University of Toronto
Department of Electrical and Computer Engineering
ECE356S System and Signal Analysis II
Spring 1999
Problem Set #5

1. Sketch the root locus of the feedback system with the loop transfer function $L(s) = \frac{N(s)}{D(s)}$ with $N(s) = 1$, and $D(s) = s(s+3)(s^2+2s+2)$. Check your sketch using matlab.

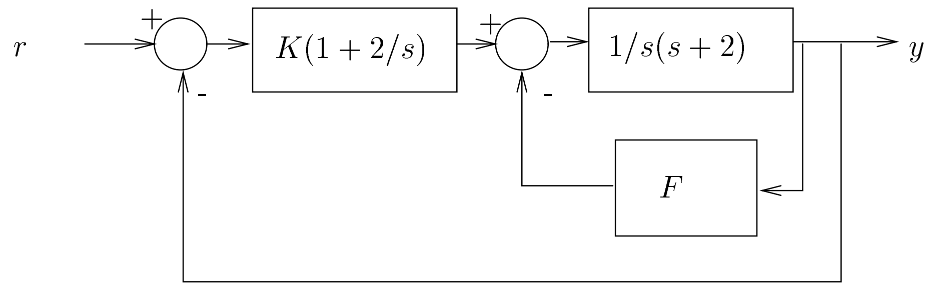
   Note: While it is not important in most cases, you can get a nicer and more accurate sketch for branches leaving imaginary open-loop poles using the so-called angle of departure γ_i at the ith pole p_i. It is given by

   $$\gamma_i = \frac{1}{n_i}\left(\sum_j \arg(p_i - z_j) - \sum_j \arg(p_i - p_j) + (2k+1)\pi\right) \quad k = 0, 1, \ldots, n_i - 1$$

   where n_i is the multiplicity of the pole p_i, p_j is the jth open-loop pole, z_j is the jth open-loop zero, and arg refers to the angle of the complex number. For simple poles, the expression simplifies to

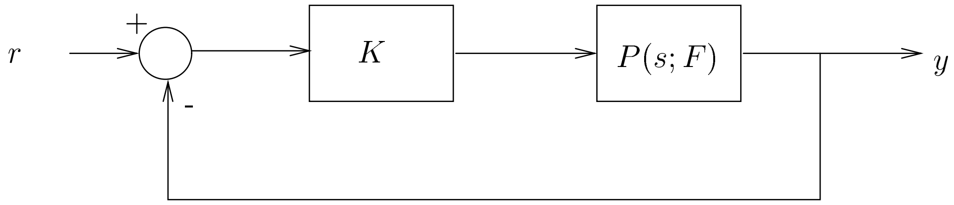
   $$\gamma_i = \sum_j \arg(p_i - z_j) - \sum_j \arg(p_i - p_j) + \pi$$

2. Consider the following feedback system



   The feedback gain F and the gain K in the forward path are both adjustable. This problem asks you to study the closed loop system behaviour in terms of K and F.

   (i) Express the feedback system in the form of



and identify $P(s;F)$. $P(s;F)$ is thus the equivalent open loop transfer function, expressed as $\dfrac{N(s)}{D(s)}$, and is a function of the feedback gain F.

(ii) Let $F = 1$ and plot the root locus of the closed loop system (CLS) as a function of K.

(iii) Discuss the open loop pole pattern (i.e. poles of $P(s;F)$) for $F > 1$. Describe in this case the 2 possible scenarios for the root locus of CLS.

(iv) One of the scenarios involves a real axis break-in point. The break-in point may be determined from the roots of $ND' - DN' = 0$. Draw the root locus of $ND' - DN'$ as a function of F. Show that this root locus has a breakaway point at $s = -\frac{2}{3}$. Show also that this occurs for $F = \frac{32}{27}$. From this, show that there is a real axis break-in point for the root locus of CLS for $F < \frac{32}{27}$. Explain what happens to the real axis break-in point for $F > \frac{32}{27}$.
(Hint: Recall that if s is a point on the root locus of $1 + KL(s)$, the corresponding value of $K = \frac{1}{|L(s)|}$).

(v) Sketch the root locus of the CLS as a function of K for $F = \frac{32}{27}$ and $F = 4$.
(Hint: From the previous parts, you should be able to conclude that at $F = \frac{32}{27}$, $s = -\frac{2}{3}$ is a point on the root locus of CLS)

3. Let

$$PCF(s) = \frac{1 + 2s}{(1 + s)(1 + 4s)(1 + 5s)}$$

Sketch the Nyquist plot for this system and find the range of values of K for which the closed loop system is stable.

4. Let

$$PCF(s) = \frac{s + 3}{s(s - 1)}$$

Sketch the Nyquist plot for this system and find the range of values of K for which the closed loop system is stable.