UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATION, April 24, 2009
ECE311 - DYNAMIC SYSTEMS AND CONTROL
Exam Type: C
Calculator Type: 4

FAMILY NAME _____

GIVEN NAME(S) _____

STUDENT NUMBER _____

INSTRUCTOR (Broucke/Davison) _____

Instructions
No Calculators. A single aid sheet, both sides, permitted. Write your solutions clearly in the spaces provided below the problem statements. Use the back sides of the pages for rough work.

Problem	Mark
1	/20
2	/20
3	/20
4	/20
5	/20
Total	/100

1. You are given a nonlinear state model

$$\begin{aligned} \dot{x}_1 &= \mathrm{e}^{x_1} - u \\ \dot{x}_2 &= x_1 x_2 + 1 \\ y &= x_1 x_2 \,. \end{aligned}$$

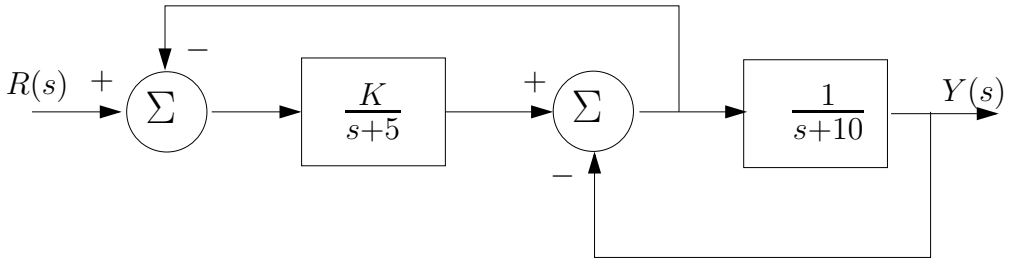
(a) Find all equilibria $\overline{x}$ when $u = \overline{u} > 0$.

(b) Using your result from part (a), linearize the nonlinear system about the equilibrium $(\overline{x}, \overline{u})$, assuming that $\overline{u} = \mathrm{e}^2$ (here $\mathrm{e} = 2.718$).

(c) Find the transfer function of the linearized state model from part (b).

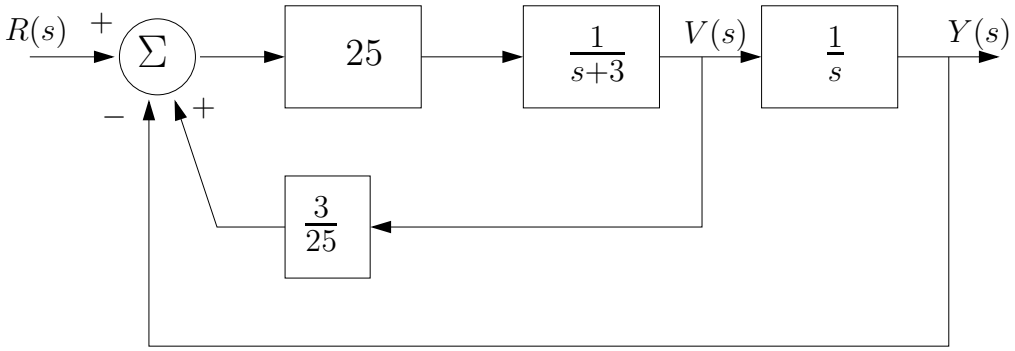
(d) Find the poles and zeros of the transfer function in part (c).

2. (a) Consider the following system



Find the range of K such that the closed loop system is asymptotically stable.

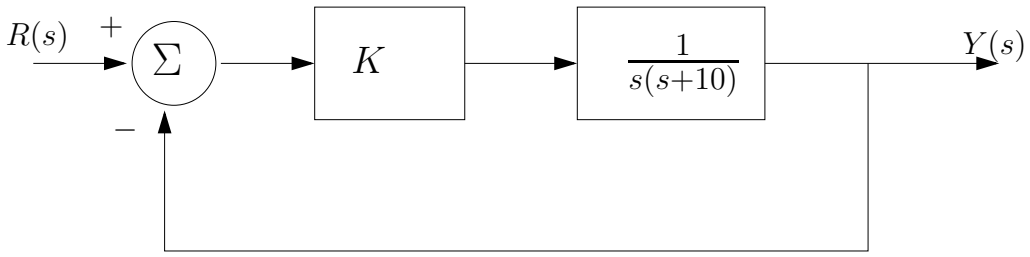
(b) Consider the following system



(i) Find a state model with state $x = (v, y)$, input r, and output y.

(ii) Compute e^{At}.

3. Consider the following system

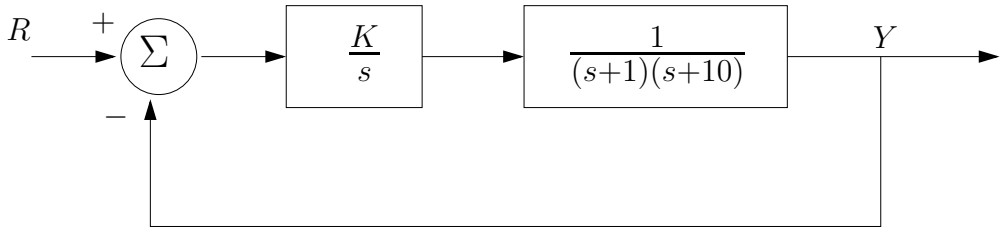


(a) Sketch the Nyquist plot of $\frac{K}{s(s+10)}$ with $K = 1$.

(b) Sketch the magnitude and phase Bode plots of $\frac{K}{s(s+10)}$ with $K = 1$.

(c) Using your Bode plots, determine the gain margin GM and phase margin PM of the closed-loop system with $K = 1$.

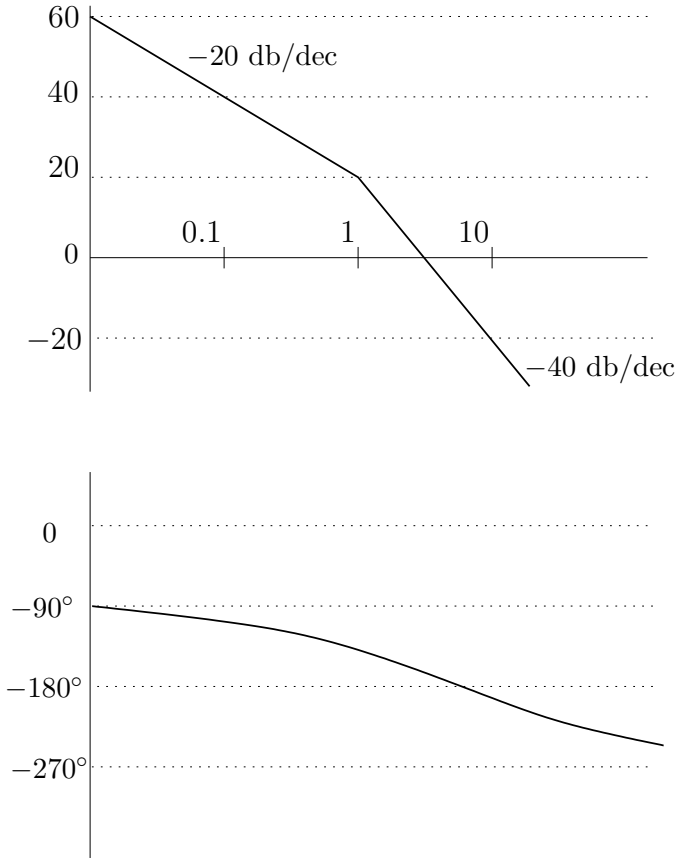
4. (a) Consider the following system



(i) Find K such that the closed loop system is asymptotically stable and the gain margin is 2.

(ii) Compute $e_{ss} = \lim_{t\to\infty} e(t)$ assuming $R(s) = \frac{r_0}{s}$.

(b) A stable system $G(s)$ has the following Bode plot



Find a controller $C(s)$ such that:

(i) The closed loop system is asymptotically stable.

(ii) The phase margin of the closed-loop system satisfies $PM > 45°$.

5. An unstable plant is described by

$$\begin{aligned} \dot{x} &= 2x - u + w \\ y &= x\,, \end{aligned}$$

where w is an unmeasurable disturbance given by

$$w(t) = w(0)\mathrm{e}^{2t}\,, \qquad\qquad t \geq 0\,,$$

and $w(0)$ is an unknown constant. Find a robust regulator which has the property that:

(i) The closed-loop system is asymptotically stable with all poles at -1.

(ii) $\lim_{t\to\infty} y(t) = 0$, for all values of $x(0)$ and $w(0)$.

Give your final controller both in state space and transfer function form.