>
</tr>
<tr>
<td>Negative Current</td>
<td>South</td>
<td>South</td>
<td>North</td>
</tr>
</tbody>
</table>
The "uvw" phase designations corresponded to the original manufacturer labelling. They related to the "abc" phase designations from the thesis in the sense that the $i_a$ current was connected to phase-w, $i_b$ to phase-u, and $i_c$ to phase-v.

Throughout the thesis experiments, each controller is applied to the $i_d$ and $i_q$ inputs of the PMLSMs. Commutation is then achieved by applying the inverse of (2.1) to the $i_d$ and $i_q$ inputs, resulting in the required 3-phase current supply.

In order for the mover of each PMLSM to be capable of maximum translational motion in both the positive and negative directions, the translational origin needs to be setup to align the permanent magnets of the mover at the exact center of the stator structure. This default position is illustrated in figure C.1.

![Figure C.1: Default translational origin of mover relative to stator](image)

Regardless of what initial position the mover of a PMLSM is started from, the encoder sensors are designed by the manufacturer to automatically reset and interpret this position as being a translational displacement of 0 mm.

Because of the magnetic orientations from table C.1, the required initial alignment from figure C.1 and the sensor reset, the current transformation based on (2.1) does not generate the proper 3-phase commutation. The problem was solved by modifying the original current relations to incorporate an offset, as depicted in (4.4). The subsections that follow depict how this modified transformation generates proper commutation.
C.1 Translational Commutation

Consider the inverse of the modified 3-phase current transformation from (4.4) shown below (with the ”abc” currents assigned to the ”uvw” phases of the manufacturer)

\[ i_a = i_w = \cos(\frac{\pi}{\tau}(d - \frac{\tau}{2}))i_d - \sin(\frac{\pi}{\tau}(d - \frac{\tau}{2}))i_q, \]
\[ i_b = i_u = \cos(\frac{\pi}{\tau}(d - \frac{\tau}{2}) - \frac{2\pi}{3})i_d - \sin(\frac{\pi}{\tau}(d - \frac{\tau}{2}) - \frac{2\pi}{3})i_q, \]
\[ i_c = i_v = \cos(\frac{\pi}{\tau}(d - \frac{\tau}{2}) + \frac{2\pi}{3})i_d - \sin(\frac{\pi}{\tau}(d - \frac{\tau}{2}) + \frac{2\pi}{3})i_q. \]  

(C.1)

To focus on translational commutation, the \(i_q\) portion of (C.1) is isolated as follows

\[ i_w = -\sin(\frac{\pi}{\tau}(d - \frac{\tau}{2}))i_q, \]
\[ i_u = -\sin(\frac{\pi}{\tau}(d - \frac{\tau}{2}) - \frac{2\pi}{3})i_q, \]
\[ i_v = -\sin(\frac{\pi}{\tau}(d - \frac{\tau}{2}) + \frac{2\pi}{3})i_q. \]  

(C.2)

Using standard trigonometric identities, equation (C.2) becomes

\[ i_w = \cos(\frac{\pi}{\tau}d)i_q, \]
\[ i_u = \cos(\frac{\pi}{\tau}d - \frac{2\pi}{3})i_q, \]
\[ i_v = \cos(\frac{\pi}{\tau}d + \frac{2\pi}{3})i_q. \]  

(C.3)

Recall the translational force model from back in chapter 2. The model indicates that for a fixed air-gap and a constant \(i_q\) input, the mover should accelerate at a constant rate, resulting in a parabolic position response. It is therefore important that the transformation from (C.3), with the chosen offset of \(\tau/2\), produce a translational commutation within the stator PMLSM coils that allows for this constant acceleration across the complete range of operation.

Table C.2 demonstrates a few of the magnetic orientations produced in the coil phases, using the translational commutation from (C.3). The mover is accelerated in the positive direction, with a constant current of \(i_q = -I\), for displacements between \([0, \tau]\).

Figure C.2 illustrates that the resulting magnetic orientations of the stator relative to the mover displacement produce the proper commutation for constant acceleration.
Table C.2: Magnetic orientations produced using translational commutation

<table>
<thead>
<tr>
<th>d</th>
<th>$i_u$</th>
<th>$i_w$</th>
<th>$i_v$</th>
<th>U</th>
<th>W</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\frac{i}{2}$</td>
<td>$-I$</td>
<td>$\frac{i}{2}$</td>
<td>North</td>
<td>North</td>
<td>North</td>
</tr>
<tr>
<td>$\frac{\tau}{3}$</td>
<td>$-\frac{I}{4}$</td>
<td>$-\frac{I}{\sqrt{2}}$</td>
<td>$I$</td>
<td>South</td>
<td>North</td>
<td>North</td>
</tr>
<tr>
<td>$\frac{2\tau}{3}$</td>
<td>$-I$</td>
<td>$\frac{I}{\sqrt{2}}$</td>
<td>$\frac{I}{4}$</td>
<td>South</td>
<td>South</td>
<td>North</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$-\frac{I}{2}$</td>
<td>$I$</td>
<td>$-\frac{I}{2}$</td>
<td>South</td>
<td>South</td>
<td>South</td>
</tr>
</tbody>
</table>

Figure C.2: Illustration of proper translational commutation relative to mover position
C.2 Normal Commutation

Focussing now on the commutation relating to the normal force, the $i_d$ portion of the current transformation from (C.1) is isolated as follows

\[
\begin{align*}
    i_w &= \cos\left(\frac{\pi}{\tau}(d - \frac{\tau}{2})\right)i_d, \\
    i_u &= \cos\left(\frac{\pi}{\tau}(d - \frac{\tau}{2}) - \frac{2\pi}{3}\right)i_d, \\
    i_v &= \cos\left(\frac{\pi}{\tau}(d - \frac{\tau}{2}) + \frac{2\pi}{3}\right)i_d.
\end{align*}
\] (C.4)

Using standard trigonometric identities, equation (C.4) becomes

\[
\begin{align*}
    i_w &= \sin\left(\frac{\pi}{\tau}d\right)i_d, \\
    i_u &= \sin\left(\frac{\pi}{\tau}d - \frac{2\pi}{3}\right)i_d, \\
    i_v &= \sin\left(\frac{\pi}{\tau}d + \frac{2\pi}{3}\right)i_d.
\end{align*}
\] (C.5)

Several thesis experiments show that when a constant $i_d$ current is applied to either contactless positioning device, the platform under consideration can be held in place by the PMLSM magnetic fields at some equilibrium air-gap position. It is important to ensure that the transformation from (C.5), with the chosen offset of $\tau/2$, produce a normal commutation within the PMLSM coils such that the levitation equilibrium can be maintained despite any translational displacement of the mover.

Table C.3 demonstrates a few of the magnetic orientations produced in the coil phases, using the normal commutation from (C.5). The resulting magnetic orientations assumes that the platform is held in place at a low enough air-gap to require a constant negative current of $i_d = -I$. At a higher air-gap, the results are similar, but with the magnetic orientations reversed. The orientation data corresponds to a mover displacement between $[0, \tau]$. Note also that the table symbol $\emptyset$ signifies no magnetic field.

Figure C.3 illustrates that the resulting magnetic orientations of the stator relative to the mover position produce the proper normal commutation to ensure levitation equilibrium in the presence of translational displacement.
Table C.3: Magnetic orientations produced using normal commutation

<table>
<thead>
<tr>
<th>d</th>
<th>$i_u$</th>
<th>$i_w$</th>
<th>$i_v$</th>
<th>U</th>
<th>W</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>0</td>
<td>$-\frac{\sqrt{3}}{2}$</td>
<td>North</td>
<td>$\emptyset$</td>
<td>South</td>
</tr>
<tr>
<td>$\frac{\tau}{3}$</td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>$-\frac{\sqrt{3}}{2}$</td>
<td>0</td>
<td>North</td>
<td>North</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$\frac{2\tau}{3}$</td>
<td>0</td>
<td>$-\frac{\sqrt{3}}{2}$</td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>$\emptyset$</td>
<td>North</td>
<td>North</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$-\frac{\sqrt{3}}{2}$</td>
<td>0</td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>South</td>
<td>$\emptyset$</td>
<td>North</td>
</tr>
</tbody>
</table>

Figure C.3: Illustration of proper normal commutation relative to mover position
C.3 Full PMLSM Commutation

Based on the results presented for the translational and normal commutation, it is possible to obtain an intuitive sense of how the full PMLSM commutation allows for the proper functioning of the 2-DOF and 3-DOF positioning devices.

Consider again the full 3-phase current transformation from (C.1) that is used to generate full PMLSM commutation. As discussed, the $i_q$ portion of (C.1) generates the translational commutation required for constant acceleration of the mover over the complete translational range of operation, while the $i_d$ portion of (C.1) generates the normal commutation required to maintain uniform levitation in the presence of translational mover displacement. When the normal commutation is superimposed over the translational commutation, the translational acceleration is still achieved. At the same time, the normal commutation produces enough magnetic imbalance in the phases of the PMLSM stator to allow for the required attractive (or equivalently repulsive) levitation force. With full PMLSM commutation in place, the controllers developed within the thesis can actuate each positioning system by varying the $i_q$ and $i_d$ currents.

The importance of proper PMLSM commutation has therefore been made clear. If (C.1) does not produce a correct 3-phase current transformation of the $i_q$ and $i_d$ control inputs, it is not possible to actuate the PMLSMs within each positioning system over the complete translational range of motion. There would reach a point in the displacement where the magnetic orientations of the PMLSM stator become out of synchronization with the PMLSM mover, resulting in complete system failure regardless of what type of controller is implemented.
Appendix D

List of MATLAB and Simulink Experimentation Files

The MATLAB and Simulink files used throughout the thesis experimentation are summarized in the tables that follow. This may prove useful for any future work that may be undertaken.

Table D.1: Miscellaneous Experimental Files

<table>
<thead>
<tr>
<th>File</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1m_coef.m</td>
<td>Numerical Approximation of $B_{pmy1}$ for 2-DOF experiments</td>
</tr>
<tr>
<td>b1m_coef_3dof.m</td>
<td>Numerical Approximation of $B_{pmy1}$ for 3-DOF experiments</td>
</tr>
<tr>
<td>init_systemID_3dof.m</td>
<td>General Initialization file for 3-DOF system ID</td>
</tr>
<tr>
<td>parameters_2dof.m</td>
<td>2-DOF model parameter initialization file</td>
</tr>
<tr>
<td>parameters_3dof.m</td>
<td>3-DOF model parameter initialization file</td>
</tr>
</tbody>
</table>
## Table D.2: Files used in 2-DOF system ID & model verification

<table>
<thead>
<tr>
<th>File</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CalibrationProcedure_2DOF.mdl</td>
<td>Estimates $C_1$, $C_2$, $C_3$, $C_4$ Simultaneously</td>
</tr>
<tr>
<td>Cogging_Estimator.mdl</td>
<td>Obtains cogging force data for procedure 3</td>
</tr>
<tr>
<td>Estimator1_FixedAirGap.mdl</td>
<td>Estimates $L_1$ at fixed air-gap</td>
</tr>
<tr>
<td>Estimator1_FixedHorizontal.mdl</td>
<td>Estimates $C_2$, $C_3$ and $C_4$ at fixed $x_3$</td>
</tr>
<tr>
<td>Analyze_HorizontalID.m</td>
<td>Generates plots relating to procedure 1</td>
</tr>
<tr>
<td>Analyze_VerticalID.m</td>
<td>Generates plots relating to procedure 2</td>
</tr>
<tr>
<td>curvefit.m</td>
<td>Used to fit parabola to horizontal position data</td>
</tr>
<tr>
<td>Offset_Test.m</td>
<td>Generates comparison between $x_3$ offset positions</td>
</tr>
<tr>
<td>Offset_Test3.m</td>
<td>Generates cogging force plot for procedure 3</td>
</tr>
</tbody>
</table>

## Table D.3: Files used in 2-DOF control experiments

<table>
<thead>
<tr>
<th>File</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>FinalController_2DOF_MAGLEV.mdl</td>
<td>2-DOF nonlinear controller implementation</td>
</tr>
<tr>
<td>LinearServo_2DOF_MAGLEV.mdl</td>
<td>2-DOF linear controller implementation</td>
</tr>
<tr>
<td>SystemID_AcquireData.mdl</td>
<td>2-DOF PID controller implementation</td>
</tr>
<tr>
<td>Get_Maglev_Data.m</td>
<td>Acquires system ID data using PID controller</td>
</tr>
<tr>
<td>initialize_fullcontroller.m</td>
<td>Initialization of 2-DOF nonlinear controller</td>
</tr>
<tr>
<td>initialize_linearservo.m</td>
<td>Initialization of 2-DOF linear controller</td>
</tr>
<tr>
<td>initialize_linearservo2.m</td>
<td>Initialization of 2-DOF linear controller</td>
</tr>
</tbody>
</table>
### Table D.4: Files used in 3-DOF system ID & model verification

<table>
<thead>
<tr>
<th>File</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CalibrationProcedure_3DOF.mdl</td>
<td>Estimates $C_1, \ldots, C_5$ Simultaneously</td>
</tr>
<tr>
<td>Cogging_Estimator_3DOF.mdl</td>
<td>Used to obtain 3-DOF cogging force data</td>
</tr>
<tr>
<td>Estimator1_3DOF_FixedAirGap.mdl</td>
<td>Estimates $L_1$ or $L_2$ at fixed air-gap</td>
</tr>
<tr>
<td>Estimator2_3DOF_FixedAirGap.mdl</td>
<td>Estimates both $L_1$ and $L_2$ at fixed air-gap</td>
</tr>
<tr>
<td>Estimator1_3DOF_FixedHorizontal.mdl</td>
<td>Estimates $C_3$, $C_4$, $C_5$ at fixed $x$ &amp; $z$</td>
</tr>
<tr>
<td>Analyze_HorizontalID_3DOF.m</td>
<td>Generates plots relating to procedure 1</td>
</tr>
<tr>
<td>Analyze_VerticalID_3DOF.m</td>
<td>Generates plots relating to procedure 2</td>
</tr>
<tr>
<td>curvefit_3DOF.m</td>
<td>Used to fit parabola to $x$ &amp; $z$ position data</td>
</tr>
<tr>
<td>Offset_Test_3DOF.m</td>
<td>Compares $x$ &amp; $z$ offset positions</td>
</tr>
<tr>
<td>Predict_CoggingForce_3DOF.m</td>
<td>Generates 3-DOF cogging force plot</td>
</tr>
</tbody>
</table>

### Table D.5: Files used in 3-DOF control experiments

<table>
<thead>
<tr>
<th>File</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain_Scheduling_Controller_3DOF.mdl</td>
<td>3-DOF gain-scheduling implementation</td>
</tr>
<tr>
<td>LinearServo_3DOF.mdl</td>
<td>3-DOF linear controller implementation</td>
</tr>
<tr>
<td>NonlinearController_3DOF.mdl</td>
<td>3-DOF nonlinear controller implementation</td>
</tr>
<tr>
<td>SystemID_AcquireData_3DOF.mdl</td>
<td>3-DOF PID controller implementation</td>
</tr>
<tr>
<td>initialize_GainSchedule_3DOF.m</td>
<td>Initialization of 3-DOF gain-scheduling</td>
</tr>
<tr>
<td>initialize_linearservo_3DOF.m</td>
<td>Initialization of 3-DOF linear controller</td>
</tr>
<tr>
<td>initialize_multilinearservo_3DOF.m</td>
<td>Initialization of 3-DOF linear controller</td>
</tr>
<tr>
<td>initialize_nonlinearcontroller_3DOF.m</td>
<td>Initialization of 3-DOF nonlinear controller</td>
</tr>
</tbody>
</table>
Bibliography


1.1
