University of Toronto
Department of Electrical and Computer Engineering
ECE356S System and Signal Analysis II
Spring 1999
Problem Set #1

1. A model describing the competition between 2 species is given by

$$\begin{aligned}\dot{N}_1 &= rN_1(1-\frac{N_1}{K}) - \alpha N_1 N_2 \\ \dot{N}_2 &= sN_2(1-\frac{N_2}{L}) - \beta N_1 N_2\end{aligned}$$

where r, s, α, and β are positive constants. Suppose that $s < \beta K$ and $r < \alpha L$. Determine the equilibrium points of the above nonlinear system of differential equations, and linearize the nonlinear equation about each of the equilibrium points.

2. Let the matrix A be given by

$$A = \begin{bmatrix} \sigma & \omega \\ -\omega & \sigma \end{bmatrix}$$

Determine the transition matrix e^{At} using the following 3 different methods:

(a) Diagonalization method (extending the scalar field to the complex numbers)

(b) Laplace transform

(c) Writing $A = \sigma I + \omega B$, where

$$B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

3. The dynamic behaviour of linear systems can be interpreted using the eigenvectors of the A matrix. We assume that A has distinct eigenvalues.

(a) Show that if v is an eigenvector with eigenvalue λ,

$$e^{At}v = e^{\lambda t}v$$

(b) Let v_k, $k = 1, \ldots, n$ be the eigenvectors of A, with corresponding eigenvalues λ_k. Show that for any x_0, we can write the solution of

$$\dot{x} = Ax$$

in the form of

$$x(t) = \sum_{k=1}^{n} c_k e^{\lambda_k t} v_k$$

How are the coefficients c_k determined? This representation is referred to as the modal decomposition of $x(t)$.

(c) For

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$$

and $x_0 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, find the modal decomposition of $x(t)$.