Problem Set 5 Solutions

Problem 1

Combine G_6 and G_7 yielding G_6G_7. Add G_4 and obtain the following diagram:

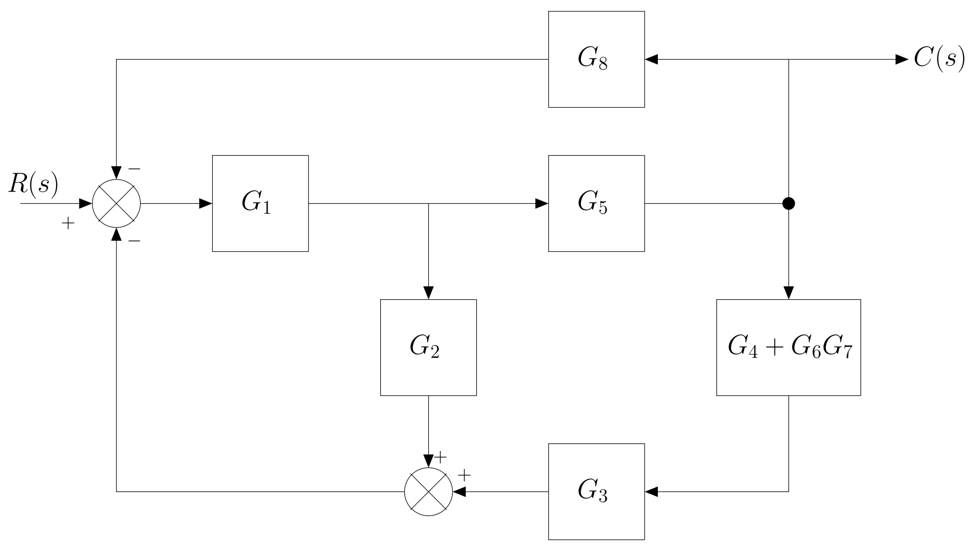

Figure 1: Initial block diagram.

Next combine G_3 and $G_4 + G_6G_7$, as shown in Figure 2. Push G_5 to the left past the pickoff point. See Figure 3 for this arrangement.

Notice that the feedback is in parallel form. Thus the equivalent feedback, $H_{eq}(s) = \frac{G_2}{G_5} + G_3(G_4 + G_6G_7) + G_8$. Since the forward path transfer function is $G(s) = G_{eq}(s) = G_1G_5$, the closed-loop transfer function is

$$T(s) = \frac{G_{eq}(s)}{1 + G_{eq}(s)H_{eq}(s)}.$$

Hence,

$$T(s) = \frac{G_5G_1}{1 + G_1(G_8G_5 + G_7G_6G_5G_3 + G_5G_4G_3 + G_2)}$$

Problem 2

Combine the feedback with G_6 and combine the parallel G_2 and G_3. Displayed below in Figure 4.
Move $G_2 + G_3$ to the left past the pickoff point. Shown is Figure 5.
Combine feedback and parallel pair in the forward path yielding and equivalent forward-path transfer function of:

$$G_e(s) = \left(\frac{G_2 + G_3}{1 + G_1(G_2 + G_3)}\right)\left(G_5 + \frac{G_4}{G_2 + G_3}\right)\left(\frac{G_6}{1 + G_6}\right)$$

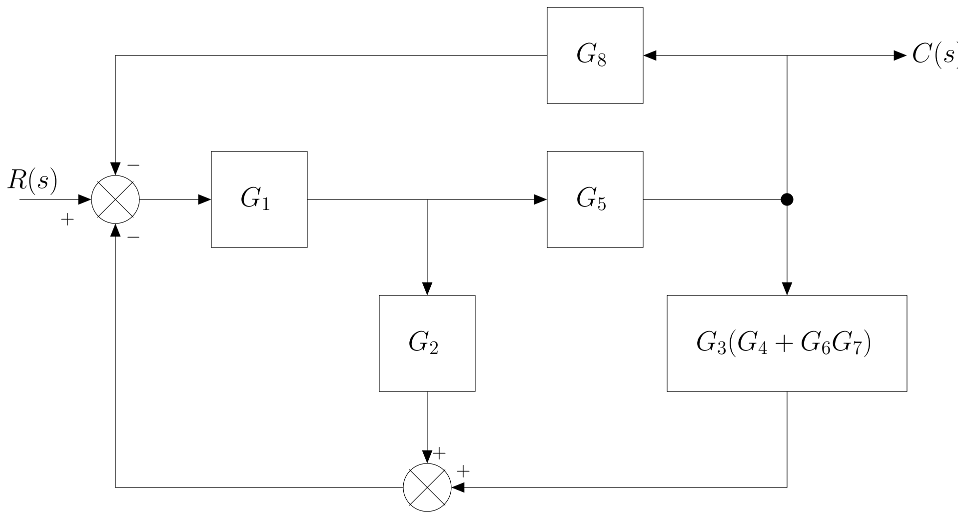

Figure 2: Combine G_3 and $G_4 + G_6G_7$.

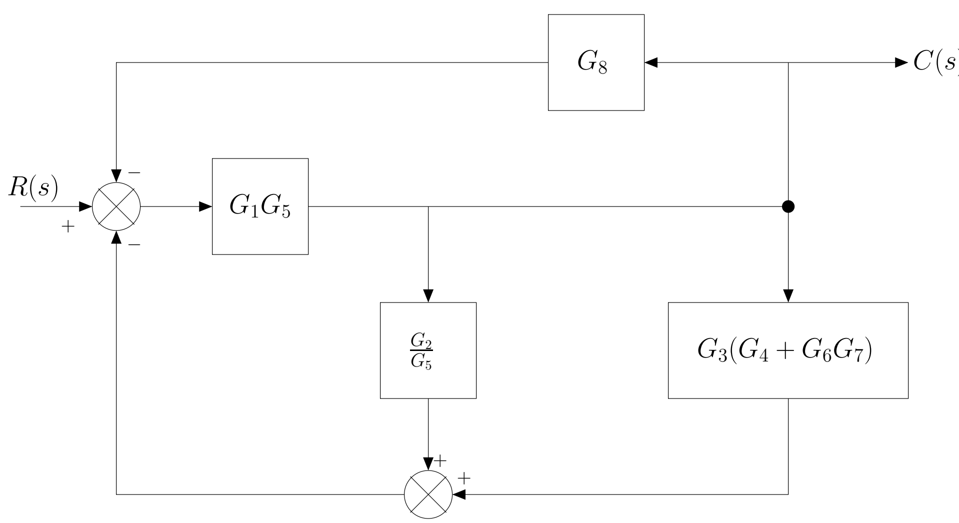

Figure 3: Push G_5 left, past the pickoff point.

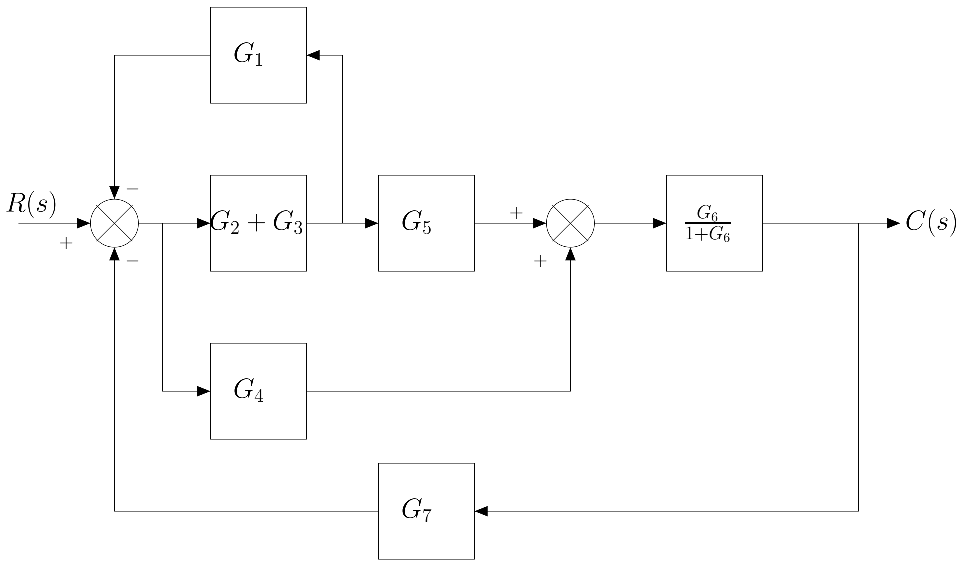

Figure 4: Feedback combined with G_6 and G_2 & G_3 in parallel.

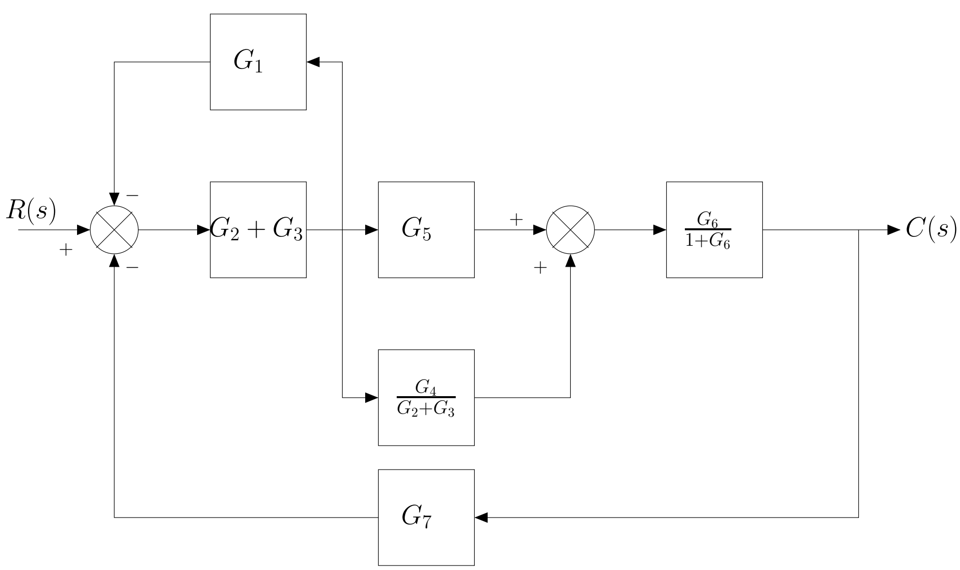

Figure 5: $G_2 + G_3$ moved left past pickoff point.

But, $T(s) = \frac{G_e(s)}{1+G_e(s)G_7(s)}$. Thus,

$$T(s) = \frac{G_6(G_4 + G_5G_3 + G_5G_2)}{G_6(G_7G_4 + G_7G_5G_3 + G_7G_5G_2 + G_3G_1 + G_2G_1 + 1) + G_1(G_3 + G_2) + 1}$$

Problem 3

Equation for T_s: $\sigma = \zeta\omega_n = \frac{4}{T_s} = 4$;
Equation for overshoot: $\zeta = \frac{-\ln(12.3/100)}{\sqrt{\pi^2+\ln^2(12.3/100)}} = 0.5549$;

$$\sigma = \zeta\omega_n = 0.5549\omega_n = 4; \quad \therefore \omega_n = 7.21$$

$$\therefore G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{51.96}{s^2 + 8s + 51.96}$$

Problem 4

Use the following equations:

$$1)\ KVL \qquad V_i - RC\frac{dV_c}{dt} - L\frac{di_L}{dt} - V_c = 0$$

$$2)\ KCL \qquad C\frac{dV_c}{dt} = i_L$$

Calculate the Laplace transforms and derive the TF:

$$\frac{V_c(s)}{V_i(s)} = \frac{\frac{1}{Cs}}{R + Ls + \frac{1}{Cs}} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Also, $T_s = 2*10^{-}3 = \frac{4}{\sigma} = \frac{4}{\frac{R}{2L}} = \frac{8L}{R}$. Thus, $\frac{R}{L} = 4000$. Using Eq. (4.39) with 15% overshoot, $\zeta = 0.5169$. But, $2\zeta\omega_n = \frac{R}{L}$. Thus, $\omega_n = 3869 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{L(10^{-5})}}$. Therefore, $L = 6.7$ mH and $R = 26.72\ \Omega$.