SIGNALS AND SYSTEMS II
Assignment 4 Solutions

1

i) Construct the Routh array for the polynomial $p(s) = s^3 + 7s^2 + 17s + 15$.

$$\begin{array}{c|cc} s^3 & 1 & 17 \\ s^2 & 7 & 15 \\ s^1 & \frac{104}{7} & 0 \\ s^0 & 15 & 0 \end{array}$$

Since there is no change in sign in the first column all roots of $p(s)$ are in the left-half plane.

ii) $p(s)$ has all its roots to the left of $Re(s) = -1$ iff all roots of $p(s-1)$ are in LHP.

$$p(s-1) = (s-1)^3 + 7(s-1)^2 + 17(s-1) + 15 = s^3 + 4s^2 + 6s + 4$$

$$\begin{array}{c|cc} s^3 & 1 & 6 \\ s^2 & 4 & 4 \\ s^1 & 5 & 0 \\ s^0 & 4 & 0 \end{array}$$

Thus all roots of $p(s)$ are to the left of $Re(s) = -1$.

ii)

$$p(s-2) = (s-2)^3 + 7(s-2)^2 + 17(s-2) + 15 = s^3 + s^2 + s + 1$$

$$\begin{array}{c|cc} s^3 & 1 & 1 \\ s^2 & 1 & 1 \\ s^1 & 0 & 0 \\ s^0 & & \end{array}$$

Observe that $p(s-2)$ has a $s^2 + 1$ factor and therefore not all roots of $p(s)$ are to the left of $Re(s) = -2$. In fact roots of $p(s)$ are -3 and $-2 \pm j$.

2

a) We have:

$$\begin{aligned} Y(s) &= \frac{\frac{K}{s}\frac{s+1}{(s+2)(s+3)}}{1 + \frac{K}{s}\frac{s+1}{(s+2)(s+3)}} R(s) \\ &= \frac{K(s+1)}{s(s+2)(s+3) + K(s+1)} \frac{1}{s} \end{aligned}$$

If the system is stable one can use the Final Value Theorem to compute the asymptotic value of y:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \frac{K}{K} = 1$$

if $K \neq 0$. So the asymptotic tracking is achieved conditioned on the stability of the system, i.e. roots of $s^3 + 5s^2 + (K+6)s + K$ being in the left-half plane.

$$\begin{array}{c|cc} s^3 & 1 & K+6 \\ s^2 & 5 & K \\ s^1 & \frac{4K+30}{5} & 0 \\ s^0 & K & 0 \end{array}$$

Roots are in LHP iff there is no sign change in the first column, i.e.:

$$\begin{aligned} &(K > 0) \wedge (K > -\frac{30}{4}) \\ &= \quad K > 0 \end{aligned}$$

b)

$$\begin{aligned} Y(s) &= \frac{\frac{K}{s}\frac{s+4}{(s+1)(s+2)}}{1 + \frac{K}{s}\frac{s+4}{(s+1)(s+2)}} R(s) \\ &= \frac{K(s+4)}{s(s+1)(s+2) + K(s+4)} \frac{1}{s} \end{aligned}$$

Like before, if $K \neq 0$ and the system is stable, asymptotic tracking is achieved.

$$p(s) = s^3 + 3s^2 + (K+2)s + 4K$$

$$\begin{array}{c|cc} s^3 & 1 & K+2 \\ s^2 & 3 & 4K \\ s^1 & \frac{6-K}{3} & 0 \\ s^0 & 4K & 0 \end{array}$$

The system is stable iff:

$$\begin{aligned} &(K > 0) \wedge (K < 6) \\ &= \quad 0 < K < 6 \end{aligned}$$

3

With the suggested $C(s)$ we have:

$$
\begin{aligned}
Y &= \frac{P}{1+PC}D \\
&= \frac{\frac{1}{(s+3)(s+4)}}{1+\frac{(s+1)(s+2)}{D_c(s)}\frac{1}{(s+3)(s+4)}}D(s) \\
&= \frac{D_c(s)}{D_c(s)(s+3)(s+4)+(s+1)(s+2)}\frac{1}{s^2+1}
\end{aligned}
$$

To remove the sinusoid from the steady-state output, the factor s^2+1 should be canceled from the denominator. Thus we need $D_c(s) = A(s)(s^2+1)$ where $A(s)$ is a polynomial in s with roots in LHP and it must be chosen so that the overall system is stable. We try $A(s) = 1$.

$$p(s) = s^4 + 7s^3 + 14s^2 + 10s + 14$$

$$
\begin{array}{c|lll}
s^4 & 1 & 14 & 14 \\
s^3 & 7 & 10 & 0 \\
s^2 & \frac{88}{7} & 14 & 0 \\
s^1 & \frac{194}{88} & 0 & 0 \\
s^0 & 14 & 0 & 0
\end{array}
$$