University of Toronto
Department of Electrical and Computer Engineering
ECE356S System and Signal Analysis II
Spring 1999
Problem Set #3

1. The following 2 items both involve a system (possibly different) modelled by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u, \quad y = \mathbf{C}\mathbf{x} + Du,$$

where $\dim u = \dim y = 1$, $\dim \mathbf{x} = 2$.

(a) Given $u = 0$ and the initial-state-responses

$$\mathbf{x}(0) = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} \implies y(t) = \mathrm{e}^{-t} - 0.5\mathrm{e}^{-2t}$$

$$\mathbf{x}(0) = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \implies y(t) = -0.5\mathrm{e}^{-t} - \mathrm{e}^{-2t},$$

find the initial-state-response for $\mathbf{x}(0) = \begin{bmatrix} 2 \\ 0.5 \end{bmatrix}$.

(b) Now suppose that for

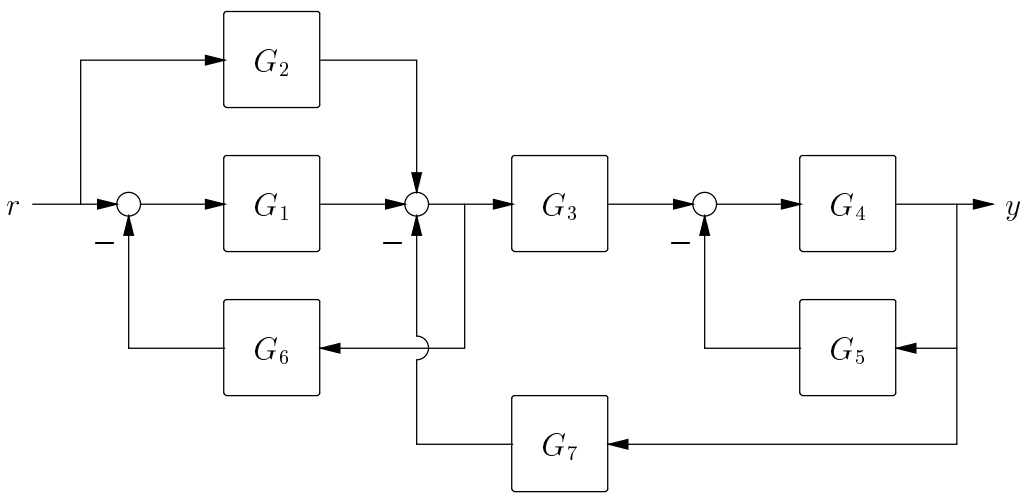
$$\mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \ u \text{ a unit step} \implies y(t) = 0.5 - 0.5\mathrm{e}^{-t} + \mathrm{e}^{-2t}$$

and that for

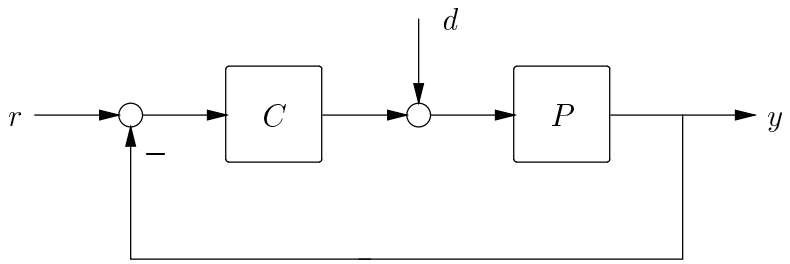
$$\mathbf{x}(0) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \ u \text{ a unit step} \implies y(t) = 0.5 - \mathrm{e}^{-t} + 1.5\mathrm{e}^{-2t}.$$

Find $y(t)$ when u is a unit step and $\mathbf{x}(0) = 0$.

2. Determine the transfer function from r to y for the system given below:

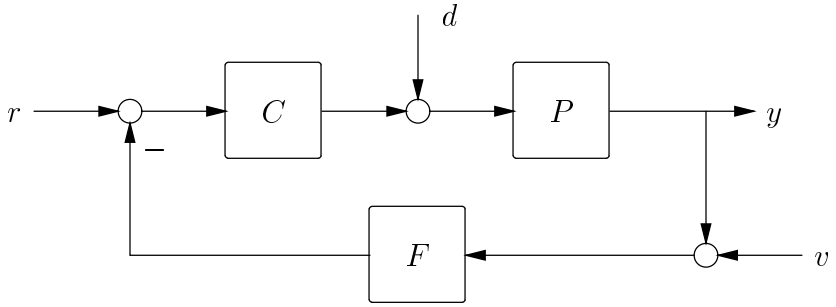


3. Prove that for the single loop feedback system



with $P(s) = \dfrac{N_p(s)}{D_p(s)}$, $C(s) = \dfrac{N_c(s)}{D_c(s)}$, internal stability implies that the characteristic polynomial $N_p(s)N_c(s) + D_p(s)D_c(s)$ must have roots in $Re\ s < 0$.

4. Generalize the internal stability theorem for the single loop feedback system



You only need to give the defnition of interval stability and the theorem statement. You don't need to prove it.