University of Toronto
Department of Electrical and Computer Engineering
ECE356S System and Signal Analysis II
Spring 1999
Problem Set #2

1. The Laplace transform method requires the computation of $(sI - A)^{-1}$. For matrices of dimension ≥ 3, sometimes the following recursive method is useful. First note that

$$(sI - A)^{-1} = \frac{adj(sI - A)}{det(sI - A)}$$

where adj(M) is the adjoint matrix of the matrix M. Now we can express

$$adj(sI - A) = B_1 s^{n-1} + B_2 s^{n-2} + \cdots + B_n$$

and

$$det(sI - A) = s^n + a_1 s^{n-1} + \cdots + a_n$$

Assume that the coefficients a_i have already been determined, derive a set of recursive equations for the matrices B_i, $i = 1, \ldots, n$ by writing

$$det(sI - A)I = (sI - A)adj(sI - A)$$

and equating coefficients of the powers of s. Using this method, determine $(sI - A)^{-1}$ for

$$A = \begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix}$$

(In fact, it is also true that the coefficients a_i can be determined recursively using the equation

$$a_i = -\frac{1}{i} tr(AB_i)$$

which you can verify for the above example.)

2. (a) Determine the transfer function from u to y for the linear system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & -b \\ 1 & -a \end{bmatrix} x + \begin{bmatrix} 1 \\ c \end{bmatrix} u \\ y &= \begin{bmatrix} 0 & 1 \end{bmatrix} x \end{aligned}$$

(b) Repeat for the linear system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 \\ -b & -a \end{bmatrix} x + \begin{bmatrix} c \\ 1 - ac \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} x \end{aligned}$$

and verify that the 2 transfer functions are the same.

(c) The above results show that the choice of state variables is not unique. Indeed, both state representations describe the second order scalar differential equation

$$\ddot{y} + a\dot{y} + by = u + c\dot{u}$$

Determine how the state variables from parts 1 and 2 are defined in terms of y and u.

3. Consider the linear system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 0 & 1 \end{bmatrix} x \end{aligned}$$

(a) Determine its transfer function from u to y, and verify that it is stable.

(b) Show that in general, the total response, which includes the initial condition response, results in the output y increasing exponentially without bound even when the input u is bounded. This shows one must be careful in interpreting results based on input-output transfer functions.