A DYNAMIC MODEL FOR TRAFFIC FLOW ON AUTOMATED HIGHWAY SYSTEMS

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Abstract: This paper presents a dynamic model for automated traffic flow. The model is based on the abstraction of vehicle activities derived from a vehicle's automatic control laws by the space and time taken up by the vehicle engaged in the activity. The paper develops the laws of conservation of vehicles and velocity dynamics, and defines static, manuever, and interaction capacity obtainable with the model.

Keywords: automated highway system, traffic flow models, capacity of automated highways

1. INTRODUCTION

In this paper we describe a model for obtaining realistic estimates of highway capacity. The model has features of a traditional traffic flow model; it also has features that allow it to differentiate among activities performed by vehicles and thereby capture the capacity limits imposed by those activities. Activities are used to characterized vehicle behavior that results from vehicle-borne control laws. Activities provide a means to abstract vehicle behavior without requiring individual simulation of vehicles, thus yielding a meso-scale simulation model. This paper is an extension of the work in (Broucke and Varaiya, 1996) which presented theory for a one-lane automated highway with stationary input flows.

2. THE DYNAMIC ACTIVITY MODEL

We assume that the AHS consists of several lanes, with some lanes including entrances and exits, and the length of the highway divided into sections. Referring to Figure 1, we label the sections $i = 1, ..., I$ with length $L(i)$. Each section can have at most one entrance and one exit. Vehicles have types indexed by θ which may stand for their origin and destination and all other distinguishing characteristics of interest; in particular, vehicle body type: passenger, truck, bus, etc. The model has two states: $n(i, t, \theta)$ is the number of vehicles of type θ in section i at time t and $v(i, t)$ is the average speed of vehicles in section i at time t, measured in meters/sec. It is required that $v(i, t) \leq V(i, \theta)$, the maximum permissible or free flow speed. Thus, the *state* of the system at time t is $x(t) = \{n(i, t, \theta), v(i, t)\}$. Time is indexed $t = 0, 1, ...$ Each time period is T seconds long.

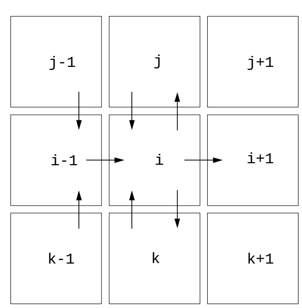

Fig. 1. The highway configuration.

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2.1 *Activities*

At each instant of time every vehicle is engaged in one of a finite number of activities such as cruising, changing a lane, entering the highway, exiting the highway, etc. The passage of a vehicle through the automated highway can be summarized by the sequence of activities that the vehicle executes. The set of vehicle activities is indexed by α. An *activity plan* is an array of non-negative numbers $\pi = \{\pi(\alpha, i, t, \theta)\}$ denoting the fraction of $n(i, t, \theta)$ vehicles that are engaged in activity α. Thus, π satisfies for every i, t, θ $\sum_\alpha \pi(\alpha, i, t, \theta) \equiv 1$.

While it is engaged in a particular activity, a vehicle's motion is governed by a feedback control law which ensures that this activity is carried out safely; the feedback laws guarantee that a vehicle will "occupy" some safe distance s meters of a highway lane for some duration d. This space and time is determined by the open-loop profile of the feedback law. We associate with each activity α the space and time (in meter-sec) $\lambda(\alpha) > 0$ utilized by each vehicle engaged in that activity.

For example, if the vehicle control law is tracking a desired velocity profile $v(t)$ for a time τ, then the instantaneous space usage can be approximated, assuming the tracking time constant is large, as $s(t) = \int_0^\tau v(t)dt$ and $\lambda(\alpha) = \int_0^\tau s(t)dt$. The control law may not be formulated as an open-loop specification of a desired velocity profile. In a feedback formulation of the control law (see (Frankel *et al.*, 1995)) an envelope of safe speed as a function of the separation between two vehicles is found. What is known about the maneuver is the initial separation x_0, the final separation x_f, and the allowed speed as a function of separation $v(x)$. The space-time can be found as follows. First compute the elapsed time as a function of separation: $t(x) = \int_{x_0}^x v(x)dx$. Next integrate the elapsed time from initial to final separation: $\lambda(\alpha) = \int_{x_0}^{x_f} t(x)dx$.

For activities involving vehicles in two lanes, as happens during a lane change and in some implementations of entry or exit, the vehicle occupies a minimum safety distance in both lanes. We denote λ_l (λ_r) to be the space taken up in the adjacent left (right) lane for vehicles engaged in a left (right) lane change activity. If adjacent lanes have different speeds, then extra space-time must be allowed for vehicles either to slow down or speed up to the desired speed. We assume that the coordination policy of the automated lanes is that faster lanes slow down to accomodate slower incoming vehicles and faster vehicles slow down in their starting lane to enter a slower lane. For example, if the starting lane has speed v_1 and the receiving lane has speed $v_2 \leq v_1$ and the deceleration of vehicles is $-a$, then the extra space needed to deceleration in the originating lane is $\frac{(v_1-v_2)^2}{2a}$ for a duration of $(v_1 - v_2)/a$ seconds. If the lane change requires λ_1 m-sec in the originating lane and λ_2 m-sec in the destination lane when the lane speeds are the same, then the space-time including lane speed differentials for a right lane change is

$$\lambda(lc) = \lambda_1 + \frac{(v_1 - v_2)^3}{2a^2}$$
$$\lambda_r(lc) = \lambda_2 \ .$$

When a vehicle engaged in activity α leaves a section, its $\lambda(\alpha)$ space is available for use by another vehicle from the upstream section. If the activities that vehicles are executing are highly space consuming, the speed in upstream sections may be forced below the maximum or free flow speed. In this manner the model captures the effect of queueing.

Two vehicles with the same (i, t, θ) index and engaged in the same activity cannot be further distinguished within the model. We also assume that vehicles of the same flow type are distributed uniformly within a section.

2.2 *Flow control*

The flow is controlled by means of three control parameters: the activity proportions $\pi(\alpha, i, t, \theta)$, the section desired velocities $v_d(i, t)$, and the entry flows $f(i, t, \theta)$. The *activity plan*, as defined above, is an array of non-negative numbers $\pi = \pi(\alpha, i, t, \theta)$ denoting the fraction of vehicles of flow type θ that will perform activity α in time period t in section i. The *velocity plan* is an array of nonnegative numbers $v_d = \{v(i, t)\}$ (in meters/sec), denoting the desired average velocity of vehicles in section i for time period t. The vehicles are assumed to be equipped with longitudinal controllers capable of tracking a desired average velocity. The *entry plan* is an array $f = \{f(i, t, \theta)\}$, denoting the number of vehicles of type θ that enter the highway in section i in period t. Each entry point on the AHS is assumed to be equipped with metering capability. We call $u(t) = [\pi(t), v^d(t), f(t)],\ t \geq 0$, a TMC *plan*.

2.3 *Conservation of vehicles*

Consider a section which has no entrance or exit. Suppose we are given the state $x(t)$ and an activity plan π at time t. Conservation of vehicles provides that for all t and $1 \leq i \leq I$,

$$n(i, t+1, \theta) = n(i, t, \theta) + \phi^{in}(i, t, \theta) - \phi^{out}(i, t, \theta) + f(i, t, \theta). \qquad (1)$$

$\phi^{in}(i,t,\theta)$ is the flow of vehicles entering section i, $\phi^{out}(i,t,\theta)$ is the flow of vehicles leaving section i, both in vehicles/sec, and $f(i,t,\theta)$ is the number of vehicles incoming via an entrance during the period t.

First we will compute $\phi^{out}(i,t,\theta)$ which comprises all vehicles that leave section i in the interval t to $t+T$. To simplify notation, let α_r (α_l) index the set of activities that turn right (left) and π_r (π_l) be the proportion of vehicles that turn right (left). Also, π_s is the proportion of vehicles that go straight. For example, $\pi_r(i,t,\theta) = \sum_{\{\alpha_r\}} \pi(\alpha,i,t,\theta)$ and $\pi_r + \pi_l + \pi_s = 1$. We will assume that π_r and π_l incorporate all vehicles that succeed in making the lane change. This means that the necessary coordination between lanes is included a priori in the value of π.

Considering vehicles that go straight, we define $\rho(i,t)$ to be the fraction of vehicles in section i at time t that remain in the section at time $t+1$. Using the assumption of uniform spatial distribution of vehicles of the same flow type within a section, we have:

$$\rho(i,t) := 1 - \frac{v(i,t) \times T}{L(i)} \ . \tag{2}$$

$[1-\rho(i,t)]$ is the fraction of vehicles in section i at time t that leave the section at the end of that period. $\rho(i,t)$ must be positive to ensure that vehicles cannot cross a section in less than one time period.

We can express $\phi^{out}(i,t,\theta)$ as:

$$\begin{aligned}\phi^{out}(i,t,\theta) &= n(i,t,\theta)\pi_r(i,t,\theta) + n(i,t,\theta)\pi_l(i,t,\theta) \\ &\quad + n(i,t,\theta)\pi_s(i,t,\theta)[1-\rho(i,t)] \ .\end{aligned}$$

This equation says that all vehicles changing lanes move out of the section, while the proportion $[1-\rho(i,t)]$ of those going straight will move out.

Vehicles that contribute to $\phi^{in}(i,t,\theta)$ come from upstream and the right and left sections. The number of vehicles that leave section $i-1$ is $n(i-1,t,\theta)\pi_s(i-1,t,\theta)[1-\rho(i-1,t)]$. Vehicles can change right or left into section $i-1$ and then travel downstream into section i during the interval t to $t+T$. We will assume that those vehicles adopt the speed of section $i-1$ after they change into it. Therefore, the number of vehicles doing a lane change from sections $j-1$ into section $i-1$ and then moving downstream into section i is given by $n(j-1,t,\theta)\pi_r(j-1,t,\theta)[1-\rho(i-1,t)]$. Similarly for vehicles from section $k-1$. Finally, some vehicles changing into section i from sections j and k will stay in section i at the end of the period. The number of vehicles staying in section i after changing lanes from section j is given by $n(j,t,\theta)\pi_r(j,t,\theta)\rho(i,t)$. Adding up these contributions we obtain:

$$\begin{aligned}&\phi^{in}(i,t,\theta) = \\ &\quad n(i-1,t,\theta)\pi_s(i-1,t,\theta)[1-\rho(i-1,t)] \\ &\quad + n(j-1,t,\theta)\pi_r(j-1,t,\theta)[1-\rho(i-1,t)] \\ &\quad + n(k-1,t,\theta)\pi_l(k-1,t,\theta)[1-\rho(i-1,t)] \\ &\quad + n(j,t,\theta)\pi_r(j,t,\theta)\rho(i,t) \\ &\quad + n(k,t,\theta)\pi_l(k,t,\theta)\rho(i,t) \ .\end{aligned}$$

We can simplify the conservations equations just derived by breaking up the movement of vehicles in two steps: first move vehicles laterally; second, move them downstream. We let $n_{long}(i,t,\theta)$ be the number of vehicles in section i at time t of flow type θ after lane changes are done, given by:

$$\begin{aligned}n_{long}(i,t,\theta) &= n(i,t,\theta)\pi_s(i,t,\theta) + \\ &\quad n(j,t,\theta)\pi_r(j,t,\theta) + n(k,t,\theta)\pi_l(k,t,\theta).\end{aligned} \tag{3}$$

If we apply this definition to Equation 1 we obtain

$$\begin{aligned}n(i,t+1,\theta) &= n_{long}(i,t,\theta)\rho(i,t) + \\ &\quad n_{long}(i-1,t,\theta)[1-\rho(i-1,t)] + f(i,t,\theta).\end{aligned} \tag{4}$$

Equations (3) and (4) constitute the conservation of vehicles law for multi-lane flow.

2.4 *Velocity dynamics*

For each period $[t, t+1)$ and every section, the TMC commands a set of activities summarized by $\pi(\alpha,i,t,\theta)$, a desired average velocity $v_d(i,t)$. and an input flow $f(i,t,\theta)$. Each vehicle will attempt to track $v_d(i,t)$ as the nominal velocity of each activity. The dynamics of the tracking law will determine whether there is any delay in tracking v_d. Here we assume that the tracking time constants of the longitudinal controller are much faster than the time scale of maneuvers so that by the end of a time period (after maneuvers are completed) vehicles have achieved the nominal velocity (in free flow conditions). The velocity in section i is limited by the space available in the downstream section. Let $v_s(i,t)$ be the maximum speed in section i so as not to exceed the space available in section $i+1$. Finally, the speed achieved in a section, which can be no larger than the desired velocity and the space-filling velocity can be reduced by interactions among vehicles. This loss in speed due to interactions (see (Prigogine, 1971) for a related treatment), called δv, is subtracted from the speed achievable in a section in non-interacting flow. Thus, the speed with which vehicles move on average over period t is

$$v(i,t) = \min\{v_d(i,t), v_s(i,t)\} - \delta v(i,t) \ .$$

Space-filling velocity The space-filling velocity is found by a backward recursion through the sections, by computing the space freed in each section and the space demanded by the upstream sections. Once the space-filling velocity of section i is known, the space-filling velocity of section $i-1$ can be computed. We derive the expression for $v_s(i-1,t)$ given $v_s(i,t)$.

Each exit section, i.e., $i \in I_{exit}$, has a maximum exit flow $g_{max}(i)$ in vehicles/sec. We assume that exit sections cannot have entry flows. The flow that exits over period t consists of vehicles traveling straight and those that change lanes into the section, i.e., $n_{long}(i,t)$. Vehicles changing into the section are assumed to adopt the destination lane speed. The exit flow is given by $g(i,t) = \sum_\theta \frac{n_{long}(i,t,\theta)v(i,t)}{L(i)}$ Letting $n_{long}(i,t) = \sum_\theta n_{long}(i,t,\theta)$ we find $v_s(i,t) = \frac{g_{max}(i)L(i)}{n_{long}(i,t)}$.

Next, consider all other sections $i \in I \setminus I_{exit}$. To determine $v_s(i-1,t)$, we need to know the free space available in section i at the end of time period t. This is the total free space $L(i)T$ minus the space-time used by vehicles that stay in section i for the time period t. The vehicles that stay in the section are either going straight or they are changing lanes into the section but do not travel out of the section in the time period. We let $\lambda(\alpha)$ be the space-time used by vehicles in the section they currently occupy and $\lambda_r(\alpha)$ ($\lambda_l(\alpha)$) be the space-time used by vehicles in the section to the right (left), if α involves a right (left) lane change. The free space is

$$\begin{aligned} S_{fr}(i,t) &= L(i)T \qquad (5)\\ &-\sum_\theta \sum_\alpha n(i,t,\theta)\pi(\alpha,i,t,\theta)[1-\frac{v_s(i,t)T}{L(i)}]\lambda(\alpha)\\ &-\sum_\theta \sum_{\alpha_r} n(j,t,\theta)\pi(\alpha,j,t,\theta)[1-\frac{v_s(i,t)T}{L(i)}]\lambda_r(\alpha)\\ &-\sum_\theta \sum_{\alpha_l} n(k,t,\theta)\pi(\alpha,k,t,\theta)[1-\frac{v_s(i,t)T}{L(i)}]\lambda_l(\alpha). \end{aligned}$$

Next, we need the space used by vehicles from upstream and incoming via entrances into section i. The vehicles arriving from upstream sections travel at their current speed of $v(i-1,t)$ and continue doing the activity plan commanded at time t for section $i-1$, inspite of the fact that they can enter section i during the time period. We assume that x percent of the available space in section i can be used by upstream vehicles, and y percent can be used by the entry flow, where $x+y=1$. We call $S_{in}(i,t)$ the space-time used by the entry flow and $S_{up}(i,t)$ the space-time used by upstream vehicles. The objective is to balance S_{fr} with S_{in} and S_{up} by adjusting $v(i-1,t)$ and $f(i,t)$.

The incoming space taken by upstream vehicles assuming they travel at maximum speed V is given by:

$$\begin{aligned} &S_{up}(i,t) = \qquad (6)\\ &\sum_\theta \sum_\alpha n(i-1,t,\theta)\pi(\alpha,i-1,t,\theta)\frac{VT}{L(i-1)}\lambda(\alpha) +\\ &\sum_\theta \sum_{\alpha_r} n(j-1,t,\theta)\pi(\alpha,j-1,t,\theta)\frac{VT}{L(i-1)}]\lambda_r(\alpha) +\\ &\sum_\theta \sum_{\alpha_l} n(k-1,t,\theta)\pi(\alpha,k-1,t,\theta)\frac{VT}{L(i-1)}\lambda_l(\alpha). \end{aligned}$$

The incoming space taken by entry flows $S_{in}(i,t)$ is analogous to the expression for S_{up} and is omitted for the sake of brevity.

Finally, to compute $v_s(i,t)$, we fill the downstream section, i.e.,

$$\begin{aligned} S_{up}(i,t) &= x \cdot S_{fr}(i,t),\\ S_{in}(i,t) &= y \cdot S_{fr}(i,t). \end{aligned}$$

Defining $\gamma(i-1,t) = \min\{1, \frac{x \cdot S_{fr}(i,t)}{S_{up}(i,t)}\}$ we obtain the result $v_s(i-1,t) = \gamma(i-1,t)V$.

Speed loss from interactions

At each instant of time, the automated flow consists of vehicles and the safety distance they reserve by their longitudinal control laws. Each section has some average free space in meters over the period t that is not reserved by an activity, given by: $L(i) - \frac{1}{T}\sum_\alpha \sum_\theta n(i,t,\theta)\pi(\alpha,i,t,\theta)\lambda(\alpha)$. We adopt the convention that free space appears immediately downstream from the safety gap in front of a vehicle. Vehicles that require free space to perform a new activity (that uses more space, on average, than the last activity) always obtain the space from upstream by slowing down. This causes a disturbance in velocity to vehicles that are caught between the vehicle demanding the free space and the desired free space.

A pair of activities (α_1, α_2) is said to be *interacting* at time t if vehicle η performs the activity α_1 in period $t-1$, activity α_2 in period t, and

$$\begin{array}{ll} \lambda(\alpha_2) > \lambda(\alpha_1) & \text{if } \alpha_1 \in \alpha_s\\ \lambda(\alpha_2) > \lambda_r(\alpha_1) & \text{if } \alpha_1 \in \alpha_r\\ \lambda(\alpha_2) > \lambda_l(\alpha_1) & \text{if } \alpha_1 \in \alpha_l \end{array}$$

We call α_{int} the set of interacting activity pairs. The pair (α_1, α_2) is said to be *non-interacting* if it is not interacting. Non-interacting activities result in new free space in period t.

We can think of the activity plan $\pi(\alpha,i,t,\theta)$ as assigning a probability that a vehicle of type θ will perform activity α in section i at time t. The probability that a vehicle will perform the activity pair (α_1,α_2) is $\pi(\alpha_2,h,t,\theta)\pi(\alpha_1,i-1,t-1,\theta)$

where $h \in \{i-1, i, j, k\}$. We define $N_{int}(i,t)$ as the number of vehicles that perform interacting activity pairs in section i at time t. For brevity, we omit the expression for $N_{int}(i,t)$ here.

To investigate the effect of interacting activity pairs, we employ a queueing model developed in (Broucke and Varaiya, 1996). Consider one interacting activity pair (α_1, α_2). Let ΔS be the extra space in meters needed on average by vehicle η in the time period t; that is, $\Delta S = \frac{1}{T}[\lambda(\alpha_2) - \lambda(\alpha_1)]$. Suppose that the free space distance x_j is an exponentially distributed random variable with mean μ^{-1}, i.e., it has the probability density, $p(x) = \mu e^{-\mu x}$, $x \geq 0$. If we label the free space distance directly upstream from vehicle η as x_1, the next free space distance from vehicle η as x_2, and so forth, then the number of vehicles upstream affected by a disturbance ΔS is M where:

$$\sum_{j=1}^{M} x_j \leq S < \sum_{j=1}^{M+1} x_j$$

The slowdown to each vehicle k is $\Delta S - \sum_{j=1}^{k} x_j$, $k = 1, ..., M$. so the total slowdown δ to the M vehicles is

$$\delta = \sum_{k=1}^{M} [\Delta S - \sum_{j=1}^{k} x_j] = M \Delta S - \sum_{k=1}^{M} \sum_{j=1}^{k} x_j . (7)$$

Let us compute the new average speed in section i taking into consideration the disturbance from one interacting activity pair. Call this speed $v(i,t)^+$. The average velocity is found by averaging over the individual vehicle velocities:

$$v(i,t)^+ = \frac{\sum_{k=1}^{n(i,t)} v_k(i,t)}{n(i,t)}$$

Thus if M vehicles are disrupted by one interacting activity pair, the new average velocity is:

$$v(i,t)^+ = v(i,t) - \frac{\delta}{T}$$

where δ is given in Equation 7.

We have found the new speed taking into consideration one interacting activity pair. There are $N_{int}(i,t)$ such slowdowns in section i for time period t. Calling q the index of the qth interacting activity pair, and $\delta(q)$ the total slowdown from the qth interacting activity pair, we find

$$v(i,t)^+ = v(i,t) - \frac{1}{T} \sum_{q=1}^{N_{int}(i,t)} \delta(q).$$

Therefore,

$$\delta v(i,t) = \frac{1}{T} \sum_{q=1}^{N_{int}(i,t)} \delta(q).$$

3. FEASIBLE TRAJECTORIES AND PLANS

A trajectory is a sequence of states $[n(i,t,\theta), v(i,t)]$ where $n(i,t,\theta)$ is the number of vehicles in section i of type θ at time t and $v(i,t)$ is the average velocity in section i over the half-open interval $[t, t+1)$. We call the pair (n,u) a trajectory-plan.

A trajectory is said to be *feasible* if is satisfies four physical constraints. In addition to conservation of vehicles it must satisfy

$$n(i,t,\theta) \geq 0, \quad (8)$$

$$0 \leq v(i,t) \leq V(i,\theta), \quad (9)$$

$$\begin{aligned} L(i) \cdot T \geq & \quad (10) \\ & \sum_{\alpha} \sum_{\theta} n(i,t,\theta) \pi(\alpha,i,t,\theta) \lambda(\alpha) + \\ & \sum_{\alpha_r} \sum_{\theta} n(j,t,\theta) \pi(\alpha,j,t,\theta) \lambda_r(\alpha) + \\ & \sum_{\alpha_l} \sum_{\theta} n(k,t,\theta) \pi(\alpha,k,t,\theta) \lambda_l(\alpha) \end{aligned}$$

The non-negativity requirement and the limit on velocity are clear. The third constraint expresses the requirement that there is enough space in the section to safely carry out the activities assigned by the plan.

An additional constraint on lane change flows is that

$$\begin{aligned} L(i) \cdot T \geq & \quad (11) \\ & \sum_{\alpha} \sum_{\theta} n_{long}(i,t,\theta) \pi(\alpha,i,t,\theta) \lambda(\alpha). \end{aligned}$$

This constraint says that after vehicles change lanes they have sufficient space in the section they arrive in to perform the activities of that section.

Additional constraints can be imposed on the TMC plan to satisfy requirements of vehicles making their exits and entering at the appropriate section and constraints on allowed activities. These are discussed in greater detail in (Broucke and Varaiya, 1996).

We will say that a trajectory-plan (n,u) is *feasible* if the constraints (3)–(4) and (8)–(10) are satisfied. We say a trajectory-plan (n,u) is *laterally feasible* if in addition constraint (11) is satisfied. A feasible trajectory-plan $(n(t), u(t))$, $t = 0, 1, ...$ is *stationary* if the sequence $(n(t), u(t))$ does not depend on t.

4. CAPACITY

The activity model enables different types of capacity estimates of automated highways to be obtained. All capacity estimates assume quiescent conditions of the flows, but we will further distinguish among types of capacity by the level of dynamic activity included in the quiescent state. This distinction is based on the claim that capacity limits are imposed by three conditions:

(1) Highway topology, routing constraints, and headway policy,
(2) Persistent lowering of density from vehicle activities,
(3) Persistent lowering of speed from vehicle interactions.

Corresponding to these conditions we define three types of capacity: *static capacity*, *maneuver capacity*, and *interaction capacity*. We assume that the activity plan is stationary and that $n(i,t,\theta)$ and $v(i,t)$ have converged to steady-state values in each section.

We simplify notation by eliminating indices for θ and α. Define $n(i)$, the total number of vehicles in section i as $n(i) = \sum_\theta n(i,\theta)$ and $\pi(\alpha,i)$, the proportion of vehicles performing activity α as

$$\pi(\alpha,i) = \frac{\sum_\theta \pi(\alpha,i,\theta)n(i,\theta)}{\sum_\theta n(i,\theta)} .$$

Let $\lambda(i)$ be the average space-time used per vehicle in section i, so that $\lambda(i)n(i)$ is the space-time used by vehicles in section i. The maximum number of vehicles in a section $N(i)$ is $N(i) = \frac{L(i)T}{\lambda(i)}$. We obtain the steady-state flow out of section i $\phi(i) = \frac{v(i)}{L(i)}n(i)$, while the maximum flow $\overline{\phi}(i)$ is $\overline{\phi}(i) = \frac{VT}{\lambda(i)}$.

Define ϕ^* as the minimum of the maximum possible flow out of any section of a lane or

$$\phi^* = \min_i \frac{VT}{\lambda(i)} = \min_i \overline{\phi}(i). \qquad (12)$$

We showed in (Broucke and Varaiya, 1996) that ϕ^* is the capacity of a lane. Now we define network capacity.

Ford Fulkerson Theorem Suppose we are given a network of highway links consisting of one or more lanes. We construct a graph consisting of a set of nodes $\mathcal{N}$ corresponding to the intersection of links, and a set of lanes $\mathcal{L}$ that connect between nodes. A cut of the graph is a partition of the nodes into two sets $\mathcal{S}$ and $\mathcal{N}-\mathcal{S}$. The set of lanes of a cut Q, indexed by q, consists of those lanes that connect a node in $\mathcal{S}$ to a node in $\mathcal{N}-\mathcal{S}$. Then, the maximum flow from node A to node B of the network is found by taking the minimum of the sum of the flows ϕ^* over all cuts Q separating A from B. That is,

$$\phi^*_{AB} = \min_Q \sum_{q\in Q} \phi^*_q \qquad (13)$$

where ϕ^*_q is the maximum flow of lane q.

The *static capacity* C_s of a network from node A to B is given by (13) where ϕ^*_q is given in (12). Vehicles of type θ perform activity $\alpha(\theta)$. Thus,

$$\pi(\alpha,i,\theta) = \begin{cases} 1 & \text{if } \alpha = \alpha(\theta) \\ 0 & \text{otherwise.} \end{cases}$$

Also, $\pi(\alpha,i) = \frac{n(i,\theta)}{n(i)}$.

Static capacity takes into consideration the highway geometry by capturing the sum of the flows through the lanes of a link. The headway policy is summarized by $\lambda(\alpha)$ and we assume that each automated vehicle performs only one activity for the duration of its trip.

The *maneuver capacity* C_m of a network from node A to B is given by (13) where ϕ^*_q is given by (12). The activity plan is $\pi(\alpha,i,\theta)$. In this definition of capacity, vehicles will perform activities such entry, exit, lane change, and other activities required for routing, safety and load balancing. It is assumed by the activity plan being stationary, that the activities are fixed for a given section and flow type.

The *interaction capacity* C_i of a network from node A to B is given by:

$$\phi^*_{AB} = \min_Q \sum_{q\in Q} \phi^*_q \min_{i\in q} \frac{(V-\delta V(i))T}{\lambda(i)}. \qquad (14)$$

$\delta V(i) \geq 0$ is a steady-state loss in velocity in section i due to persistence vehicle interactions.

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