University of Toronto

Department of Electrical and Computer Engineering

ECE410F – Control Systems – 2008

EXPERIMENT 1

CONTROL DESIGN USING MATLAB AND SIMULINK

1 Purpose

The purpose of this simulation experiment is to familiarize you with basic tools for analysis and design of control systems using Matlab and Simulink. In Part I, a helicopter system is used to illustrate tools for linear systems. In Part II, a magnetic levitation system is used to illustrate simple nonlinear systems analysis and control design.

2 Part I: Control of a Helicopter System

Introduction

A state space model for longitudinal motion of a helicopter near hover is given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -0.4 & 0 & -0.01 \\ 1 & 0 & 0 \\ -1.4 & 9.8 & -0.02 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 6.3 \\ 0 \\ 9.8 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

where x_1 is the pitch rate, x_2 is the pitch angle, x_3 is the horizontal velocity, and u is the rotor tilt angle.

Designing a controller for the helicopter system is challenging since it is an open loop unstable system. The design specifications are:

(a) The closed loop system is stable;

(b) The closed loop system asymptotically tracks a step input.

Once you have obtained a controller which achieves the above design specs, you can try to improve the response by tuning the various control parameters.

3 Preparation

You will be using Matlab and Simulink to design controllers in this experiment. The experimental work in Part I involves mostly design work using Simulink. In your preparation, you will build up preliminary data to facilitate your experimental work. You should also familiarize yourself with the libraries of Simulink, especially “Continuous”, “Sinks”, and “Sources”.

1. Enter the helicopter system matrices A, B, and C into Matlab. Use the command “ss2tf” to produce the transfer function $G(s)$. Matlab uses the object “sys” to represent a system block which can then be composed using the commands “series” and “feedback”. Use the command “tf” to generate the helicopter “sys” object. Determine the helicopter poles and zeros and show that it is open loop

unstable. Determine the eigenvalues of A and verify that they are the same as the poles of $G(s)$. Label this as preparation for Part I to be handed in with preparation for Part II.

2. A Simulink diagram for the helicopter system, heli.mdl, is supplied. You can display it by starting Matlab and typing heli. Note that the input and output are not yet connected to a source or sink, and no control law is in place. So the Simulink diagram represents the **open loop** system. Study heli.mdl carefully to gain better understanding of Simulink modelling. For example, you can click each block and look at the block parameters.

4 Experiment

4.1 Control Design for Helicopter System

You will learn different approaches to control design in the course. For now, since you have learned some classical control design techniques in ECE311S, we shall build a controller that achieves the design specifications using root locus ideas.

1. Start heli.mdl. Connect a step function as the input. Connect x_3, which is the output y, to a scope. We shall refer to the open loop system as $G(s)$.

2. From your preparation, you know that the system is unstable. To control the flight of the helicopter, you need to stabilize the system. Furthermore, to achieve asymptotic tracking, we need an integrator at the origin (review ECE311 material, if necessary, to reinforce your understanding). Consider the the feedback system described in the following diagram:

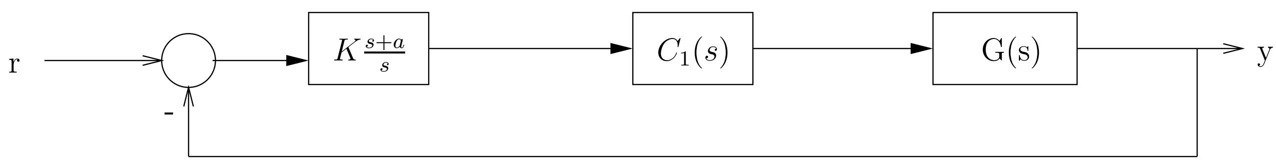

Figure 1: Complete controller design

where the reference input r is a step. The controller $C_1(s)$ serves to stabilize the closed loop system, while the controller $C_2(s) = \frac{s+a}{s}$ provides asymptotic tracking. Motivated by root locus considerations, choose as your starting point the controller

$$C_1(s) = \frac{s + 0.65}{s + 10} \frac{s + 0.2}{s + 11}$$

(You should reflect on and try to understand why based on root locus and the poles and zeros of $G(s)$, a second order controller is chosen). Add $C_1(s)$ and $C_2(s)$ to your Simulink diagram for $G(s)$ and complete the feedback system shown in Figure 1. Choose a small value for a initially so that $C_2(s) \approx 1$. Choose some value for the gain K. You will adjust it in your simulation to get a stable closed loop system. Click "Simulation", "Simulation parameters", "Solver", and set "Stop time" to 180. Start the simulation and look at the output on the scope. If it is not stable, adjust the gain K so that a finite steady state value is reached. Note that once the system is stabilized, the tracking error should tend to 0. However, the error dynamics may be slow depending on the choice of a and K. Enter in the space below the first set of values of a and K when you successfully stabilized the closed loop system. Do a rough sketch of the step response and label your sketch.

3. Tune the various control parameters to see if you can get a better step response. You do not have to try to get the best response possible. You should record how the step response changes as a result of your tuning to gain insight on, for example, the effects of placement of controller poles and zeros. For your final control configuration, compute the maximum overshoot, and the 2% settling time for the step response. Put your final control parameters and experimental observations and results in the space provided below. Print your Simulink diagram for the final continuous time controller you have designed, label it, and insert it after this page. Sketch or print the step responses on the Simulink scope and include them with your Simulink diagram. Show this to your TA, and demonstrate your simulation.

5 Part II: Control of a Magnetically Levitated Ball

5.1 Introduction

Many industrial systems contain nonlinearities. It is therefore important to understand how to apply control design procedures based on linear systems to nonlinear systems and their limitations. A magnetic levitation (maglev) system to suspend a ball is often used as an illustrative model for control design of a nonlinear system.

The simplified differential equation for the system is given by

$$\ddot{y} = g - k\left(\frac{u^2}{y^2}\right)$$

where u the control input is the current through the electromagnet, y the vertical displacement of the ball from the magnet, g is the gravitational acceleration, and k is a constant determined by the physical dimensions and material of the system. For the purposes of this experiment, $g = 9.8$, and $k = 1$. The objective of the control problem is to keep the ball suspended at some fixed distance from the electromagnet. This corresponds to the specifications that asymptotically, the position $y(t)$ reaches an equilibirum point $\bar{y}$ and the equilibrium velocity $\dot{y}$ is 0.

6 Preparation: The Nonlinear Maglev Model and Its Linearization

In your preparation, you will derive the state equations of the nonlinear maglev system and linearize it about an equilibrium point.

6.1 The Nonlinear Model

1. Take the state vector $x = [y\ \dot{y}]^T$, corresponding to the position and velocity of the ball. Write down the nonlinear differential equation for x. You will have a nonlinear function on the right hand side of the differential equation, so that the state equation is of the form

$$\dot{x} = f(x, u)$$

where $f(x,u) = [f_1(x,u) \;\; f_2(x,u)]^T$ is a 2 dimensional vector with component functions $f_1(x,u)$ and $f_2(x,u)$.

2. Let the desired equilibrium position be $\bar{y} = 0.5$. The equilibrium position is maintained by a steady state input current $\bar{u}$. Thus at equilibrium, $y = \bar{y}$, and $u = \bar{u}$, and there should be no change in the ball's position. Determine $\bar{u}$, taking it to be positive. Write down $f(x,u)$ and $\bar{u}$.

6.2 The Linearized Model

The majority of control system designs are based on linear models. In preparation for the controller design expeirment, you will first linearize the nonlinear model you have built about the equilibrium point

1. Linearize the system around the equilibrium point $x^* = \begin{bmatrix} \bar{y} & 0 \end{bmatrix}^T$, $u^* = \bar{u}$, where $\bar{y} = 0.5$ and $\bar{u}$ was determine in the nonlinear model preparation. Let

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}(x^*,u^*) & \frac{\partial f_1}{\partial x_2}(x^*,u^*) \\ \frac{\partial f_2}{\partial x_1}(x^*,u^*) & \frac{\partial f_2}{\partial x_2}(x^*,u^*) \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{\partial f_1}{\partial u}(x^*,u^*) \\ \frac{\partial f_2}{\partial u}(x^*,u^*) \end{bmatrix}$$

The linearized system is given by

$$\frac{d}{dt}\delta x = A\delta x + B\delta u$$
$$\delta y = C\delta x$$

where $\delta x = x - x^*$ denotes the deviation of the state from the equilibrium. Similarly, $\delta u = u - u^* = u - \bar{u}$. Note that $\delta x_1 = y - \bar{y} = \delta y$ and $\delta x_2 = x_2 = \dot{y}$. Record the resulting matrices A and B in your preparation.

7 Experiment

7.1 Controller Design for the Linearized System

We shall first design a controller for the linearized system.

1. Draw the Simulink diagram for the linearized system.

2. Note that when $\delta x = 0$, $y = \bar{y}$ and $x_2 = \dot{y} = 0$ so that the ball is balanced at the desired set point. When the ball is initially placed under the electromagnet at $t = 0$, $\delta x(0) \neq 0$. The control objective can now be recast as finding a control law such that $\delta x(t) \underset{t\to\infty}{\longrightarrow} 0$ when initially $\delta x(0)$ is nonzero. Let $\delta x(0) = \begin{bmatrix} 0.3 & 0.2 \end{bmatrix}^T$. Consider a control law of the form

$$\delta u = -K\delta x$$

The closed loop system is given by

$$\frac{d}{dt}\delta x = (A - BK)\delta x$$

We know that if the eigenvalues of $(A-BK)$ have negative real parts, we will have $\delta x(t) \underset{t\to\infty}{\longrightarrow} 0$, achieving the design objective. Find $K = [k_1 \; k_2]$ so that the eigenvalues of $(A - BK)$ are approximately at −0.2 and −0.3. You can do this analytically, or use the Matlab command "place". In using this control law, we are assuming that both the position and velocity of the ball can be measured. We call the control law a state feedback control law. Add this controller to the Simulink diagram for the linearized system, and show on the Simulink scope that δx converges to 0. In the space below, record your control gains and sketch or tape the response trajectories of $\delta x(t)$ and $\delta u(t)$ from the Simulink scope.

7.2 Control Design for the Nonlinear Maglev System

Although we have designed a controller for the linearized system, we can try using it on the original **nonlinear** system. We can expect that if the deviations from the equilibrium point are not large, the controller design based on linearization should still work well.

1. First draw the Simulink diagram for the nonlinear system, defining a suitable nonlinear function in your model using the Simulink **fcn** block from the nonlinear library. You may also find the **Mux** block from the "Signals and Systems" library useful.

2. In terms of the original control signal u, the feedback law is

$$u = \bar{u} - K\delta x = \bar{u} - k_1(y - \bar{y}) - k_2\dot{y}$$

   Again, since both y and $\dot{y}$ are used in feedback, we refer to this as a state feedback law. Add this controller to the Simulink diagram for the original **nonlinear** system. Note that for the simulation of the nonlinear system, you need to set the initial condition to $x(0) = x^* + \delta x(0)$. Tune the parameters k_1, k_2, if necessary, and check that the design specs are satisfied on the Simulink scope. Print your Simulink diagram implementing the state feedback controller and the closed loop responses of $x(t)$ and $u(t)$ as displayed on the Simulink scope. Label your printout "State feedback control design for maglev system" and insert it after this page.

3. Now that you have a basic working design, you can try to speed up the response by tuning k_1 and k_2. Find at least one K so that the linearized system eigenvalues are real and further away from 0, and another K so that the linearized system eigenvalues are complex with negative real parts. Try these gains in your controller and observe the response of the nonlinear system. Change the initial conditions to see when the controller fails to achieve the required specs. In the space below, describe your observations and discuss the performance of the various controllers, especially in connection with their ability to control the nonlinear maglev system over the range of initial conditions.

- *Name:*
- *Student No.:*

- *Name:*
- *Student No.:*

- *Name:*
- *Student No.:*

- *Lab Section, Date, Time:*