University of Toronto
Department of Electrical and Computer Engineering
ECE410F Control Systems
Fall 2008
Problem Set #5

1. The system is described by the state equation

$$\dot{x} = \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix} x + \begin{bmatrix} \alpha \\ 1 \end{bmatrix} u \quad \alpha \neq 1$$

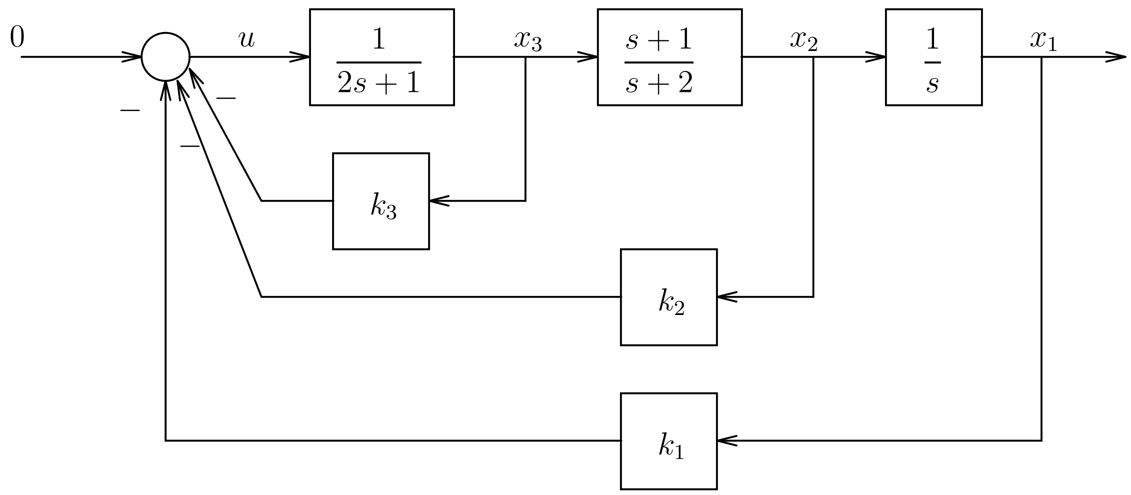
Find a state feedback gain k^T in terms of the parameter α such that the eigenvalues of the closed loop system are at $-2, -3$.

2. Let $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

(i) Check that (A, B) is controllable.

(ii) Find a feedback law $u = -k^T x$ such that the closed-loop poles are all at -1.

3. Find constants k_1, k_2, k_3 such that the closed-loop poles of the following system are assigned to $\{-1, -3 \pm j\}$.



4. Given

$$A = \begin{bmatrix} 5 & 2 & 1 \\ -2 & 1 & -1 \\ -4 & 0 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} -2 & 2 \\ 2 & -1 \\ 2 & -2 \end{bmatrix}$$

(i) Verify that the system is controllable, but not from a single column of B.

(ii) Using the multivariable pole assignment algorithm described in Chapter 3, find a feedback gain K such that the closed-loop poles are assigned to $\{-1, -2 \pm i\}$.

5. The pair (A, B) are given by

$$A = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

(a) Design a feedback gain K, using the multivariable algorithm from Chapter 3, to place the closed loop poles at $-2, -3, -4$.

(b) Without using the multivariable algorithm from Chapter 3, find a different feedback gain K such that the closed loop poles are at $-2, -3, -4$ (Hint: Examine the structure of (A, B) to write down the answer right away!).

(c) Use the Matlab command "place" to generate possibly yet another K which achieves the pole assignment.

(d) You can compare the sizes of the 3 solutions by computing the norm of the gain K in each case (Use the Matlab command "norm"). The one that has the largest norm roughly corresponds to the most "expensive" solution. Which K has the largest norm?

This problem illustrates that there are many ways to do multivariable pole placement. The method described in Chapter 3 is guaranteed to work, but there may well exist much simpler constructions.

6. For the system $\dot{x} = Ax + Bu$ with

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$$

(i) Verify that the system is not controllable.

(ii) Determine whether or not the system is stabilizable using the PBH test.