

Plant after gain adjustment is

$$K_P(s) = 20/s(s+2)$$

From the attached Bode plot, the straight line approximation gives the phase margin estimate to be  $29^\circ$ . To boost the PM to  $60^\circ$ , we need to add  $31^\circ$  but owing to the gain of the lead compensator, we need to add some extra phase. Since the straight line approximation under-estimates the phase lag after the breakpoint, we add an extra  $12^\circ$  to give  $43^\circ$  as the design value.

The parameter  $a$  is now determined by

$$a = \frac{1 + \sin(43)}{1 - \sin(43)} = 5.29$$

The gain contributed by  $C$  at the maximum phase frequency  $\omega_m$

$$= 10 \log(5.29) = 7.234$$

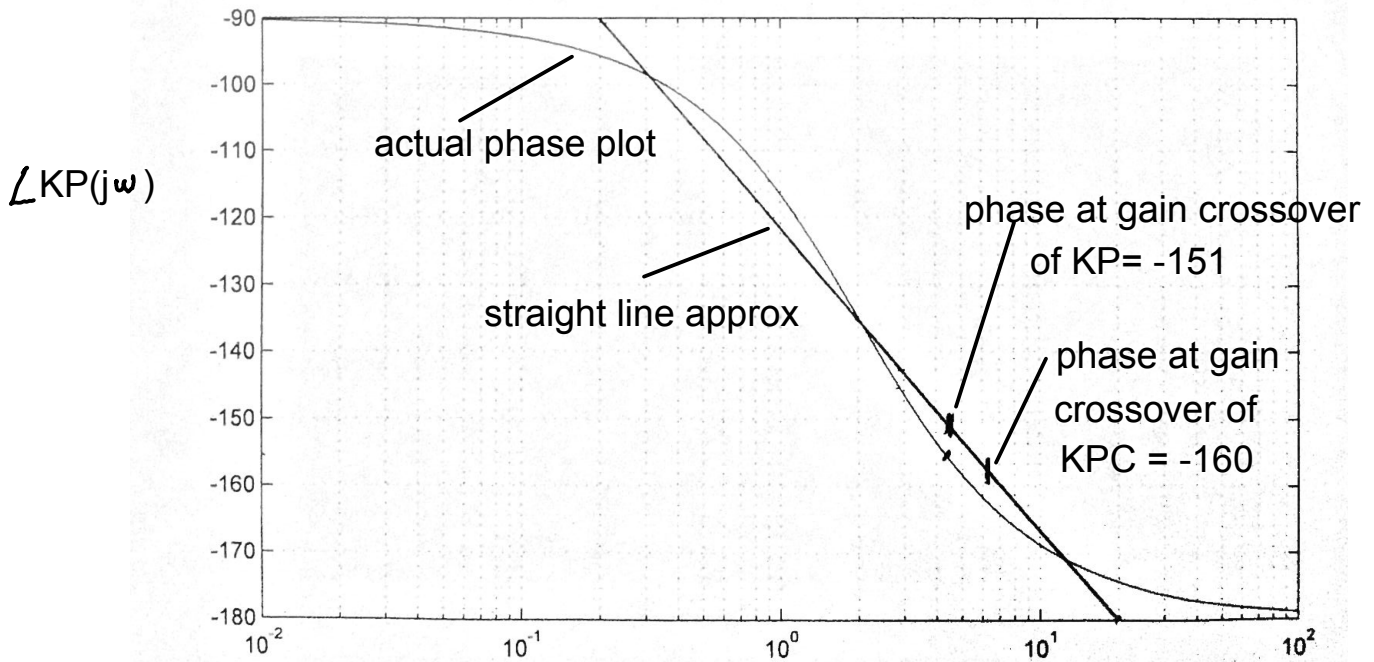
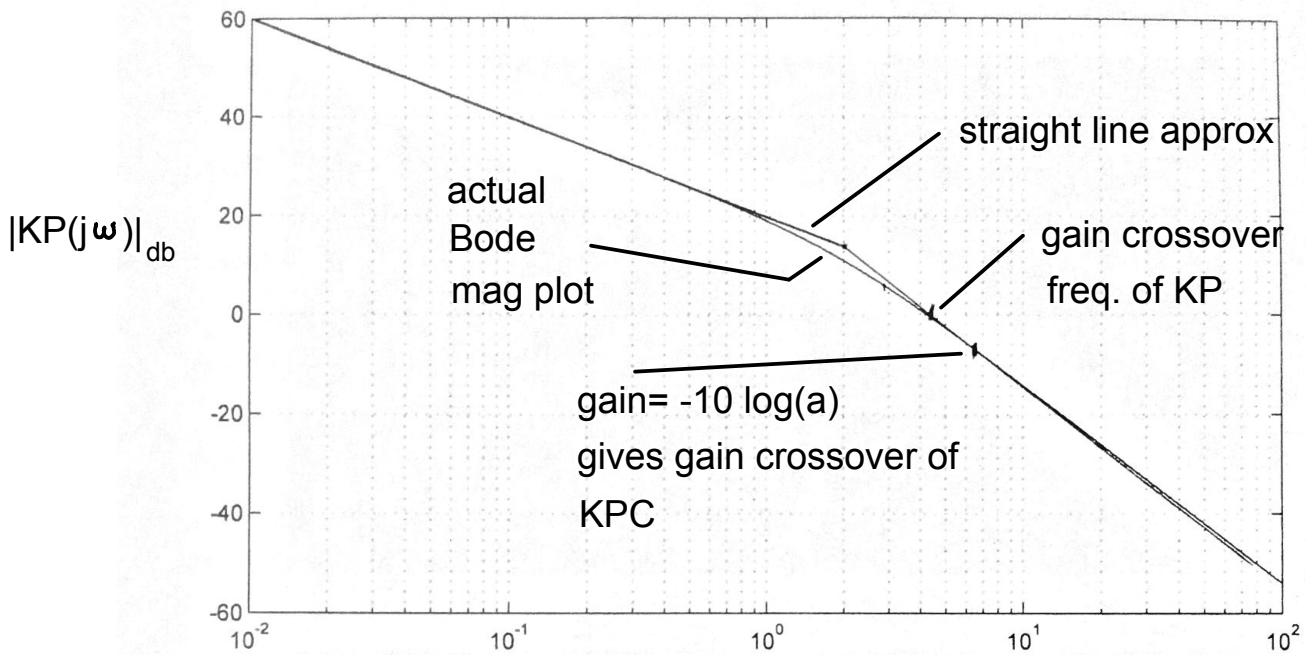
The frequency at which  $K_P(j\omega)$  has a gain of  $-7.234$  is estimated to be  $6.4$  by the straight line approximation. This gives the  $T$  parameter to be

$$T = \frac{1}{\sqrt{a} \omega_m} = \frac{1}{2.3 \times 6.4} = 0.0679$$

giving the lead compensator the transfer function

$$C(s) = \frac{1 + 5.29 \times 0.0679s}{1 + 0.0679s}$$

Bode plot of  $20/s(s+2)$



PM of compensated system is estimated to be  $20+43=63$ .

The design is therefore satisfactory.