UNIVERSITY OF TORONTO
Department of Electrical and Computer Engineering
ELE 356S Spring 1999
Problem Set #8

Due: April 9, 1999

1. Recall the ideal phase shifting filter $H_d(e^{i\omega})$ given by

$$H_d(e^{i\omega}) = \begin{cases} -i & 0 \leq \omega \leq \pi \\ i & -\pi \leq \omega \leq 0 \end{cases}$$

   Design FIR digital filters using Kaiser windows for the following specs:

   (a) $\delta = 2\%$ and N=5

   (b) $\delta = 0.056\%$ and N=5

   Determine the filter coefficients in each case and compare the designs to the one based on the Hamming window. Give qualitative comments on how the three designs compare to each other.

2. In Butterworth filter design, we have said that the following procedure will yield a filter which exactly satisfies the specs in the passband, but exceeds the specs in the stopband:

   (i) Compute $N_c = \dfrac{\log(\frac{A^2-1}{\epsilon^2})^{\frac{1}{2}}}{\log \frac{\omega_s}{\omega_p}}$ and round the result up to the next integer. This is N, the order of the Butterworth filter.

   (ii) Determine the critical frequency ω_c using the formula:

$$\omega_c = \frac{\omega_p}{(\epsilon^2)^{\frac{1}{2N}}}$$

   In this problem, you are asked to establish these claims. Let

$$\omega_1(n) = \frac{\omega_p}{(\epsilon^2)^{\frac{1}{2n}}} \tag{1}$$

$$\omega_2(n) = \frac{\omega_s}{(A^2-1)^{\frac{1}{2n}}} \tag{2}$$

   (a) Show that $\dfrac{\omega_1(N_c)}{\omega_2(N_c)} = 1$.

   (b) In any reasonable spec, $A^2 - 1 > \epsilon^2$. Show that $\omega_c = \omega_1(N) < \omega_2(N)$.

   (c) Finally, show that

$$1 + (\frac{\omega_s}{\omega_c})^{2N} \geq A^2$$

   so that the stopband specs are exceeded.

   Remark: If we take $\omega_c = \omega_2(N)$, similar calculations show that the stopband specs are exactly satisfied, but the passband specs are exceeded.

3. Suppose it is desired to design a low pass filter such that the passband is constant to within 1 db for $\omega \leq 0.2\pi$, and the stopband attenuation is greater than 15 db for frequencies between 0.3π and π. Design a Butterworth filter satisfying the specs using the bilinear transformation $s = \dfrac{z-1}{z+1}$.

4. Another way to transform analog filter design into digital filter design is by matching the impulse response. It is useful if the analog filter $H_a(\omega)$ is bandlimited, say to $\mid \omega \mid \leq \pi$. Let us assume this to be true. Suppose we choose the impulse response of the digital filter h_k to match the analog impulse response $h_a(t)$ at $t = k$, i.e., set $h_k = h_a(k)$.

   (i) How are the digital filter and analog filter frequency responses related?

   (ii) Now suppose the analog filter has transfer function $H_a(s) = \dfrac{2\omega_p^2}{(s+\omega_p)^2 + \omega_p^2}$. Write it in the form of $H_a(s) = \frac{A_1}{s-s_1} + \frac{A_2}{s-s_2}$. Since then $h_a(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$, impulse response matching yields $h_k = A_1 e^{s_1 k} + A_2 e^{s_2 k}$. The digital filter frequency response $H(e^{i\omega})$ corresponding to the analog filter $H_a(s)$ can now be obtained by taking Fourier transform of h_k. Determine $H(e^{i\omega})$ explicitly for the $H_a(s)$ considered.