UNIVERSITY OF TORONTO
Department of Electrical and Computer Engineering
ECE356S Spring 1999
Problem Set #6

Due: March 18, 1999

1. (a) The sequence x_k, $-\infty < k < \infty$, is given by

$$\begin{aligned} x_k &= (\frac{1}{2})^k & k \geq 0 \\ &= (\frac{1}{3})^{-k} & k < 0 \end{aligned}$$

Determine the Fourier transform $X(e^{i\omega})$ of x_k.

(b) Carry out the Fourier inversion integral for the Fourier transform $X(e^{i\omega})$ in 1(a), and verify that the sequence is indeed x_k.

2. (a) Suppose a sequence x_k, $k \geq 0$, has a z-transform $X(z) = \dfrac{1}{(1-az^{-1})(1-bz^{-1})}$ with region of convergence given by $|z| > \max(|a|, |b|)$. Compute x_k using the inversion integral.

(b) Consider $X(z)$ in part (a) as a ratio of polynomials in z^{-1} and expand $X(z)$ in a partial fraction expansion

$$X(z) = \alpha_1 \frac{1}{z^{-1} - a^{-1}} + \alpha_2 \frac{1}{z^{-1} - b^{-1}} \tag{2.1}$$

Determine α_1 and α_2. Using the fact that a^k has z-transform $\dfrac{1}{1-az^{-1}}$, $|z| > |a|$, determine x_k using (2.1), and verify that it is the same as that determined in 2(a).

3. (a) If the sequence $\{x_k\}$ has a z-transform $X(z)$, with region of convergence $|z| > R_x$, determine the z-transform of

   (i) $\{kx_k\}$

   (ii) $\{k^2 x_k\}$

(b) If $X(z)$ and $Y(z)$ are the z-transforms of $\{x_k\}$ and $\{y_k\}$, with regions of convergence $|z| > R_x$ and $|z| > R_y$ respectively, show that $\{x_k y_k\}$ has a z-transform given by

$$\frac{1}{2\pi i} \oint X\left(\frac{z}{v}\right) Y(v) \frac{dv}{v}$$

and determine the associated region of convergence.

(c) What are the regions of convergence of the 2 sequences in 3(a)? Explain your answers.

4. Sometimes it is necessary to consider z-transforms for sequences defined on $-\infty < k < \infty$. The z-transform for such a sequence x_k is defined to be

$$X(z) = \sum_{-\infty}^{\infty} x_k z^{-k}$$

It is sometimes referred to as the bilateral z-transform. It exists if the 2 series

$$\sum_{-\infty}^{-1} x_k z^{-k} \quad \text{and} \quad \sum_{0}^{\infty} x_k z^{-k}$$

both converge. Since the first sum converges for $|z|$ sufficiently small while the second sum converges for $|z|$ sufficiently large, the region of convergence is an annulus.

(a) Let

$$\begin{aligned} x_k &= 3^k \qquad k < 0 \\ &= 2^k \qquad k \geq 0 \end{aligned}$$

Determine the bilateral z-transform of x_k, and determine its region of convergence.

(b) Given the z-transform $X(z) = \frac{1}{z-4}$, determine the sequence x_k if

(i) the region of convergence is $|z| < 4$ (Hint: Expand in a power series in $\frac{z}{4}$);

(ii) the region of convergence is $|z| > 4$ (Hint: Expand in a power series in $4z^{-1}$).