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ECE410F Control Systems
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Problem Set #1

1. Consider the following systems of differential equation involving the variables y_1, y_2, y_3 and the inputs u_1, u_2, and u_3:

$$\begin{aligned} \dddot{y}_1 + \ddot{y}_1 + 2(\dot{y}_1 + \dot{y}_2) + 2(y_1 - y_3) &= u_1 \\ \ddot{y}_2 + 3(\dot{y}_2 - \dot{y}_1 + 2\dot{y}_3) + y_2 - y_1 &= u_2 \\ \dot{y}_3 + y_3 - y_1 &= u_3 \end{aligned}$$

Let $x_1 = y$, $x_2 = \dot{y}_1$, $x_3 = \ddot{y}_1$, $x_4 = y_2$, $x_5 = \dot{y}_2$, $x_6 = y_3$. Write the state equation for $x = [x_1\ x_2\ \cdots\ x_6]^T$, where the superscript T denotes transposition. Express $y = [y_1\ y_2\ y_3]^T$ in terms of x.

2. Let A be given by

$$A = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}$$

Find the eigenvalues and eigenvectors of A. Hence find the diagonalizing matrix P such that $P^{-1}AP = \Lambda$, where Λ is a diagonal matrix consisting of the eigenvalues of A. Use these results to compute e^{At}.

3. Let A be given by

$$A = \begin{bmatrix} -3 & -2 & 2 \\ 1 & 1 & -1 \\ -3 & -2 & 2 \end{bmatrix}$$

Find the eigenvalues and eigenvectors of A. Hence find the diagonalizing matrix P such that $P^{-1}AP = \Lambda$, where Λ is a diagonal matrix consisting of the eigenvalues of A. Determine e^{At}.

4. Let the matrix A be given by

$$A = \begin{bmatrix} \sigma & \omega \\ -\omega & \sigma \end{bmatrix}$$

Use the diagonalization method (extending the scalar field to complex numbers) to determine e^{At}.

5. In the previous problem, we saw that when

$$A = \begin{bmatrix} \sigma & \omega \\ -\omega & \sigma \end{bmatrix}$$

its matrix exponential is easy to write down. This problem demonstrates that a matrix with distinct complex eigenvalues can be transformed into the above form using a real nonsingular transformation. Let the matrix A be given by

$$A = \begin{bmatrix} -1 & -4 \\ 1 & -1 \end{bmatrix}$$

(a) Determine the eigenvalues and eigenvectors of A, noting that they form complex conjugate pairs. Let the first eigenvalue be written as $a + ib$ with the corresponding eigenvector $v_1 + iv_2$. v_1 and v_2 are 2-dimensional vectors.

(b) Set

$$P = [v_1 \quad v_2]$$

Determine $D = P^{-1}AP$. Hence determine e^{At}.

You can think of this as an alternative way of computing e^{At} when A has complex eigenvalues. Instead of using a complex nonsingular matrix to transform A into diagonal form with complex eigenvalues along the diagonal (the method of Problem 4), this method transforms A, using a real nonsingular P, into block diagonal form with each 2×2 block of the form $\begin{bmatrix} \sigma & \omega \\ -\omega & \sigma \end{bmatrix}$.