On control policies for automated traffic

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In automated traffic vehicles perform a sequence of activities that are implemented through vehicle control laws. The characteristics of the control law are abstracted in a flow model by the space-time of the activity. The space-time abstraction is used to set an upper limit on the density of traffic flow. If all vehicles perform an activity α and the space-time for this activity is $\lambda(\alpha) = s \cdot \tau$, with s in meters and τ in seconds then the maximum density is $k = \frac{1}{s}$. The activity flow model [1] addresses the case when s, τ, and the choice of activity α are variable.

Activity Model The highway is divided into sections indexed i which are one lane wide and of length $L(i)$. Sections j and k are to the right and left, respectively, of section i, and section $i+1$ is downstream of section i. Time is discretized with a time period of T seconds. Flow types, indexed θ, distinguish the destination and vehicle body type. The states of the model are $n(i,t,\theta)$, the number of vehicles in section i at time t and of flow type θ, and $v(i,t)$, the average velocity in i at time t. The control inputs are $v_d(i,t)$, the desired average speed, $f(i,t)$, the volume of entry flow, and $\pi(\alpha,i,t,\theta)$, the proportion of vehicles of type θ in section i at time t that will perform activity α. $u(t) = [\pi(t), v^d(t), f(t)]$ is called a Traffic Management Center (TMC) plan.

The activity model uses a conservation of vehicles law and velocity dynamics equation to update the states. The conservation law consists of two steps: lateral motion of vehicles doing a lane change activity, followed by longitudinal motion. We let α_r (α_l) denote the set of activities that turn right (left) and π_r (π_l) be the proportion of vehicles that turn right (left). π_s is the proportion of vehicles that go straight. (Note $\pi_r(i,t,\theta) + \pi_l(i,t,\theta) + \pi_s(i,t,\theta) = 1$ for each i,t,θ.) π_r and π_l represent successful lane changes. Considering vehicles that go straight, we define $\rho(i,t)$ to be the fraction of vehicles in section i at time t that remain in the section at time $t+T$. Assuming a uniform spatial distribution of vehicles of the same flow type within a section we have:

$$\rho(i,t) := 1 - \frac{v(i,t) \times T}{L(i)} . \tag{1}$$

Let $n_{long}(i,t,\theta)$ be the number of vehicles in section i at time t of type θ after lane changes are done, given by:

$$n_{long}(i,t,\theta) = n(i,t,\theta)\pi_s(i,t,\theta) + n(j,t,\theta)\pi_r(j,t,\theta) + n(k,t,\theta)\pi_l(k,t,\theta). \tag{2}$$

Then, the conservation of vehicles law is:

$$n(i,t+T,\theta) = n_{long}(i,t,\theta)\rho(i,t) + n_{long}(i-1,t,\theta)[1-\rho(i-1,t)] + f(i,t,\theta). \tag{3}$$

The velocity in a section i is limited by the space available in the downstream section. Let $v_s(i,t)$ be the maximum speed in section i so as not to exceed the space available in section $i+1$. Then the speed achieved in a section can be no larger than $v_d(i,t)$ and $v_s(i,t)$ and the velocity law gives the average speed over period t as

$$v(i,t) = \max\{0, \min\{v_d(i,t), v_s(i,t)\}\} . \tag{4}$$

Finally, the flows and activities are constrained by the maximum available space-time in a section over one period. The space-time for an activity can be computed using a specification of the space as a function of time, given by $s(t)$, and the duration of the activity, given by τ. The space-time is

$$\lambda(\alpha) = \int_0^\tau s(t)dt$$

in section i over period t. The space-time constraint is

$$\begin{aligned} L(i) \cdot T \geq \\ \sum_{\alpha}\sum_{\theta} n(i,t,\theta)\pi(\alpha,i,t,\theta)\lambda(\alpha) + \\ \sum_{\alpha_r}\sum_{\theta} n(j,t,\theta)\pi(\alpha,j,t,\theta)\lambda_r(\alpha) + \\ \sum_{\alpha_l}\sum_{\theta} n(k,t,\theta)\pi(\alpha,k,t,\theta)\lambda_l(\alpha). \end{aligned} \tag{5}$$

Control It is necessary to develop adaptive TMC policies as typically the demanded input flows are not stationary and congestion can develop when, for example, many automated vehicles have the same exit at the same time.

We consider a single lane with sections $i = 1, ...I$ and simplify notation by eliminating indices for θ and α.

Define $n(i) = \sum_\theta n(i, \theta)$, the total number of vehicles in section i, and $\pi(\alpha, i)$, the proportion of vehicles performing activity α by

$$\pi(\alpha, i) = \frac{\sum_\theta \pi(\alpha, i, \theta) n(i, \theta)}{\sum_\theta n(i, \theta)} . \qquad (6)$$

Then $\lambda(i)$, the average space-time used per vehicle in section i, is $\lambda(i) = \sum_\alpha \lambda(\alpha)\pi(\alpha, i)$. $\lambda(i)n(i)$ is the space-time used by vehicles in section i. The maximum number of vehicles in a section is $N(i) = \frac{L(i)T}{\lambda(i)}$. Consequently, the maximum flow for section i is $\overline{\phi}(i) = \frac{VT}{\lambda(i)}$, where V is the maximum speed, and the link or lane capacity is $\phi^* = \min_i \overline{\phi}(i)$. We assume $\phi(I) = \phi^*$.

A maximum throughput, minimum time policy is one that achieves $\phi(i) = \phi^*$ and $v(i, t) = V$ for all i. In this case, $n(i, t)$ must satisfy

$$n(i, t) \leq \frac{\phi^* L(i)}{V} = N^*(i).$$

To ensure section i does not exceed $N^*(i)$ we artificially increase $\lambda(i)$ to $\lambda^* = \frac{L(i)T}{N^*(i)} = \frac{VT}{\phi^*}$. Applying a ϕ^*-filling velocity policy for v_d results in [1]:

$$v_d(i-1, t) = \min\{V, \tfrac{L(i)L(i-1)}{n(i-1,t)\lambda^*} - [1 - \tfrac{v(i,t)T}{L(i)}]\tfrac{n(i,t)L(i-1)}{n(i-1,t)T}\}\}. \qquad (7)$$

The entry flow policy is:

$$f(t) = \frac{L(1)}{\lambda^*} - \frac{n(1,t)}{T} + \frac{v(1,t)n(1,t)}{L(1)}. \qquad (8)$$

Theorem 1 Using a ϕ^*-filling policy (7), (8) and assuming $v(I) = V, \forall t$, then for every t, i, either $v(i, t) = V$, or $\phi(i, t) \geq \phi^*$.

Proof In [1] . □

The theorem suggests an adaptive approach for estimating ϕ^*, particularly when it suddenly drops to an unknown value. If vehicles observe $v(i, t) < V$, then at some time $\phi(i, t) \geq \phi^*$. Let ϕ^k be the kth guess of ϕ^*.

Lemma 1 Using a ϕ^k-filling policy, $\phi^k \leq \phi^*$, and $v(I, t) = V, \forall t$, then $v(i, t)$ converges to V in finite time, for all i.

Proof The ϕ^k-filling policy uses $\lambda^k = \frac{VT}{\phi^k}$ and substituting in (3) and (4), gives $n^k(i) = \frac{L(i)T}{\lambda^k}, \forall i$. By induction, assume $v(i, t) = V$, some t. Substituting $n^k(i)$, and λ^k in (7) we obtain $v(i-1, t) = V$. □

Lemma 2 Using a ϕ^k-filling policy, $\phi^k > \phi^*$, and $v(I, t) = V, \forall t$, then $v(i, t)$ converges to $\frac{\lambda^k}{\lambda^*}V$ in finite time, for all $i < I$.

Proof Apply the same steps as in Lemma 1. □

In principle, assuming steady-state conditions, vehicles could estimate $\phi^* = \frac{v^k(i)T}{\lambda^k(i)}$. Since this approach is not robust we employ an adaptive scheme. To add realism to the model, we assume λ has (discrete) dynamics given by

$$\lambda^{k+1}(i) = (1 - T\tau)\lambda^k(i) + T\tau\lambda_{ss}(i) \qquad (9)$$

where τ(sec) is the time constant, $T < \tau$ and λ_{ss} is the steady-state value.

Theorem 2 Using a ϕ^k-filling policy, $\phi^k > \phi^*$, $v(I, t) = V, \forall t$, and adaptation rule $\lambda_{ss}(i) = \lambda^k(i)\frac{V}{v^k(i)}$, the system (3), (4), (7), (8) converges to the equilibrium solution $\lambda(i) = \lambda^* + \epsilon(i)$, $\phi = \phi^*$, $v(i) = V$, $n(i) = \frac{\lambda^*}{\lambda(i)}N^*(i)$, in a finite number of steps, $\forall i < I$.

Proof Substituting $\lambda_{ss}(i)$ in (9),

$$\lambda^{k+1}(i) = (1 - T\tau + T\tau\frac{V}{v^k(i)})\lambda^k(i).$$

Consider section $I - 1$. Since $v(I) = V$ and $\lambda(I) = \lambda^*$, substituting in (7), (4) we obtain $v^k(i) = \min\{V, \frac{\lambda^k(i)}{\lambda^*}V\}$. If $\lambda^k(i) < \lambda^*$, then $v^k(i) < V$ and $\lambda^k(i)$ increases. For some $k > K$, $\lambda^k(i) = \lambda^* + \epsilon(i)$, $\epsilon(i) > 0$ and $v^k(i) = V$. Therefore, $\lambda^{k+1}(i) = \lambda^k(i)$. Using Lemma 2, $v^k(i)$, $i < I$ converges to V in a finite number of steps. Therefore, $\lambda^k(i)$ converges to $\lambda(i)$ in a finite number of steps, and $n(i) = \frac{L(i)T}{\lambda(i)} = \frac{\lambda^*}{\lambda(i)}N^*(i)$. □

Implementation The above scheme works well when the flow is homogeneous as in an ACC architecture: each vehicle uses $\lambda(i)$. In a non-homogeneous flow such as platooning, only lead vehicles can adjust their space-time. We consider the platooning architecture with vehicles performing the activities $\alpha_l = leader$ and $\alpha_f = follower$. Let $\lambda_l(i) = \lambda(\alpha_l)\pi(\alpha_l, i)$ and $\lambda_f(i) = \lambda(\alpha_f)\pi(\alpha_f, i)$. Then $\lambda(i) = \lambda_l(i) + \lambda_f(i)$. Now $\lambda^k(i)$ is adjusted by changing $\lambda_l(i)$. In particular we propose the adaptation rule

$$\lambda_{ss}(i) = \lambda_l(i)\frac{V}{v^k(i)} + \lambda_f(i).$$

That is, leaders use space $\lambda^k(\alpha_l) = \lambda(\alpha_l)\frac{V}{v^k(i)}$. Assuming $\pi(\alpha_l, i) > 0, \forall i < I$, a result similar to Theorem 2 can be obtained in this case.

The policies discussed can be implemented directly using (7) and (8) if ϕ^* is communicated by a link layer controller to all vehicles in the link. Finally, an important extension is to include velocity dynamics.

References

[1] M. Broucke and P. Varaiya. A theory of traffic flow in automated highway systems. *Transportation Research-C*, vol. 4, no. 4, pp. 181-210, 1996.