University of Toronto
Department of Electrical and Computer Engineering
ECE410F Systems Control
Fall 2008
Problem Set 7

1. Consider the linear system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} x \end{aligned}$$

   (i) Suppose the control design objective is closed loop stability and that the output y asymptotically tracks a constant reference set point y_d. Determine, using feed-forward control, a state feedback controller which will place the closed loop poles at -1 and -2 and achieves asymptotic output tracking.

   (ii) Now assume that the state is not measured. Design a full order observer to estimate the state that is not directly measured, placing the observer poles at -4 and -5.

   (iii) Using the separation principle, combine the results of (i) and (ii) to design an output feedback controller which, together with the feedforward of the reference input, ensures that the design objectives are satisfied. Determine the DC-gain and the closed loop transfer function from y_d to y, and verify that indeed asymptotic tracking is achieved.

2. Again consider the linear system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} x \end{aligned}$$

   (i) Augment the system dynamics with the equation

$$\dot{\xi} = y - y_d$$

   Design a state feedback law of the form

$$u = -Kx - K_I \xi$$

   to place the poles of the closed loop system at -1,-1,-2. Determine the closed transfer function from y_d to y and verify that asymptotic tracking is achieved.

(ii) Now suppose that system matrix is perturbed to

$$A_p = \begin{bmatrix} 0 & 1 \\ 2.1 & 1 \end{bmatrix}$$

Assume that the same control law as in (i) is used. Verify that the closed loop system is still stable and that asymptotic tracking is maintained despite the perturbation.

(iii) Now use the full order observer contructed in problem 1 to design an output feedback law to achieve the design objectives. Determine the DC-gain and the closed loop transfer function from y_d to y, and verify that asymptotic tracking is achieved.

3. We have studied 2 methods to achieve asymptotic set point tracking. In this problem we consider combining both methods to see if there are any additional advantages to be gained.

(i) Consider the simple scalar system

$$\begin{aligned} \dot{x} &= 2x + u \\ y &= x \end{aligned}$$

Again the objective is stabilization and for y to track y_d. First we add an integrator

$$\dot{\xi} = y - y_d$$

Now determine a feedback law $u = -Kx - K_I\xi$ such that the closed loop poles are located at -1 and -2.

(ii) Now add also the feedforward term Ny_d to the controller so that $u = -Kx - K_I\xi + Ny_d$. Determine the closed loop transfer function from y_d to y as a function of the gain N, and verify that asymptotic tracking is achieved for any N. What role can N play?

(iii) To see the effect of N, use the matlab command lsim in the form

$$[y, x] = lsim(num, den, u, T)$$

Take for example, $y_d = 1$, $T = 0 : 0.001 : 10$, $u = ones(length(T), 1) * y_d$. Run lsim, and plot(T,y), varying N from 0 to 2.5 What can you conclude from the plots?