

ECE311S: Dynamic Systems and Control

Problem Set 3

Problem 1

For the system described by the ODE

$$\ddot{y} + 3\dot{y} - 2y = \ddot{u} - 3\dot{u} + 2u,$$

find the transfer function from u to y . Next, find a state space representation of the system.

Problem 2

For the system with state space representation

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u$$
$$y = [1 \ 0 \ 0]\mathbf{x},$$

find the transfer function from u to y .

Problem 3

Part 1. You are given a linear system with input u and output y . When¹ $u(t) = \sin t \cdot \mathbf{1}(t)$ and all initial conditions are set to zero, it is found experimentally that $y(t) = \frac{1}{2} [e^{-t} + \sin t - \cos t] \mathbf{1}(t)$. Find the transfer function of the system from u to y .

Part 2. Consider the system with state space representation

$$\dot{x}_1 = x_2 + u$$
$$\dot{x}_2 = u$$
$$y = x_1 + x_2 + u.$$

Find the transfer function from u to y .

¹ $\mathbf{1}(t)$ denotes the unit step function, i.e., $\mathbf{1}(t) = 1$ for $t \geq 0$ and $\mathbf{1}(t) = 0$ for $t < 0$

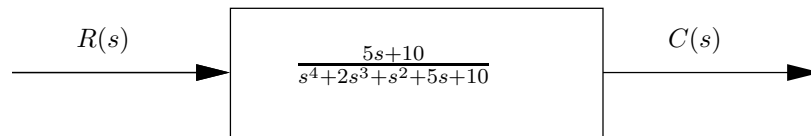
Problem 4

For the following transfer function, write the corresponding differential equation.

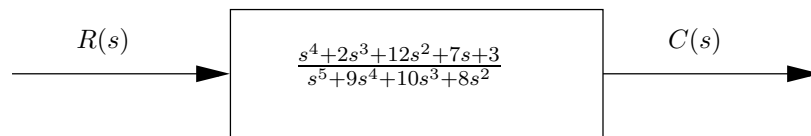
$$\frac{X(s)}{F(s)} = \frac{s + 2}{s^3 + 8s^2 + 9s + 15}.$$

Problem 5

For each system shown below, write the state equations and the output equation for the phase-variable representation.



(a)



(b)

Problem 6

Find the transfer function, $G(s) = Y(s)/R(s)$, for the following system represented in state space.

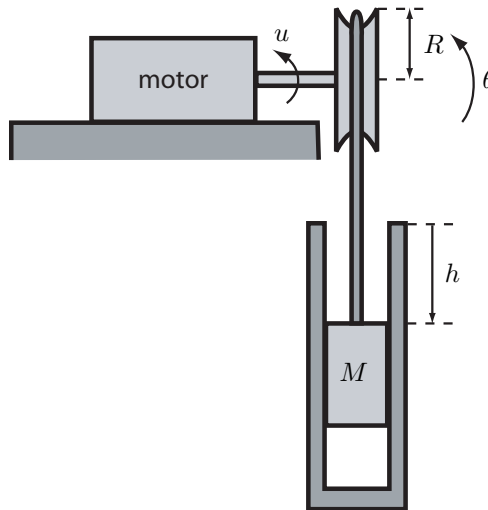
$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -5 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} r$$

$$\mathbf{y} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \mathbf{x}$$

Problem 7

A pulley of radius R and moment of inertia I is connected to the shaft of a DC motor. The pulley is connected via a cable to a piston of mass M . The rotation of the pulley in the clockwise or counterclockwise direction makes the piston slide vertically upward or downward in a cylinder. The contact between piston

and cylinder is subject to viscous friction with coefficient K_v .



Assume that

- The torque (moment) u imparted by the DC motor is the control input (in other words, neglect the dynamics of the DC motor)
- The cable is *rigid*, in other words neglect any issue concerning its elasticity and its tension.

Do the following:

- Find the equations of motion of the system.
- Suppose that the position of the cylinder is measured by means of an optical encoder. Let $\tilde{u} := u - RMg$. Find the transfer function from \tilde{u} to h .
- Suppose now that the angular velocity of the shaft, $\dot{\theta}$, is measured by means of a tachometer. Find the transfer function from \tilde{u} to $\dot{\theta}$.