Numerical Values Used for Simulations in "An Automated Parallel Parking Strategy Using Reach Control Theory"

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Abstract

This document contains raw numerical values omitted from [1] for lack of space. It gives the dimensions of the vehicle, coordinates of the obstacles, and the location of initial and desired goal sets used in the parallel parking strategy presented in that paper. Additionally, this document lists the coordinates of vertices, simplices and polytopes used in [1], as well as controls assigned at each vertex and each simplex of the state space. For details on the proposed parking maneuver, we invite the reader to consult [1].

1 Vehicle Size

Dimensions of the car:

wheelbase	2.468m,
width	1.952m,
distance of the front end from the front axle	0.911m,
distance of the back end from the rear axle	0.819m.

2 Obstacle Positions

Unless otherwise stated, all values in the remainder of this file are given in metres.

Position of the front parked car:

rear left corner	$(2, 0)$,
rear right corner	$(2, -1.952)$,
front left corner	$(6.198, 0)$,
front right corner	$(6.198, -1.952)$.

Position of the rear parked car:

rear left corner	$(-10.198, 0)$,
rear right corner	$(-10.198, -1.952)$,
front left corner	$(-6, 0)$,
front right corner	$(-6, -1.952)$.

Position of the curb: $\{(x, y) \in \mathbb{R}^2 \mid y < -2.002\}$.

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3 Positions of the Initial and Ending Box

Coordinates of the initial box $\mathcal{I}$:

top left corner	$(2.319, 1.976)$,
bottom left corner	$(2.319, 1.726)$,
top right corner	$(2.819, 1.976)$,
bottom right corner	$(2.819, 1.726)$.

Coordinates of the ending box $\mathcal{E}$:

top left corner	$(-5.181, -0.676)$,
bottom left corner	$(-5.181, -0.976)$,
top right corner	$(-1.379, -0.676)$,
bottom right corner	$(-1.379, -0.976)$.

4 Polytope Construction

Coefficients Δx_k, Δy_k, w_{kx}, and w_{ky} used in the construction of polytopes $\mathcal{P}_k$, $k \in \{1, \dots, 8\}$, are as follows:

k	Δx_k	Δy_k	w_{kx}	w_{ky}
1	-0.850	-0.094	0	0.09375
2	-0.834	-0.286	0	0.12
3	-0.785	-0.445	0	0.12
4	-0.707	-0.592	0	0.12
5	-0.707	-0.592	0	0.12
6	-0.785	-0.445	0	0.12
7	-0.834	-0.286	0	0.12
8	-0.850	-0.094	0	0.09375

In total, polytopes $\mathcal{P}_k$ contain 40 vertices, labeled by v_i, $i \in \{1, \dots, 40\}$. Vertices of polytopes $\mathcal{P}_k$ for $k \in \{1, 2, 3, 4\}$ are $v_{4(k-1)+1}, \dots, v_{4(k+1)}$, and vertices of polytopes $\mathcal{P}_k$ for $k \in \{5, 6, 7, 8\}$ are $v_{4k+1}, \dots, v_{4(k+2)}$. Vertices $v_{4k+1}, \dots, v_{4(k+1)}$ are vertices of exit facets $\mathcal{F}_k^{\text{out}}$ for $k \in \{1, 2, 3, 4\}$, and vertices $v_{4(k+1)+1}, \dots, v_{4(k+2)}$ are vertices of exit facets $\mathcal{F}_k^{\text{out}}$ for $k \in \{5, 6, 7, 8\}$. Vertices $v_{17}, \dots, v_{20}$ and $v_{21}, \dots, v_{24}$, respectively, coincide, but are treated as separate in the remainder of the calculations. The coordinates (x, y, θ) of vertices v_i are as follows:

i	x	y	θ (in rad)
1	2.319	1.976	0.000
2	2.819	1.976	0.000
3	2.819	1.726	0.000
4	2.319	1.726	0.000
5	1.469	1.976	0.196
6	1.969	1.976	0.196
7	1.969	1.539	0.196
8	1.469	1.539	0.196
9	0.635	1.820	0.393
10	1.135	1.820	0.393

i	x	y	θ (in rad)
11	1.135	1.123	0.393
12	0.635	1.123	0.393
13	-0.150	1.505	0.589
14	0.350	1.505	0.589
15	0.350	0.547	0.589
16	-0.150	0.547	0.589
17	-0.857	1.043	0.785
18	-0.357	1.043	0.785
19	-0.357	-0.175	0.785
20	-0.857	-0.175	0.785

i	x	y	θ (in rad)	i	x	y	θ (in rad)
21	-0.857	1.043	0.785	31	-1.849	-1.472	0.393
22	-0.357	1.043	0.785	32	-2.349	-1.472	0.393
23	-0.357	-0.175	0.785	33	-3.182	0.109	0.196
24	-0.857	-0.175	0.785	34	-2.682	0.109	0.196
25	-1.563	0.580	0.589	35	-2.682	-1.888	0.196
26	-1.063	0.580	0.589	36	-3.182	-1.888	0.196
27	-1.063	-0.897	0.589	37	-4.032	0.109	0.000
28	-1.563	-0.897	0.589	38	-3.532	0.109	0.000
29	-2.349	0.265	0.393	39	-3.532	-2.076	0.000
30	-1.849	0.265	0.393	40	-4.032	-2.076	0.000

5 Polytope Triangulation

In addition to vertices $v_1, \ldots, v_{40}$, triangulations of polytopes $\mathcal{P}_k$, $k \in \{1, \ldots, 8\}$, make use of 8 additional points denoted by v_i, $i \in \{41, \ldots, 48\}$. Their coordinates are given as follows:

i	x	y	θ (in rad)
41	2.144	1.804	0.098
42	1.302	1.614	0.295
43	0.493	1.249	0.491
44	-0.253	0.730	0.687
45	-0.960	0.138	0.687
46	-1.706	-0.381	0.491
47	-2.516	-0.747	0.295
48	-3.357	-0.936	0.098

Each polytope $\mathcal{P}_k$, $k \in \{1, \ldots, 8\}$, is triangulated into 12 simplices which are denoted by $\mathcal{S}_{12(k-1)+1}, \ldots, \mathcal{S}_{12k}$. Each simplex $\mathcal{S}_j$, $j \in \{1, \ldots, 96\}$, is a convex hull of four vertices $v_{j_1}, \ldots, v_{j_4}$, with indices $j_1, \ldots, j_4 \in \{1, \ldots, 48\}$. The vertices of all simplices are given as follows:

j	j_1	j_2	j_3	j_4	j	j_1	j_2	i_3	j_4	j	j_1	j_2	i_3	j_4
1	41	6	1	5	13	42	10	5	9	25	43	14	11	10
2	41	3	6	7	14	42	5	8	9	26	43	9	14	10
3	41	1	6	2	15	42	8	12	9	27	43	14	9	13
4	41	6	3	2	16	42	12	8	11	28	43	9	12	13
5	4	3	41	7	17	42	5	10	6	29	43	11	14	15
6	4	41	8	7	18	42	10	7	6	30	43	12	11	15
7	4	8	41	5	19	42	7	10	11	31	43	12	16	13
8	4	41	1	5	20	42	8	7	11	32	43	16	12	15
9	41	7	6	5	21	42	11	10	9	33	43	11	9	10
10	41	8	7	5	22	42	12	11	9	34	43	11	12	9
11	4	41	3	2	23	42	7	5	6	35	43	15	14	13
12	4	1	41	2	24	42	7	8	5	36	43	16	15	13

j	j_1	j_2	i_3	j_4
37	44	16	15	19
38	44	15	18	19
39	44	16	20	17
40	44	20	16	19
41	44	13	18	14
42	44	18	15	14
43	44	13	16	17
44	44	18	13	17
45	44	20	19	17
46	44	19	18	17
47	44	16	13	14
48	44	15	16	14
49	45	26	21	22
50	45	23	26	22
51	45	21	26	25
52	45	24	21	25
53	45	26	23	27
54	45	23	24	27
55	45	28	24	25
56	45	24	28	27

j	j_1	j_2	i_3	j_4
57	45	27	28	25
58	45	26	27	25
59	45	24	23	21
60	45	21	23	22
61	30	46	25	29
62	30	25	46	26
63	28	32	46	31
64	28	46	32	29
65	28	25	46	29
66	27	28	46	31
67	27	46	30	31
68	27	30	46	26
69	30	32	46	29
70	30	46	32	31
71	27	46	28	25
72	27	46	25	26
73	34	47	29	33
74	34	29	47	30
75	32	36	47	35
76	32	47	36	33

j	j_1	j_2	i_3	j_4
77	32	29	47	33
78	31	32	47	35
79	31	47	34	35
80	31	34	47	30
81	34	36	47	33
82	34	47	36	35
83	31	47	32	29
84	31	47	29	30
85	38	48	33	37
86	38	33	48	34
87	38	48	35	34
88	38	35	48	39
89	36	48	40	37
90	36	33	48	37
91	36	40	48	39
92	36	48	35	39
93	38	40	48	37
94	38	48	40	39
95	36	48	33	34
96	36	35	48	34

6 Proposed Control Law

Speeds and front wheel angles assigned at each simplex vertex v_i, $i \in \{1, \ldots, 48\}$, are given as follows:

i	v (in m/s)	φ (in rad)
1	-1.654	-0.484
2	-1.391	-0.141
3	-2.225	-0.273
4	-1.747	-0.499
5	-1.773	-0.505
6	-2.319	-0.095
7	-2.447	-0.509
8	-1.711	-0.533
9	-1.815	-0.506
10	-2.348	-0.077
11	-2.289	-0.488
12	-1.674	-0.518
13	-1.818	-0.504
14	-2.358	-0.071
15	-2.227	-0.506
16	-1.667	-0.516

i	v (in m/s)	φ (in rad)
17	-1.822	-0.459
18	-2.198	0.244
19	-2.159	-0.504
20	-1.760	-0.493
21	-1.854	0.466
22	-2.264	0.136
23	-2.186	0.511
24	-1.767	0.493
25	-1.818	0.504
26	-2.358	0.071
27	-2.227	0.506
28	-1.667	0.516
29	-1.815	0.506
30	-2.348	0.077
31	-2.289	0.488
32	-1.674	0.518

i	v (in m/s)	φ (in rad)	i	v (in m/s)	φ (in rad)
33	-1.773	0.505	41	-1.909	-0.380
34	-2.319	0.095	42	-2.047	-0.404
35	-2.447	0.509	43	-2.025	-0.398
36	-1.711	0.533	44	-2.001	-0.351
37	-1.643	0.495	45	-2.018	0.400
38	-1.406	-0.195	46	-2.025	0.398
39	-2.198	0.166	47	-2.047	0.404
40	-1.734	0.502	48	-1.904	0.326

If $\mathcal{S}_j$ is a simplex with vertices $v_{j_1}, \ldots, v_{j_4}$, the feedback control $u_j(x) = K_j x + g_j$ is obtained by affinely extending the control values assigned to $v_{j_1}, \ldots, v_{j_4}$ above (with x_1 corresponding to speed v, and x_2 corresponding to front wheel angle ϕ). This produces the following feedback controls at each simplex $\mathcal{S}_j$:

j	$u_j(x) = K_j x + g_j$
1	$\begin{pmatrix} -1.092 & -0.456 & -5.332 \\ 0.820 & 0.526 & 3.442 \end{pmatrix} x + \begin{pmatrix} 1.779 \\ -3.425 \end{pmatrix}$
2	$\begin{pmatrix} -1.510 & 0.293 & -7.388 \\ 0.605 & 0.946 & 2.324 \end{pmatrix} x + \begin{pmatrix} 1.526 \\ -3.612 \end{pmatrix}$
3	$\begin{pmatrix} 0.527 & -0.456 & -2.446 \\ 0.685 & 0.526 & 3.202 \end{pmatrix} x + \begin{pmatrix} -1.976 \\ -3.112 \end{pmatrix}$
4	$\begin{pmatrix} -2.081 & 3.338 & -13.736 \\ 0.684 & 0.528 & 3.194 \end{pmatrix} x + \begin{pmatrix} -2.120 \\ -3.112 \end{pmatrix}$
5	$\begin{pmatrix} -0.957 & 1.097 & -4.228 \\ 0.451 & 0.721 & 1.440 \end{pmatrix} x + \begin{pmatrix} -1.419 \\ -2.788 \end{pmatrix}$
6	$\begin{pmatrix} -1.473 & 1.097 & -5.146 \\ 0.048 & 0.721 & 0.721 \end{pmatrix} x + \begin{pmatrix} -0.224 \\ -1.853 \end{pmatrix}$
7	$\begin{pmatrix} -0.621 & -0.142 & -2.643 \\ 0.500 & 0.063 & 2.052 \end{pmatrix} x + \begin{pmatrix} -0.061 \\ -1.767 \end{pmatrix}$
8	$\begin{pmatrix} -0.525 & 0.369 & -2.879 \\ 0.499 & 0.059 & 2.053 \end{pmatrix} x + \begin{pmatrix} -1.165 \\ -1.759 \end{pmatrix}$
9	$\begin{pmatrix} -1.092 & 0.293 & -6.643 \\ 0.820 & 0.946 & 2.707 \end{pmatrix} x + \begin{pmatrix} 0.557 \\ -4.110 \end{pmatrix}$
10	$\begin{pmatrix} -1.473 & -0.142 & -8.498 \\ 0.048 & 0.063 & -1.059 \end{pmatrix} x + \begin{pmatrix} 2.339 \\ -0.491 \end{pmatrix}$
11	$\begin{pmatrix} -0.957 & 3.338 & -6.011 \\ 0.451 & 0.528 & 1.593 \end{pmatrix} x + \begin{pmatrix} -5.287 \\ -2.456 \end{pmatrix}$
12	$\begin{pmatrix} 0.527 & 0.369 & -1.003 \\ 0.685 & 0.059 & 2.385 \end{pmatrix} x + \begin{pmatrix} -3.605 \\ -2.190 \end{pmatrix}$

j	$u_j(x) = K_j x + g_j$
13	$\begin{pmatrix} -1.065 & -0.047 & -4.775 \\ 0.858 & 0.397 & 3.952 \end{pmatrix} x + \begin{pmatrix} 0.823 \\ -3.325 \end{pmatrix}$
14	$\begin{pmatrix} -1.173 & -0.142 & -5.307 \\ 0.478 & 0.063 & 2.077 \end{pmatrix} x + \begin{pmatrix} 1.272 \\ -1.740 \end{pmatrix}$
15	$\begin{pmatrix} -1.189 & -0.203 & -5.287 \\ 0.466 & 0.017 & 2.092 \end{pmatrix} x + \begin{pmatrix} 1.385 \\ -1.654 \end{pmatrix}$
16	$\begin{pmatrix} -1.231 & -0.165 & -5.388 \\ 0.060 & 0.375 & 1.124 \end{pmatrix} x + \begin{pmatrix} 1.410 \\ -1.418 \end{pmatrix}$
17	$\begin{pmatrix} -1.092 & -0.047 & -4.821 \\ 0.820 & 0.397 & 3.889 \end{pmatrix} x + \begin{pmatrix} 0.871 \\ -3.258 \end{pmatrix}$
18	$\begin{pmatrix} -1.478 & 0.293 & -6.190 \\ 0.197 & 0.946 & 1.680 \end{pmatrix} x + \begin{pmatrix} 1.228 \\ -2.682 \end{pmatrix}$
19	$\begin{pmatrix} -1.380 & -0.084 & -5.233 \\ 0.290 & 0.589 & 2.586 \end{pmatrix} x + \begin{pmatrix} 1.427 \\ -2.494 \end{pmatrix}$
20	$\begin{pmatrix} -1.473 & -0.165 & -5.798 \\ 0.048 & 0.375 & 1.103 \end{pmatrix} x + \begin{pmatrix} 1.845 \\ -1.396 \end{pmatrix}$
21	$\begin{pmatrix} -1.065 & -0.084 & -4.699 \\ 0.858 & 0.589 & 3.550 \end{pmatrix} x + \begin{pmatrix} 0.859 \\ -3.517 \end{pmatrix}$
22	$\begin{pmatrix} -1.231 & -0.203 & -5.576 \\ 0.060 & 0.017 & -0.670 \end{pmatrix} x + \begin{pmatrix} 1.526 \\ -0.311 \end{pmatrix}$
23	$\begin{pmatrix} -1.092 & 0.293 & -3.568 \\ 0.820 & 0.946 & 5.910 \end{pmatrix} x + \begin{pmatrix} -0.047 \\ -4.739 \end{pmatrix}$
24	$\begin{pmatrix} -1.473 & -0.142 & -5.816 \\ 0.048 & 0.063 & 1.345 \end{pmatrix} x + \begin{pmatrix} 1.813 \\ -0.963 \end{pmatrix}$
25	$\begin{pmatrix} -1.175 & -0.084 & -4.888 \\ 0.323 & 0.589 & 2.272 \end{pmatrix} x + \begin{pmatrix} 1.058 \\ -2.408 \end{pmatrix}$
26	$\begin{pmatrix} -1.065 & -0.150 & -4.555 \\ 0.858 & 0.266 & 3.891 \end{pmatrix} x + \begin{pmatrix} 0.924 \\ -3.063 \end{pmatrix}$
27	$\begin{pmatrix} -1.081 & -0.150 & -4.578 \\ 0.867 & 0.266 & 3.905 \end{pmatrix} x + \begin{pmatrix} 0.943 \\ -3.074 \end{pmatrix}$
28	$\begin{pmatrix} -1.168 & -0.203 & -5.010 \\ 0.455 & 0.017 & 1.857 \end{pmatrix} x + \begin{pmatrix} 1.263 \\ -1.555 \end{pmatrix}$
29	$\begin{pmatrix} -1.162 & -0.137 & -4.728 \\ 0.358 & 0.455 & 2.672 \end{pmatrix} x + \begin{pmatrix} 1.040 \\ -2.454 \end{pmatrix}$
30	$\begin{pmatrix} -1.231 & -0.179 & -5.132 \\ 0.060 & 0.275 & 0.952 \end{pmatrix} x + \begin{pmatrix} 1.325 \\ -1.238 \end{pmatrix}$

j	$u_j(x) = K_j x + g_j$
31	$\begin{pmatrix} -1.156 & -0.157 & -5.051 \\ 0.454 & 0.013 & 1.860 \end{pmatrix} x + \begin{pmatrix} 1.221 \\ -1.551 \end{pmatrix}$
32	$\begin{pmatrix} -1.119 & -0.179 & -4.969 \\ 0.021 & 0.275 & 0.895 \end{pmatrix} x + \begin{pmatrix} 1.190 \\ -1.191 \end{pmatrix}$
33	$\begin{pmatrix} -1.065 & -0.084 & -4.168 \\ 0.858 & 0.589 & 5.771 \end{pmatrix} x + \begin{pmatrix} 0.651 \\ -4.389 \end{pmatrix}$
34	$\begin{pmatrix} -1.231 & -0.203 & -5.102 \\ 0.060 & 0.017 & 1.283 \end{pmatrix} x + \begin{pmatrix} 1.340 \\ -1.078 \end{pmatrix}$
35	$\begin{pmatrix} -1.081 & -0.137 & -4.612 \\ 0.867 & 0.455 & 3.411 \end{pmatrix} x + \begin{pmatrix} 0.943 \\ -3.068 \end{pmatrix}$
36	$\begin{pmatrix} -1.119 & -0.157 & -4.809 \\ 0.021 & 0.013 & -0.975 \end{pmatrix} x + \begin{pmatrix} 1.084 \\ 0.054 \end{pmatrix}$
37	$\begin{pmatrix} -1.119 & -0.162 & -4.278 \\ 0.021 & 0.292 & 1.162 \end{pmatrix} x + \begin{pmatrix} 0.774 \\ -1.357 \end{pmatrix}$
38	$\begin{pmatrix} -0.838 & -0.032 & -2.789 \\ 0.720 & 0.614 & 4.863 \end{pmatrix} x + \begin{pmatrix} -0.273 \\ -3.958 \end{pmatrix}$
39	$\begin{pmatrix} -1.040 & -0.051 & -4.401 \\ 0.554 & 0.028 & 2.215 \end{pmatrix} x + \begin{pmatrix} 0.797 \\ -1.753 \end{pmatrix}$
40	$\begin{pmatrix} -0.798 & -0.162 & -3.940 \\ -0.021 & 0.292 & 1.117 \end{pmatrix} x + \begin{pmatrix} 0.623 \\ -1.338 \end{pmatrix}$
41	$\begin{pmatrix} -1.081 & -0.013 & -3.108 \\ 0.867 & 0.406 & 5.678 \end{pmatrix} x + \begin{pmatrix} -0.130 \\ -4.330 \end{pmatrix}$
42	$\begin{pmatrix} -0.811 & -0.137 & -2.427 \\ 0.762 & 0.455 & 5.413 \end{pmatrix} x + \begin{pmatrix} -0.439 \\ -4.210 \end{pmatrix}$
43	$\begin{pmatrix} -1.067 & -0.157 & -4.233 \\ 0.550 & 0.013 & 2.238 \end{pmatrix} x + \begin{pmatrix} 0.752 \\ -1.760 \end{pmatrix}$
44	$\begin{pmatrix} -0.753 & -0.013 & -2.763 \\ 1.406 & 0.406 & 6.245 \end{pmatrix} x + \begin{pmatrix} -0.284 \\ -4.583 \end{pmatrix}$
45	$\begin{pmatrix} -0.798 & -0.051 & -2.917 \\ -0.021 & 0.028 & -1.320 \end{pmatrix} x + \begin{pmatrix} -0.162 \\ 0.531 \end{pmatrix}$
46	$\begin{pmatrix} -0.753 & -0.032 & -2.700 \\ 1.406 & 0.614 & 5.584 \end{pmatrix} x + \begin{pmatrix} -0.313 \\ -4.281 \end{pmatrix}$
47	$\begin{pmatrix} -1.081 & -0.157 & -4.247 \\ 0.867 & 0.013 & 2.572 \end{pmatrix} x + \begin{pmatrix} 0.758 \\ -1.909 \end{pmatrix}$
48	$\begin{pmatrix} -1.119 & -0.137 & -4.324 \\ 0.021 & 0.455 & 0.859 \end{pmatrix} x + \begin{pmatrix} 0.787 \\ -1.268 \end{pmatrix}$

j	$u_j(x) = K_j x + g_j$
49	$\begin{pmatrix} -0.818 & -0.132 & 3.737 \\ -0.660 & -0.196 & 3.170 \end{pmatrix} x + \begin{pmatrix} -5.353 \\ -2.385 \end{pmatrix}$
50	$\begin{pmatrix} -1.001 & -0.064 & 4.236 \\ -0.359 & -0.308 & 2.348 \end{pmatrix} x + \begin{pmatrix} -5.881 \\ -1.515 \end{pmatrix}$
51	$\begin{pmatrix} -1.081 & -0.132 & 4.014 \\ -0.867 & -0.196 & 3.388 \end{pmatrix} x + \begin{pmatrix} -5.796 \\ -2.733 \end{pmatrix}$
52	$\begin{pmatrix} -0.920 & -0.072 & 3.293 \\ -0.399 & -0.022 & 1.293 \end{pmatrix} x + \begin{pmatrix} -5.154 \\ -0.868 \end{pmatrix}$
53	$\begin{pmatrix} -0.995 & -0.089 & 4.116 \\ -0.362 & -0.295 & 2.411 \end{pmatrix} x + \begin{pmatrix} -5.789 \\ -1.564 \end{pmatrix}$
54	$\begin{pmatrix} -0.838 & -0.031 & 3.338 \\ 0.035 & -0.147 & 0.438 \end{pmatrix} x + \begin{pmatrix} -5.112 \\ 0.154 \end{pmatrix}$
55	$\begin{pmatrix} -0.928 & -0.102 & 3.206 \\ -0.395 & -0.008 & 1.333 \end{pmatrix} x + \begin{pmatrix} -5.098 \\ -0.894 \end{pmatrix}$
56	$\begin{pmatrix} -1.119 & -0.031 & 3.634 \\ -0.021 & -0.147 & 0.497 \end{pmatrix} x + \begin{pmatrix} -5.585 \\ 0.059 \end{pmatrix}$
57	$\begin{pmatrix} -1.119 & -0.102 & 4.383 \\ -0.021 & -0.008 & -0.966 \end{pmatrix} x + \begin{pmatrix} -6.090 \\ 1.045 \end{pmatrix}$
58	$\begin{pmatrix} -1.081 & -0.089 & 4.207 \\ -0.867 & -0.295 & 2.943 \end{pmatrix} x + \begin{pmatrix} -5.935 \\ -2.414 \end{pmatrix}$
59	$\begin{pmatrix} -0.838 & -0.072 & 3.207 \\ 0.035 & -0.022 & 0.836 \end{pmatrix} x + \begin{pmatrix} -5.016 \\ -0.137 \end{pmatrix}$
60	$\begin{pmatrix} -0.818 & -0.064 & 3.111 \\ -0.660 & -0.308 & 4.200 \end{pmatrix} x + \begin{pmatrix} -4.933 \\ -3.078 \end{pmatrix}$
61	$\begin{pmatrix} -1.065 & -0.072 & 4.366 \\ -0.858 & -0.134 & 3.635 \end{pmatrix} x + \begin{pmatrix} -6.013 \\ -2.900 \end{pmatrix}$
62	$\begin{pmatrix} -1.081 & -0.072 & 4.388 \\ -0.867 & -0.134 & 3.648 \end{pmatrix} x + \begin{pmatrix} -6.051 \\ -2.923 \end{pmatrix}$
63	$\begin{pmatrix} -1.231 & -0.057 & 5.126 \\ -0.060 & -0.129 & 0.611 \end{pmatrix} x + \begin{pmatrix} -6.664 \\ -0.053 \end{pmatrix}$
64	$\begin{pmatrix} -1.154 & -0.081 & 4.889 \\ -0.454 & -0.007 & 1.828 \end{pmatrix} x + \begin{pmatrix} -6.425 \\ -1.276 \end{pmatrix}$
65	$\begin{pmatrix} -1.160 & -0.102 & 4.789 \\ -0.454 & -0.008 & 1.820 \end{pmatrix} x + \begin{pmatrix} -6.393 \\ -1.273 \end{pmatrix}$
66	$\begin{pmatrix} -1.119 & -0.057 & 4.964 \\ -0.021 & -0.129 & 0.555 \end{pmatrix} x + \begin{pmatrix} -6.393 \\ 0.041 \end{pmatrix}$

j	$u_j(x) = K_j x + g_j$
67	$\begin{pmatrix} -1.043 & -0.034 & 4.588 \\ -0.365 & -0.236 & 2.245 \end{pmatrix} x + \begin{pmatrix} -6.069 \\ -1.416 \end{pmatrix}$
68	$\begin{pmatrix} -1.028 & -0.089 & 4.204 \\ -0.350 & -0.295 & 1.838 \end{pmatrix} x + \begin{pmatrix} -5.876 \\ -1.213 \end{pmatrix}$
69	$\begin{pmatrix} -1.065 & -0.081 & 4.307 \\ -0.858 & -0.007 & 4.471 \end{pmatrix} x + \begin{pmatrix} -5.987 \\ -3.263 \end{pmatrix}$
70	$\begin{pmatrix} -1.231 & -0.034 & 4.862 \\ -0.060 & -0.236 & 1.801 \end{pmatrix} x + \begin{pmatrix} -6.525 \\ -0.678 \end{pmatrix}$
71	$\begin{pmatrix} -1.119 & -0.102 & 4.731 \\ -0.021 & -0.008 & 1.190 \end{pmatrix} x + \begin{pmatrix} -6.295 \\ -0.225 \end{pmatrix}$
72	$\begin{pmatrix} -1.081 & -0.089 & 4.549 \\ -0.867 & -0.295 & 5.223 \end{pmatrix} x + \begin{pmatrix} -6.136 \\ -3.757 \end{pmatrix}$
73	$\begin{pmatrix} -1.092 & -0.021 & 4.440 \\ -0.820 & -0.111 & 3.574 \end{pmatrix} x + \begin{pmatrix} -6.118 \\ -2.794 \end{pmatrix}$
74	$\begin{pmatrix} -1.065 & -0.021 & 4.395 \\ -0.858 & -0.111 & 3.638 \end{pmatrix} x + \begin{pmatrix} -6.038 \\ -2.908 \end{pmatrix}$
75	$\begin{pmatrix} -1.473 & 0.014 & 6.412 \\ -0.048 & -0.117 & 0.374 \end{pmatrix} x + \begin{pmatrix} -7.629 \\ 0.087 \end{pmatrix}$
76	$\begin{pmatrix} -1.303 & -0.031 & 5.787 \\ -0.434 & -0.014 & 1.796 \end{pmatrix} x + \begin{pmatrix} -7.052 \\ -1.227 \end{pmatrix}$
77	$\begin{pmatrix} -1.316 & -0.081 & 5.438 \\ -0.432 & -0.007 & 1.845 \end{pmatrix} x + \begin{pmatrix} -7.020 \\ -1.232 \end{pmatrix}$
78	$\begin{pmatrix} -1.231 & 0.014 & 6.002 \\ -0.060 & -0.117 & 0.395 \end{pmatrix} x + \begin{pmatrix} -6.902 \\ 0.050 \end{pmatrix}$
79	$\begin{pmatrix} -1.046 & 0.064 & 5.107 \\ -0.396 & -0.207 & 2.011 \end{pmatrix} x + \begin{pmatrix} -6.134 \\ -1.338 \end{pmatrix}$
80	$\begin{pmatrix} -1.020 & -0.034 & 4.212 \\ -0.388 & -0.236 & 1.743 \end{pmatrix} x + \begin{pmatrix} -5.879 \\ -1.262 \end{pmatrix}$
81	$\begin{pmatrix} -1.092 & -0.031 & 4.357 \\ -0.820 & -0.014 & 4.418 \end{pmatrix} x + \begin{pmatrix} -6.101 \\ -2.971 \end{pmatrix}$
82	$\begin{pmatrix} -1.473 & 0.064 & 5.833 \\ -0.048 & -0.207 & 1.419 \end{pmatrix} x + \begin{pmatrix} -7.422 \\ -0.288 \end{pmatrix}$
83	$\begin{pmatrix} -1.231 & -0.081 & 5.294 \\ -0.060 & -0.007 & 1.212 \end{pmatrix} x + \begin{pmatrix} -6.765 \\ -0.109 \end{pmatrix}$
84	$\begin{pmatrix} -1.065 & -0.034 & 4.520 \\ -0.858 & -0.236 & 4.934 \end{pmatrix} x + \begin{pmatrix} -6.084 \\ -3.383 \end{pmatrix}$

j	$u_j(x) = K_j x + g_j$
85	$\begin{pmatrix} 0.473 & 0.301 & -2.710 \\ -1.378 & -0.163 & 6.021 \end{pmatrix} x + \begin{pmatrix} 0.230 \\ -5.045 \end{pmatrix}$
86	$\begin{pmatrix} -1.092 & 0.301 & 0.079 \\ -0.820 & -0.163 & 5.026 \end{pmatrix} x + \begin{pmatrix} -5.297 \\ -3.074 \end{pmatrix}$
87	$\begin{pmatrix} -0.104 & 0.064 & -4.200 \\ -0.638 & -0.207 & 4.237 \end{pmatrix} x + \begin{pmatrix} -1.779 \\ -2.425 \end{pmatrix}$
88	$\begin{pmatrix} -0.159 & 0.362 & -0.926 \\ -0.646 & -0.165 & 4.700 \end{pmatrix} x + \begin{pmatrix} -2.009 \\ -2.457 \end{pmatrix}$
89	$\begin{pmatrix} -0.901 & 0.042 & 3.976 \\ -0.750 & -0.004 & 3.404 \end{pmatrix} x + \begin{pmatrix} -5.279 \\ -2.529 \end{pmatrix}$
90	$\begin{pmatrix} -0.914 & -0.031 & 3.294 \\ -0.752 & -0.014 & 3.310 \end{pmatrix} x + \begin{pmatrix} -5.326 \\ -2.536 \end{pmatrix}$
91	$\begin{pmatrix} -0.927 & 0.048 & 4.086 \\ -0.673 & -0.022 & 3.090 \end{pmatrix} x + \begin{pmatrix} -5.373 \\ -2.258 \end{pmatrix}$
92	$\begin{pmatrix} -1.473 & 0.048 & 5.058 \\ -0.048 & -0.022 & 1.974 \end{pmatrix} x + \begin{pmatrix} -7.300 \\ -0.048 \end{pmatrix}$
93	$\begin{pmatrix} 0.473 & 0.042 & -5.465 \\ -1.378 & -0.004 & 7.724 \end{pmatrix} x + \begin{pmatrix} 0.259 \\ -5.063 \end{pmatrix}$
94	$\begin{pmatrix} -0.927 & 0.362 & 0.442 \\ -0.673 & -0.165 & 4.749 \end{pmatrix} x + \begin{pmatrix} -4.722 \\ -2.555 \end{pmatrix}$
95	$\begin{pmatrix} -1.092 & -0.031 & 3.611 \\ -0.820 & -0.014 & 3.431 \end{pmatrix} x + \begin{pmatrix} -5.954 \\ -2.777 \end{pmatrix}$
96	$\begin{pmatrix} -1.473 & 0.064 & 5.212 \\ -0.048 & -0.207 & 0.179 \end{pmatrix} x + \begin{pmatrix} -7.300 \\ -0.045 \end{pmatrix}$

References

[1] M. Ornik, M. Moarref, M. E. Broucke, An automated parallel parking strategy using reach control theory, *accepted to 20th IFAC World Congress*, 2017.