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L_2 Nonlinear Control of EDFA System with Amplified Spontaneous Emission

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***Abstract*— Current control methods for Erbium Doped Fiber Amplifier (EDFA) devices lack a systematic approach to provide specified performance over the entire state space. Linearization techniques are combined with heuristic switching to attain operation around specific operating points. In this paper, a more inclusive model that takes Amplified Spontaneous Emission (ASE) into account is developed. Furthermore, a nonlinear L_2 control scheme is developed that eliminates the necessity of switching between multiple linear control approximations and replaces them by one nonlinear controller that is valid over the entire state space (or average inversion level).**

I. INTRODUCTION

Erbium Doped Fiber Amplifiers (EDFAs) are used to amplify the optical power in light signals traveling through optical cables. Signal amplification is necessary for long-haul fiber optic connections approximately every 80 to 100 kilometers [1][2]. Examples of some of these long-haul fiber optic connections include the networking of two cities separated by thousands of kilometers across a continent and intercontinental networks through transoceanic links.

An EDFA is an optical fiber that is a few meters in length that has been doped with Erbium Ions (Er^{3+})[3]. The Erbium ions absorb certain wavelengths of light energy to excite the atoms into higher energy states. The proportion of the Erbium atoms that are excited into the higher energy states is called the inversion level of the Erbium doped fiber. An atom that is not excited is at ground state. Incident signal photons will interact with the excited Er^{3+} ions to release their stored energy as photons in the 1540nm band [2]. The resulting released photon propagates with the same phase, polarity and direction as incident photon, hence, signal amplification is realized.

Amplified Spontaneous Emission (ASE) is a specific EDFA effect [2]. The excited Erbium atoms can release their energy spontaneously without the catalyst of an incident photon, resulting in photons with a random phase, polarization and direction. These random photons also become amplified by the same process that amplifies the input signal. Since ASE does not carry signal information, it contributes to noise. The effect becomes more pronounced in long connections of EDFAs because ASE can propagate and amplify itself.

Previous work on control applications to EDFA devices has ignored ASE [1][3][4][5]. However, it has been shown in [6] that ASE can have a significant effect on the output power in the EDFA. Large channel drops are one example.

Typical control schemes for EDFA devices are based on a simple system model with linear control methods as in [1][4][7]. This results in a number of defined equilibrium points where a controller is scheduled based on the measured input and output values of the system in real-time. The operating region over which a linear controller works can be very small. Having a linear controller in place for a nonlinear system can only give an approximate behaviour.

The output powers of each channel are directly affected by the average inversion level. If channel powers change the average inversion level will change. If the average inversion is kept constant, the output channel powers will remain constant [1][5]. We can minimize the variation of the average inversion through L_2 nonlinear control[8]-[12]. L_2 nonlinear control,like its linear counterpart[13]-[15], will ensure a direct attenuation between a disturbance input on the channel powers to the average inversion level. The necessity of EDFA device controllers that attenuate the changes in average inversion are further justified in [16] and [17] that show cascaded transients speed up with chains of EDFAs.

This paper is not only a further development of EDFA control, but it is a good example of a practical application of nonlinear theory[18] to a challenging model. The following will tackle a highly nonlinear and stiff[21] system, and provide it with a practical control solution. Taylor approximations will be utilized to achieve the final control results [19][8].

The paper is organized as follows. In Section II, some background on the definitions and concepts of L_2 nonlinear control theory is given. Sections III and IV derive the new EDFA model with ASE and L_2 control scheme, respectively. Section V presents the results of both the ASE model and the L_2 controller application. Section VI and VII present the conclusions and future work, respectively.

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II. L_2 NONLINEAR CONTROL BACKGROUND

L_2 nonlinear control will serve to attenuate input disturbance effects on the state and input variable. A system model is defined by a state equation that outlines the dynamics of the state variable, and an output equation. We include an additional performance output equation along with the measured output equation to apply L_2 control theory [9][8].

We define a simplified subproblem that is relevant to the development of our nonlinear controller. This subproblem is relevant to the system because of the potential availability of the state variable in practice. The Full Information Problem (FI) [9][12][15] is defined as:

$$\left\{\begin{array}{lllllll} \dot{x} &=& f(x) &+& g_1(x)w &+& g_2(x)u \\ z &=& h_1(x) &+& k_{11}(x)w &+& k_{12}(x)u \\ y &=& \begin{bmatrix} x \\ 0 \end{bmatrix} &+& \begin{bmatrix} 0 \\ I \end{bmatrix} w && \end{array}\right. \tag{1}$$

Here, w represents the disturbance input while u represents the system input. z represents the performance output that the designer would specify. The state is directly available for measurement.

For a nonlinear system as in (1)we say that the system has an L_2 gain,γ, if the following holds:

$$\int_0^T \|z(t)\|^2 dt \leq \gamma^2 \int_0^T \|w(t)\|^2 dt \tag{2}$$

where γ represents the amount of attenuation from the disturbance $\|w(t)\|$ to the defined variable $\|z(t)\|$.

The majority of the work developed herein is an application of the theoretical results developed in [8]-[12]. Theorem 3.1 of [8] (or see [9]) is the most important theorem in the analysis and is restated below as Theorem 1.

Theorem 1: The L_2 control objective is achievable for the FI problem, assuming $k_{12}(x)$ has full column rank, and $S_1(x) < 0 \quad \forall x \epsilon R^n$, where $S_1(x) = k_{11}(x)^T k_{11} - \gamma^2 I$ with feedback $u = F_{2\infty}(x)$ as long as:

1)$\exists V(x) \geq 0$, $V(0) = 0$ solution of $H_{FI}(V,x) \leq 0$ (3).

2)$\dot{x} = f(x) + g(x)F_{\infty}(x)$ is asymptotically stable, where:

$$\begin{aligned} g(x) &= [g_1(x) \quad g_2(x)] \\ F_{\infty}(x) &= \begin{bmatrix} F_{1\infty}(x) \\ F_{2\infty}(x) \end{bmatrix} \\ &= -S^{-1}(x)[\frac{1}{2} g^T(x) V_x^T(x) + k_1^T(x) h_1(x)] \\ S(x) &= k_1^T(x) k_1(x) - \begin{bmatrix} -\gamma^2 I & 0 \\ 0 & 0 \end{bmatrix} \\ k_1(x) &= [k_{11}(x) \quad k_{12}(x)] \end{aligned}$$

The Hamilton-Jacobi Inequality (HJI), $H_{FI}(V,x) \leq 0$ is defined as [9]:

$$H_{FI}(V,x) = V_x(x)f(x) - F_{\infty}^T(x)S(x)F_{\infty}(x) + h_1^T(x)h_1(x) \tag{3}$$

where $V_x(x)$ is equivalent to $\frac{dV(x)}{dx}$.

III. EDFA MODEL WITH AMPLIFIED SPONTANEOUS EMISSION

The basic mechanisms in an EDFA are stimulated emission, which amplifies the signal, and spontaneous emission, which causes noise. The pump signal is responsible for exciting the Erbium atoms, while channel signals interact with the atoms to release that energy in the form of coherent light. Amplified Spontaneous Emission (ASE) is the result of excited atoms that release this energy without stimulus from the input channel powers. This results in a random phase, polarity and direction for this new light, which causes noise[2]. The effects of ASE also compound as they propagate through chained EDFAs.

The system model developed in [4] describing the EDFA does not include any terms for ASE. The rate equation and output equations are

$$\begin{aligned} \dot{x} &= -\frac{x}{\tau} - \frac{1}{\rho s L} \sum_k [e^{[(g_k+\alpha_k)x - \alpha_k]L} - 1] Q_k^{in}(t) \\ Q_k^{out}(t) &= e^{[(g_k+\alpha_k)x-\alpha_k]L} Q_k^{in}(t) \end{aligned} \tag{4}$$

where $x = \int_0^L N_2(z,t)dz$ and $N_1(z,t) + N_2(z,t) = 1$. L is the length of the Erbium Doped fiber. x is the average inversion level of the EDFA and $N_1(z,t)$ and $N_2(z,t)$ are the ground state and excited Erbium ion concentrations compared to the total concentrations. τ is the spontaneous lifetime of the excited energy level. ρ represents the number density of active Erbium atoms. S is the fiber core cross section. Also, Q_k^{in} and Q_k^{out} represent the normalized input and output powers of the k^{th} EDFA channels respectively. They are called photon flux [3][4], and defined as $Q_k = \frac{P_k}{h\nu_k}$, where P_k denotes the power of the k^{th} beam of light in Watts.

In [3], the raw equations for EDFAs with ASE in PDE form are outlined. The equations by themselves can not be used for control. We will make certain key assumptions to simplify the general equations and a state space model with ASE will be developed. In [5], an EDFA model is derived that includes an ASE term, but it was not placed in a state space form, nor was the ASE term utilized. [20] derives a representation of the EDFA model with ASE in parallel with the model derived herein, but does not include the dynamic transient behaviour of the system for changes in state or input power.

A. EDFA System Variables

Let $n_i(r, \phi, z)$ for $i = 1, 2, t$ represent the ground state, excited state and total erbium ion populations, respectively. Here r is the radius, ϕ is the azimuth angle, and z is the distance along the EDFA fiber [3].

$P_k(z)$ will denote the power of the kth beam of light in Watts, or the channel power, as a function of distance along the EDFA fiber. The frequency of the light beam is indicated by ν_k, and centered at $\lambda_k = \frac{c}{\nu_k}$.

Among the important characteristic values that define the EDFA system, two of the more important are the absorption and gain spectra, α_k and g_k, respectively [3].

$$\alpha_k = \sigma_{ak} \int_0^{2\pi} \int_0^{\infty} i_k(r,\phi) n_t(r,\phi,z) r dr d\phi \quad (5)$$

$$g_k = \sigma_{ek} \int_0^{2\pi} \int_0^{\infty} i_k(r,\phi) n_t(r,\phi,z) r dr d\phi \quad (6)$$

$i_k = \frac{I_k(r,\phi,z)}{P_k(z)}$ is the normalized optical intensity, and $I_k(r,\phi,z)$ is the light intensity distribution. σ_{ak} and σ_{ek} are the absorption and emission cross sections, respectively.

B. Simplifying Assumptions

From [3], we can state some of the underlying standard assumptions used to simplify the EDFA model. First, Er^{3+} is assumed to be radially symmetric and decreasing monotonically from $r = 0$. Define b_{eff} as

$$b_{eff} = [\frac{1}{2} \int_0^{2\pi} \int_0^{\infty} \frac{n_t(r)}{n_t(0)} r dr d\phi]^{\frac{1}{2}} \quad (7)$$

so we get the average density of Erbium ion population as:

$$\bar{n_i}(z) = \frac{\int_0^{2\pi} \int_0^{\infty} n_i(r,\phi,z) r dr d\phi}{\pi b_{eff}^2} \text{ for } i = 1, 2, t \quad (8)$$

with this, the erbium population is now only a function of the distance along the fiber.

The above assumption can be taken one step further, such that the erbium populations are distributed uniformly in a disk of radius b, as in [3]. Also, to further simplify, we assume that the erbium populations are uniformly distributed along the length of the EDFA fiber as well. We are only interested in the average Erbium concentration as a whole. Thus, now n_i represents the average Erbium concentration along the radius, ϕ angle, and length of the entire EDFA fiber.

C. Model Derivation

The EDFA rate and propagation equations [3] are written as

$$\frac{dn_2}{dt} = \sum_k \frac{P_k i_k \sigma_{ak}}{h\nu_k} n_1(r,\phi,z) - \sum_k \frac{P_k i_k \sigma_{ek}}{h\nu_k} n_2(r,\phi,z) - \frac{n_2(r,\phi,z)}{\tau} \quad (9)$$

$$\frac{\partial P_k}{\partial z} = u_k \sigma_{ek} \int_0^{2\pi} \int_0^{\infty} i_k(r,\phi) n_2(r,\phi,z) r dr d\phi (P_k(z) + m h\nu_k \Delta\nu_k) - u_k \sigma_{ak} \int_0^{2\pi} \int_0^{\infty} i_k(r,\phi) \cdot n_1(r,\phi,z) r dr d\phi (P_k(z)) \quad (10)$$

Here, u_k represents either the forward or reverse direction of propagation through the EDFA with either $+1$ or -1, respectively. m represents the number of modes in the fiber, and is usually set to 2. $\Delta\nu_k$ represents the effective noise bandwidth, and is set to 100GHz here.

We take the average inversion level across the EDFA length. Based on the foregoing assumptions, and changing the propagation equation to use α_k,(5), and g_k, (6), we obtain

$$(\frac{\partial}{\partial t} + \frac{1}{\tau} - \frac{1}{\zeta\tau} \sum_k g_k m \Delta\nu_k) \frac{n_2}{n_t} = \frac{-1}{\zeta\tau} \sum_k u_k \frac{1}{h\nu_k} \frac{\partial P}{\partial z} \quad (11)$$

$$\frac{\partial P_k}{\partial z} = u_k P_k(z) \{(\alpha_k + g_k) \frac{n_2}{n_t} - (\alpha_k + \ell_k)\} + u_k g_k \frac{n_2}{n_t} m h\nu_k \Delta\nu_k \quad (12)$$

where $\zeta = \frac{\rho S}{\tau}$ is the saturation parameter and ℓ_k is an additional loss term in a channel. We assume $\ell_k = 0$ in the forthcoming simulations.

We apply classical solution techniques to solve the differential equation for P_k explicitly from (12). Since the inversion population, n_2, does not depend on the length z, we can define

$$Q(z) = -u_k \{(\alpha_k + g_k) \frac{n_2}{n_t} - (\alpha_k + \ell_k)\} \quad (13)$$

$$R(z) = u_k g_k \frac{n_2}{n_t} m h\nu_k \Delta\nu_k \quad (14)$$

Then, from the propagation equation (12) we derive the solution for $P_k(L)$, the power at the end of the fiber

$$P_k(L) = e^{-H(L)} \{\int_0^L e^{H(z)} R(z) dz + P_k(0)\} \quad (15)$$

where $H(z) = \int_0^z Q(p) dp$. If we substitute $z = L$ into $H(z)$, we obtain $H(L) = Q(z)L$. We can now express $P_k(L)$ explicitly as

$$P_k(L) = \frac{-g_k \frac{n_2}{n_t} m \nu_k \Delta\nu_k}{(\alpha_k + g_k)\frac{n_2}{n_t} - (\alpha_k + \ell_k)} [1 - e^{u_k \{(\alpha_k + g_k)\frac{n_2}{n_t} - (\alpha_k + \ell_k)\} L}] + e^{u_k \{(\alpha_k + g_k)\frac{n_2}{n_t} - (\alpha_k + \ell_k)\} L} P_k(0) \quad (16)$$

Finally, we explicitly integrate the rate equation (11) along the length of the fiber and divide by L. Also, we take the state variable to be $x = \frac{n_2}{n_t}$, and we normalize the power into photon flux, denoted by Q_k before. We can now state the final, state-space representation of the EDFA model with ASE included.

$$\begin{aligned} \dot{x} &= -\frac{x}{\tau} - \frac{1}{\zeta\tau L} \sum_k (-g_k m \Delta\nu_k L x \\ &+ u_k [\frac{-g_k m \Delta\nu_k x}{(\alpha_k + g_k)x - (\alpha_k + \ell_k)} \cdot (1 - e^{u_k \{(\alpha_k + g_k)x - (\alpha_k + \ell_k)\} L}) \\ &+ (e^{u_k \{(\alpha_k + g_k)x - (\alpha_k + \ell_k)\} L} - 1) Q_k^{in}]) \\ Q_k^{out} &= \frac{-g_k m \Delta\nu_k x}{(\alpha_k + g_k)x - (\alpha_k + \ell_k)} \cdot [1 - e^{u_k \{(\alpha_k + g_k)x - (\alpha_k + \ell_k)\} L}] \\ &+ e^{u_k \{(\alpha_k + g_k)x - (\alpha_k + \ell_k)\} L} Q_k^{in} \end{aligned} \quad (17)$$

D. Model Discussion

The new EDFA model derived herein shows some interesting differences between (4) and (17). We see two new terms in the state equation, and one new term in the output equation in (17). These terms are due to ASE contribution. As we can see, given a dropped channel, there will still be an ASE contribution as depicted by the output equation. Also note that in the state equation we have one linear ASE term and one nonlinear ASE term.

The model derived in the previous subsection possesses terms that are highly nonlinear in nature. Due to the wildly different changes in the magnitude of the terms that constitute the vector field for $\dot{x}$, the equations are stiff [21]. There are a large number of design variables that have to be assigned that couple with each other. These observations add to the technical challenge.

IV. L_2 CONTROL APPLICATION

Before the L_2 control design, a few observations about the system, (17), can be made. We can directly compute the value of our state x in real-time using measurements of the input and output powers in a designated channel. Theorem 1 does not require the use of the individual disturbance values in the controller, but only of the state x. The above observations indicate the significance of solving the FI problem on its own.

A. FI Problem Construction

The developed EDFA model (17) can be associated to the FI problem (1). In our model, w represents the the normalized input channel powers, and u represents the normalized input pump power. A standard nonlinear shift will be used later in the paper to denote $x = 0$, $w = 0$ and $u = 0$ as the equilibrium point to generate a specific average inversion level,x_0, with particular input powers. Any deviations from the equilibrium point will result in non-zero values of x,w and u.

Next, we must design the performance output, z, such that $k_{12}(x)$ has full column rank and $S_1(x) < 0$. The performance variable, z, represents the variable to be attenuated. We desire to attenuate the state, x, primarily, but it would also be beneficial to minimize the input, u. Note that any scalar multiples of the state and input, u, could also be used as design parameters.

One possible option is to use:

$$z = \begin{bmatrix} x \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \beta \end{bmatrix} u \qquad (18)$$

such that the L_2 gain(2) is taken from the disturbance inputs of the system directly to the state and pump power input. Here, β is a scalar of arbitrary choosing, and it is set to unity for our system. With (18) chosen, we can state $H_{FI}(V, x)$ for the EDFA system with ASE as:

$$H_{FI}(V, x) = \; A_{FI}(x)\frac{dV(x)}{dx} + \frac{1}{4}\frac{dV(x)}{dx}Q_{FI}(x)\frac{dV(x)}{dx} + R_{FI}(x) \qquad (19)$$

Where,

$$\begin{aligned} A_{FI}(x) &= f(x) && (20)\\ Q_{FI}(x) &= \frac{1}{\gamma^2}g_1(x)g_1^T(x) - g_2(x)g_2^T(x) && (21)\\ R_{FI}(x) &= x^2 && (22)\end{aligned}$$

To implement the entire state space model, we use real EDFA data with $f(x)$, $g_1(x)$ and $g_2(x)$ appropriately defined from the right hand side of (17) after the nonlinear shift from x_0.

B. Linear FI Problem Solution

We can find an explicit mathematical representation for the valid range of γ in the linearized system. It is shown in [10], that the linearized γ is valid for some neighborhood in the nonlinear system. Thus, we can take the linearization, and solve for γ directly, then extend the results to the nonlinear system.

We distinguish the linear representation of any function from its nonlinear representation by using a capital letter and dropping the function of x notation. So, for example, the nonlinear $f(x)$ term would be represented as F. This notation will be used throughout.

Taking only the linear terms of the HJE,$H_{FI}(V, x) = 0$, we obtain the linear Riccati equation

$$A^T P + PA + PQ_{FI}P + R = 0 \qquad (23)$$

From basic H_∞ theory [14], we restrict $Q_{FI} \leq 0$ and solve for $P > 0$. From this restriction on Q_{FI}, we can explicitly write out a restriction for γ.

$$\gamma \geq \sqrt{\frac{G_1 G_1^T}{G_2 G_2^T}} \qquad (24)$$

Note that here, G_1 and G_2 represent the linear parts of $g_1(x)$ and $g_2(x)$ and hence are not functions of x, but are explicit constants. The above relation would not be very useful if we were given an unsatisfactory static lower bound for γ.

To address this, we scale the pump input and disturbances. We let the scaled channel power disturbances $\Delta w = X_w w$, where X_w is the scaling factor multiplying the channel disturbances, w. We can do the same for the pump power input, $\Delta u = X_u u$, where X_u represents the scaling factor multiplying the pump power input. We re-write the EDFA system model in FI form below, including the scaling factors.

$$\begin{cases} \dot{x} = f(x) + \frac{g_1(x)}{X_w}(\Delta w) + \frac{g_2(x)}{X_u}(\Delta u) \\ z = \begin{bmatrix} x \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}(\Delta u) \\ y = \begin{bmatrix} x \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix}(\Delta w) \end{cases}$$

We treat the scaling terms X_w and X_u as positive scalar values for simplicity. However, they can be used in matrix form as well. Based on this scaling, we can manipulate the γ variable. Notice that $A_{FI}(x) = f(x)$ and $R_{FI}(x) = x^2$

are not affected by the scaling factors. With scaling, (24) becomes

$$\gamma \geq \frac{X_u}{X_w}\sqrt{\frac{G_1 G_1^T}{G_2 G_2^T}} \tag{25}$$

The γ that achieves equality in (25) represents γ_{min}, the smallest possible γ for a valid solution. We want to satisfy the expression:

$$\|x\|^2 + \|u\|^2 \leq \gamma^2 \|w\|^2 \tag{26}$$

Now, we can substitute γ_{min} from (25) into (26) to obtain

$$\|x\| \leq \left(\frac{X_u}{X_w}\sqrt{\frac{G_1 G_1^T}{G_2 G_2^T}} + \epsilon \right) \|\Delta w\| \tag{27}$$

Here, the ϵ is a non negative value. This completes the design tools necessary to solve the linear problem.

We apply the controller $u = -G_2^T P x$ from [10]. Notice in [10] that the linear controller that we applied here is a linearization of the full nonlinear controller in Theorem 1.

C. Nonlinear FI Solution

With the linear solution framework laid, the nonlinear controller can be developed. As mentioned in [10], if the linear theory is satisfied, there exists a neighborhood about the operating point such that (26) is satisfied for a nonlinear controller.

We add the positive value, $0.2x^2$, to $R_{FI}(x)$ to solve the HJI (19) as a specific Hamilton-Jacobi Equality (HJE). To facilitate this task, we extend the work from [19] and [8] which use Taylor approximations of the nonlinear equations. It can be proven inductively that each Taylor term coefficient for V(x) is calculable given the previous coefficients of V(x).

$Q_{FI}(x)$ is manipulated by increasing the γ value from its minimum value, but its multiplicative influence on $V_x(x)$ (see Theorem 1 conditions) is unclear because $V(x)$ is not known in advance. However, A few inferences can be made. First of all, we do know that since the linear part is valid, $V(x)$ will satisfy $V(x_0) = 0$ and $V(x) > 0$ at least in some region around x_0. The shape is parabolic in this region, so that $V_x(0) = 0$ and $V_x(x) < 0$ for $x < 0$, and $V_x(x) > 0$ for $x > 0$ in a neighborhood of x_0.

We have to satisfy two conditions according to Theorem 1. If we are able to find the solution of the HJI, $H_{FI}(V, x) \leq 0$,(19), the second condition is that

$$\dot{x} = f(x) + \frac{1}{2} Q_{FI}(x) V_x(x) \tag{28}$$

is asymptotically stable. The first term in (28), $f(x)$ has a stable vector field over the state space. If we can guarantee that the second term in (28) has a stable vector field, or an unstable vector field that does not have sufficient magnitude to overcome the first term, then the second condition of Theorem 1 will be satisfied. We can do this by examining the geometry of the terms over the state space.

If we can obtain a $V(x)$ that looks parabolic, all that would be necessary is to make $Q_{FI} < 0 \quad \forall x$ (assuming $\frac{1}{2} Q_{FI}(x) V_x(x)$ is stable, or of sufficiently small magnitude).

TABLE I
EDFA PARAMETERS

Core Radius	1.8	μm
Er Radius	1.7	μm
Er ion Density,ρ	4.33E+24	$ions/m^3$
Metastable Lifetime,τ	10	ms
Channel Separation (for $\Delta\nu$)	0.75	nm

TABLE II
EDFA SIGNAL AND PUMP BANDS

PUMP Band		
	absorption	emission
Wavelength (nm)	alpha (dB/m)	g* (dB/m)
980	3.229	0
Signal Band		
	absorption	emission
Wavelength (nm)	alpha (dB/m)	g* (dB/m)
1530-1565		
1530.072	6.49342764	6.3508087
		...
1565.072	1.64433299	3.29519786

If we look at the structure of Q_{FI}, we can see that the first term is positive, and the second term is negative. Hence, if the second term dominates the first, our objective is achieved. The final nonlinear solution, is obtained by tuning and iteration.

V. SIMULATIONS AND RESULTS

The following sections will provide open loop and closed loop simulation results. The first and second subsection will address the new EDFA model exclusively for steady-state simulation and dynamic channel drop cases. The third subsection will provide the Taylor series representation of the L_2 controller and closed loop performance simulations.

A. Steady State EDFA Model with ASE Simulations

The steady state analysis will plot the four major terms of the state equation (17). We can plot all of the terms logarithmically to view their individual contributions to state changes over the average inversion level. Figure 1 shows the terms graphically.

The results have been derived using the following parameters: Channels=32,Channel Powers $P_{in,k}$=42.6μW,Pump Power P_{pump}=150mW, fiber Length L=13m. The parameter data for the EDFA is listed in Table I. The signal and pump wavelengths with absorption and emission ranges are provided in Table II. Note that the channels that are utilized begin from the lower wavelengths, increasing by a 0.75nm spacing.

From Figure 1, the linear terms are represented by the two parallel looking plots, where the higher offset plot is the linear portion from (4), and the lower offset plot is the ASE.

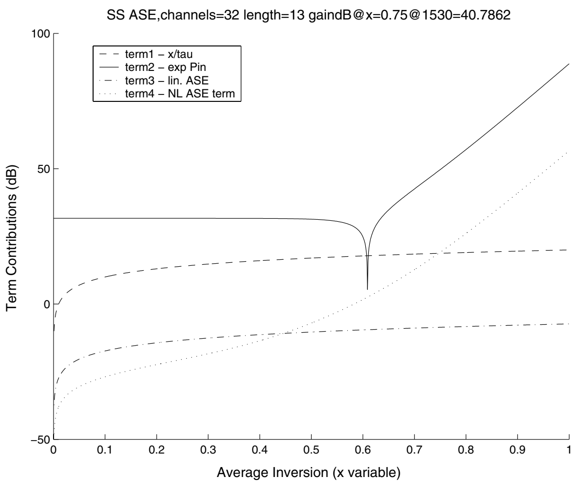

Fig. 1. ASE model State Equation Terms in Steady State

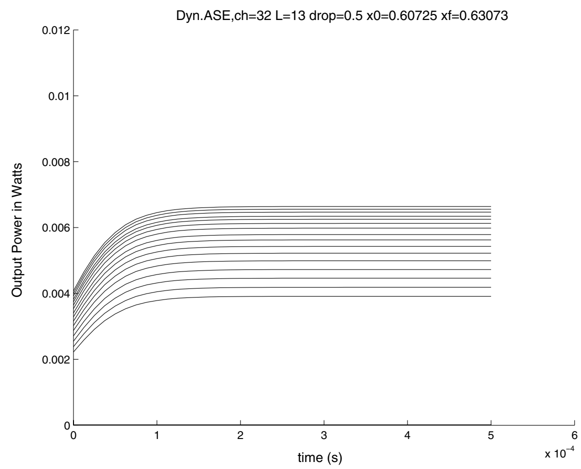

Fig. 2. Dynamic ASE model response with 50% channel drop

The linear ASE term is almost three orders of magnitude smaller than the original linear term and will have a small effect on the state.

The term with the singularity in the middle is the input power term. The singularity is simply the result of the logarithmic function onto the value 0. The plot shows the individual channel and pump terms as one consolidated sum. If the input powers increase, we should see these terms increase proportionally. This term is most dominant.

The most interesting plot is given by the nonlinear ASE term. It starts as the smallest term and then grows to be the second largest, next to the input term. Since the input term is very dependent on the input powers, we see that the nonlinear ASE term may have a significant effect on the state equation for relatively low input powers at high inversion levels. This confirms ASE experiments [6].

B. Dynamic Response for EDFA model with ASE

For consistency, we take the number of channels to be 32. The same input powers and EDFA length are taken as in the steady state analysis. We also adhere to the same parameters as defined in Tables I and II. We show two main drop channel drop situations, a 50% channel drop, or 16 channels, starting from the lower wavelengths, and a 97% channel drop, or 31 channels, are reduced to zero. Also, the full channel failure case is considered where 100%, or 32 channels, are dropped, and the transient response is analyzed.

First of all, we can note the different impact that ASE has in Figures 2, 3 and 4. The effect of ASE for a 50% drop is very small. The component due to ASE grows larger for the 97% drop case. The largest ASE magnitude manifests itself for the 100% drop case, which is much larger than either of the other cases. The reason for this can be seen from the changes in the average inversion levels, x, in each case. The larger channel drops results in a much larger increase for x, approximately $\Delta x = 0.14123$ for 97% drop and $\Delta x = 0.22223$ for 100% drop cases. This in turn results in the ASE

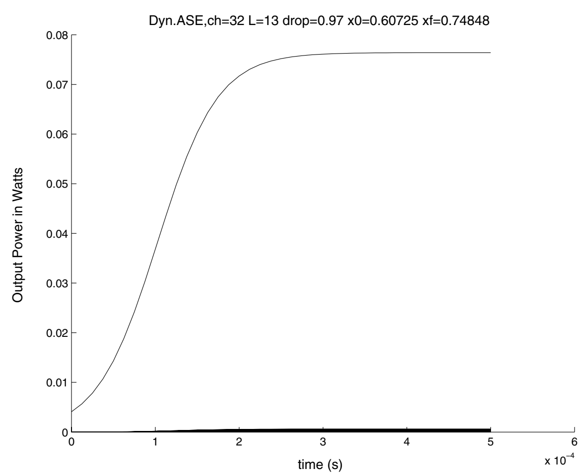

Fig. 3. Dynamic ASE model response with 97% channel drop

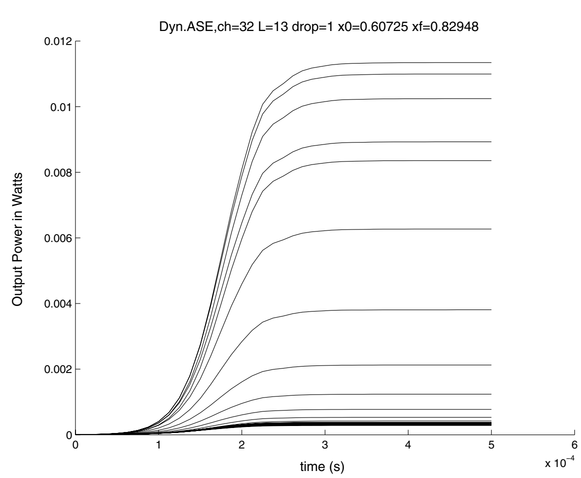

Fig. 4. Dynamic ASE model response with 100% channel failure

term being much larger. In fact, the magnitude of the ASE plot in the 100% drop case surpass that of the channel output powers for the 50% drop case. If we can control the variation in the state, we can restrict variation in ASE.

The 100% channel drop case is a special example that represents a total channel failure. It depicts the extreme increase of ASE noticeable from Figure 1. Moreover, since EDFAs are interconnected in optical networks, such a channel failure could result in a disturbance to other EDFAs in the network. If uncontrolled, the transient could propagate down an EDFA chain and increase its transient speed through the fiber length as demonstrated in [16] and [17].

We can see in Figure 2, that the power increases on the remaining channels by approximately 3 dB. Figure 3 shows an even greater power increase of approximately 12dB. This is due to the increasing of the average inversion variable, x.

C. L_2 Controller Simulations

We begin with our EDFA system model, (17). We follow the data in Tables I and II. We calculate the the operating point, $x_0 = 0.60725$, using the same inputs as the previous section. We perform a standard nonlinear shift on the state equation to set $x_0 = 0$, $\Delta w = 0$ and $\Delta u = 0$. Our disturbance is any deviation in signal input power from that which defines the operating point.

We use the linear theory of H_∞ control as a first step in the design process. We defined scaling factors X_w and X_u on the disturbance and pump inputs, respectively, as design parameters. Here, we design for a worst case scenario so the norm of the disturbance for a 100% channel drop is equal to unity. For our system, $\|w\| = 1.8716 * 10^{15} W/J$, which gives $X_w = \frac{1}{1.8716*10^{15}} = 5.343 * 10^{-16}$. Thus, $\Delta w = 1$, so our x is limited directly by the L_2 gain, (27).

Now, we have a selection for X_w. If we denote G_1 and G_2 as the linear part of $g_1(x)$ and $g_2(x)$, then we can use (27) to obtain X_u such that we obtain a proper L_2 gain from $\|\Delta w\|$ to $\|x\|$. We choose a small L_2 gain, $\gamma = 10^{-8}$ and get $X_u = 1.3723 * 10^{-26}$.

By choosing the above parameters, we satisfy the linear theory presented in [10], such that our linear controller, $u = -G_2 P x$, where P is the linear term of $\frac{dV(x)}{dt}$, must yield the designed L_2 gain. Furthermore, from [10], there exists a neighborhood around the operating point such that a nonlinear controller is globally asymptotically stable within the neighborhood. The nonlinear controller is exactly that used in Theorem 1.

If we increase the value of γ, which decreases the amount of attenuation, we can increasingly negate $Q_{FI}(x)$, to achieve an L_2 gain over the entire nonlinear state space. So we begin increasing γ by orders of 10, until we obtain the requirements for Theorem 1. An excellent choice of γ is provided if we choose $\gamma = 10^{-4}$. The final controller used is:

$$\begin{aligned} u &= -1.0954x + 3.4235(10^{-6})x^2 + 2.8757(10^{-5})x^3 \\ &- 1.0932(10^{-4})x^4 - 9.3091(10^{-4})x^5 \\ &+ 8.9275(10^{-1})x^6 + 1.5442x^7 \end{aligned} \quad (29)$$

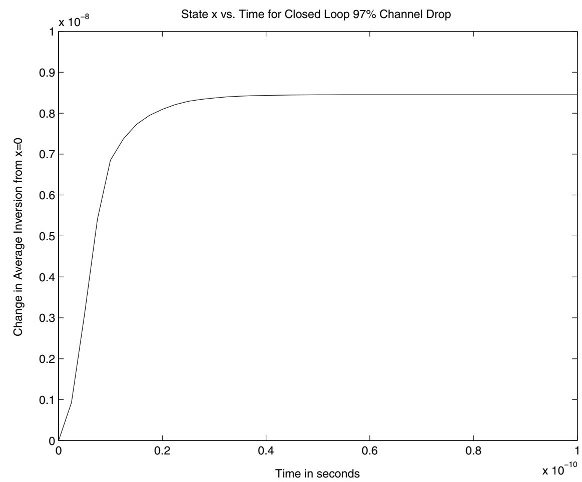

Fig. 5. Closed Loop System Response of x for 97% channel drop.

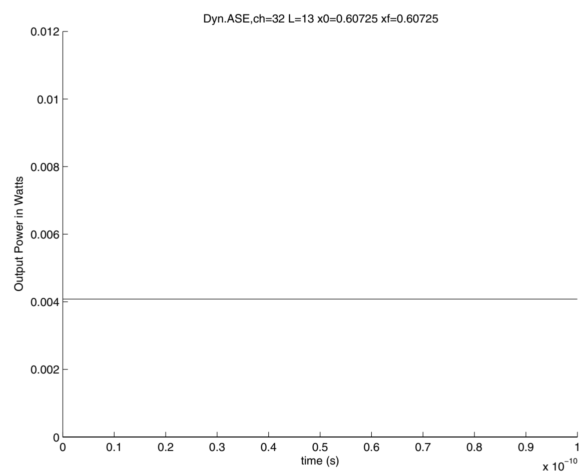

Fig. 6. Closed Loop System Output Response for 97% channel drop.

Notice that in this particular case, the linear term of the controller dominates the higher order terms since we chose to restrict our state x very tightly. The controller, however, is not necessarily linear for higher values of L_2 gain. For the worst case channel drop, the pump power does not exceed $150mW$ from the operating point value. For our chosen L_2 gain, the pump would operate on an extremely small time scale.

We see in Figure 5 that the output power is virtually constant. Notice, however, that since the value x changes so little, the system is still in the linear region. In a real-world application, we could only achieve a performance within hardware limits. We could design a controller that operates on a slower time scale at the cost of increasing the L_2 gain. The simulations of these cases yield highly nonlinear controllers where the pump power and response times are very reasonable.

VI. Conclusion

We have successfully developed a more complete system representation for the EDFA that includes ASE. We have demonstrated the significant effect that ASE can have for the situation in which a large number of channels are dropped. We simulated the steady state effects of the terms in the state equation, where we took special note of the two new ASE terms where one was linear and the other nonlinear. We noted that for high average inversion levels the nonlinear ASE term can have a significant impact in the state equation. The dynamic simulations showed the effects of the average inversion, channel powers and ASE for channel drops.

We successfully took the more complete model with ASE and devised both a linear and nonlinear controller that solves the Full Information problem since state is computable in real-time. We found a nonlinear controller that guarantees L_2 gain of 0.0001 over the entire state space. We also noted that the transient performance requirements of the pump can be reduced by increasing the L_2 gain if they are unreasonable.

VII. Future Work

Although we found a a nonlinear controller that satisfies the L_2 requirements for all of the state space, we must realize that this design was taken over Taylor approximated terms in the EDFA model. Also, future work will look for an explicit set of design laws to guarantee a solution for the nonlinear L_2 control problem. A method that does not involve tuning and iteration would be ideal. Finally, the above work can be extended into the robust control problem for the EDFA.

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