SIGNALS AND SYSTEMS II
Problem Set 9 Solutions

1. The z-transform for h_k is

$$H(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}$$

The z-transform of a unit step is

$$U(z) = \frac{1}{1 - z^{-1}}$$

$$\therefore Y(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} \, \frac{1}{1 - z^{-1}}$$

Using partial fractions expansion, we find

$$Y(z) = \frac{2}{1 - z^{-1}} - \frac{1}{1 - \frac{1}{2}z^{-1}}$$

Hence

$$y_k = 2 - \left(\frac{1}{2}\right)^k \qquad k \geq 0$$

2. We have

$$P(i\omega) = \frac{1 - \omega^2}{(4 - \omega^2)(4 + i\omega)} = \frac{(1 - \omega^2)(4 - i\omega)}{(4 - \omega^2)(16 + i\omega^2)}$$

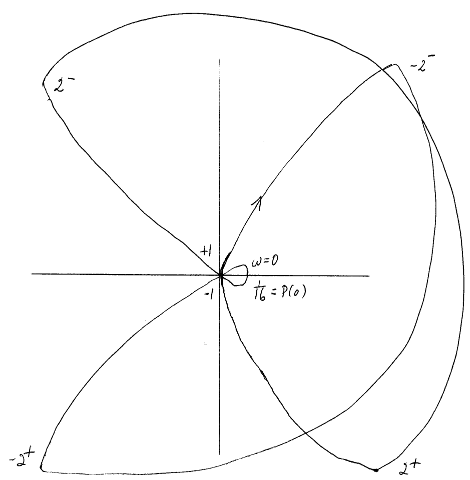
$$\begin{aligned} \Rightarrow \quad \text{Re } P(i\omega) &= \frac{1 - \omega^2)}{(4 - \omega^2)(16 + \omega^2)} \\ \text{Im } P(i\omega) &= -\frac{\omega(1 - \omega^2)}{(4 - \omega^2)(16 + \omega^2)} \end{aligned}$$

$$\angle P(i\omega) = \angle(1 - \omega^2) - \angle(4 - \omega^2) - \tan^{-1}\frac{\omega}{4}$$

At $\omega = 1$, $P(i\omega) = 0$
At $\omega = 2^-$, $|P(i\omega)| = \infty$, $\angle P(i\omega) = 180^o - \tan^{-1}\frac{1}{2}$

$$= 180^o - 26.565 = 153.435$$

The Nyquist plot is sketched below.



From the sketch, we see that there is no encirclement of $-\frac{1}{K}$ for $K > 0$. Hence the closed loop system is stable for $K > 0$.

3. Since $\delta = 0.05\% = 0.0005$, the parameter A is

$$A = -20 \log_{10} \delta = 66.02$$

Using the inequality

$$N \geq \frac{A - 7.95}{4.57\Delta}$$

we find that $N \geq 40.45$. Since for the given specification, $N \leq 35$, the design specs cannot be met.