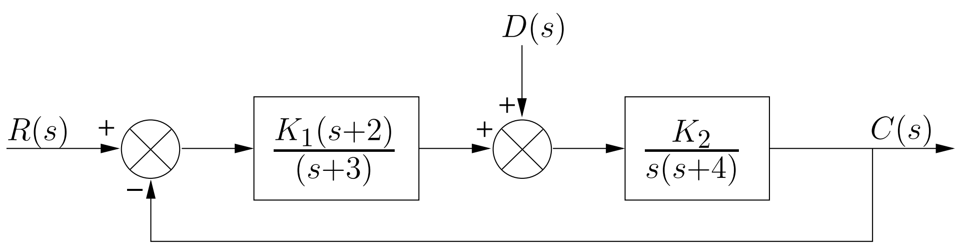
ECE311S: Dynamic Systems and Control

Problem Set 7

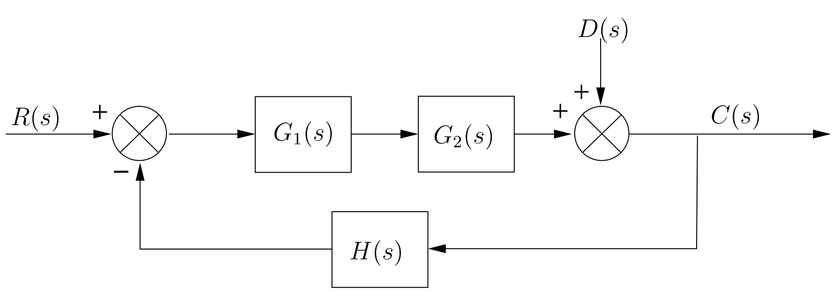
Problem 1

Design the values of K_1 and K_2 in the system below to meet the following specifications: steady-state error component due to a unit step disturbance is -0.000012; steady-state error component due to a unit ramp input is 0.003.



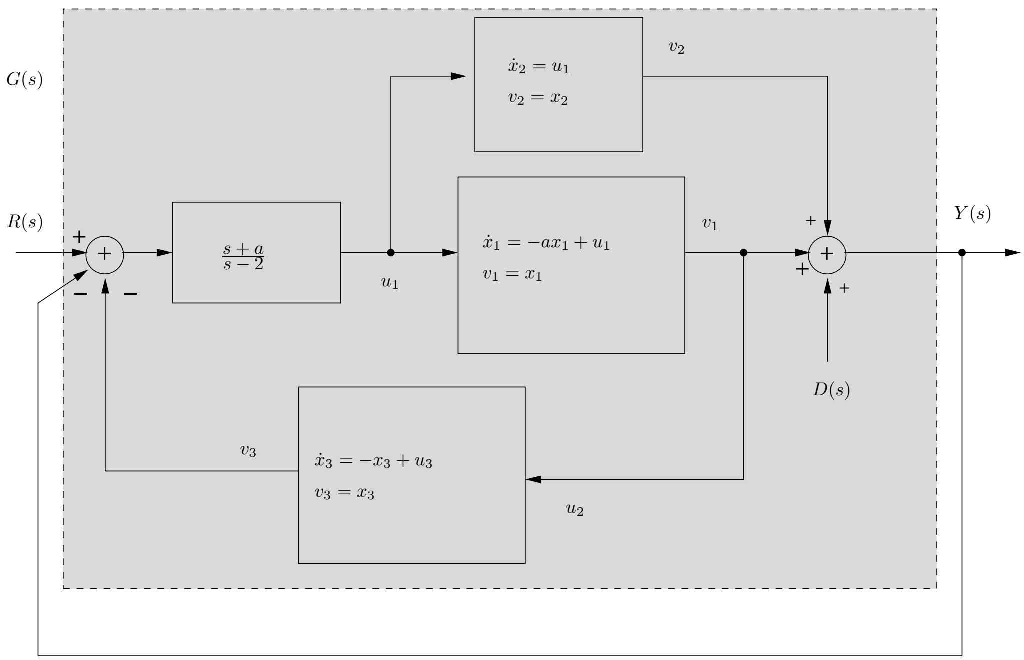
Problem 2

Given the system shown below, do the following:



a. Derive the expression for the error, $E(s) = R(s) - C(s)$, in terms of $R(s)$ and $D(s)$.

b. Derive the steady-state error, $e(\infty)$, if $R(s)$ and $D(s)$ are unit step functions.



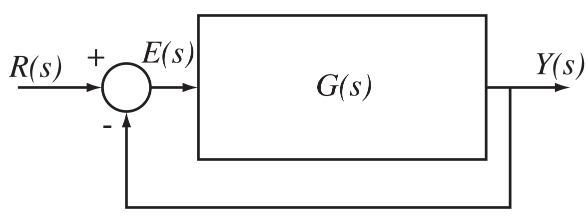
Problem 3

You are given the following system:
where a is a real number, $R(s) = \frac{2}{s}$, and $D(s) = \frac{1}{s}$. Assuming that the limit $e(\infty) := \lim_{t \to \infty} (r(t) - y(t))$ exists, find $e(\infty)$ as a function of a.

Problem 4

Consider the unity feedback control system shown below where

$$G(s) = \frac{K}{s(\tau s + 1)}.$$



Compute the steady-state tracking error due to a ramp input $r(t) = R_0 \, t \cdot \mathbf{1}(t)$ (where $\mathbf{1}(t)$ denotes the unit step).

Problem 5

Consider the following system:

$$Y(s) = \frac{K(s+a)(s+b)}{s^2(s^2+Ks+Kb)+K(s+a)(s+b)}R(s)$$

where it is assumed that $a > 0$, $b > 0$, $K > 0$.

(a) Find conditions on K, a, and b such that the system with input $R(s)$ and output $Y(s)$ is BIBO stable.

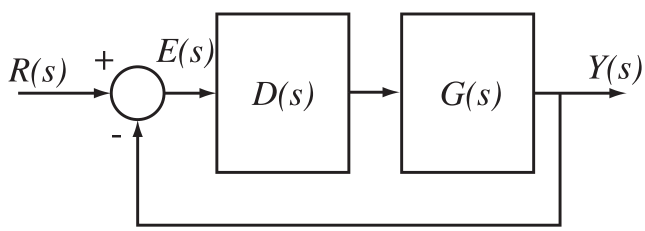
(b) Letting $R(s) = \frac{1}{s}$ and assuming K is selected such that the closed-loop system is BIBO stable, find an expression for $y_{ss} := \lim_{t\to\infty} y(t)$.

Problem 6

A controller for a satellite attitude control with transfer function $G(s) = \frac{1}{s^2}$ has been designed with a unity feedback structure and has the transfer function

$$D(s) = \frac{K(s+2)}{s+5},$$

where $K > 0$ is a parameter to be designed (see the Figure below).

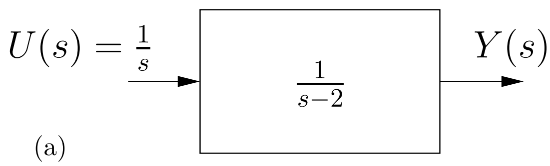


Assuming that the reference trajectory is given by $r(t) = \frac{t^2}{2}\mathbf{1}(t)$, calculate the value of K guaranteeing that the steady-state error is 0.01.

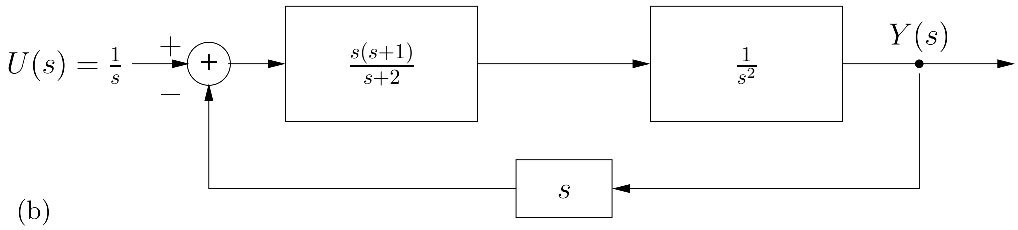
Problem 7

For each of the following block diagrams find the steady-state tracking error

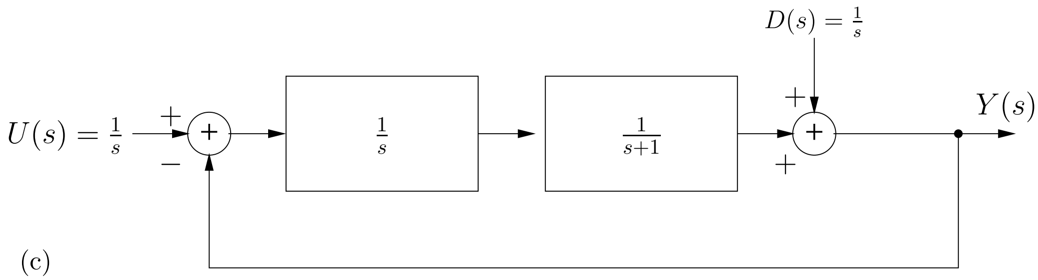
$$e(\infty) := \lim_{t\to\infty} u(t) - y(t).$$



(a)



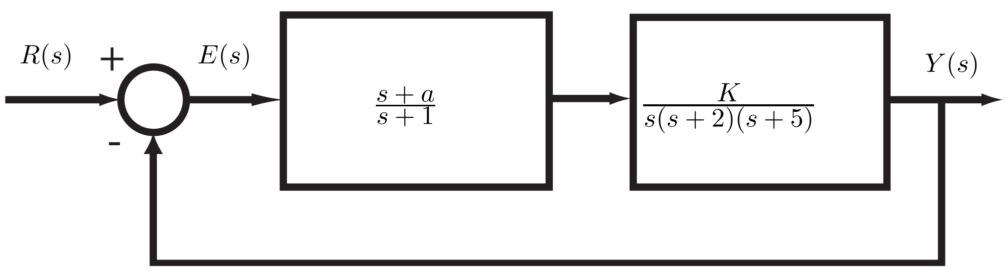
(b)



(c)

Problem 8

Consider the control system in the figure below.



(i) Find the most general conditions on the parameters $K > 0$ and $a > 0$ so that the closed-loop system with input $r(t)$ and output $y(t)$ is BIBO stable.

(ii) Let $r(t)$ be a ramp input given by $r(t) = R\,t \cdot \mathbf{1}(t)$, with $R > 0$. Assuming that $\lim_{t\to\infty} e(t)$ exists, find all values of the parameters $K > 0$ and $a > 0$ so that $\lim_{t\to\infty} e(t) \leq 0.25\,R$.

(iii) Let $a = 2$. Find all values of $K > 0$ so that the conditions you found in (i) and (ii) both hold.

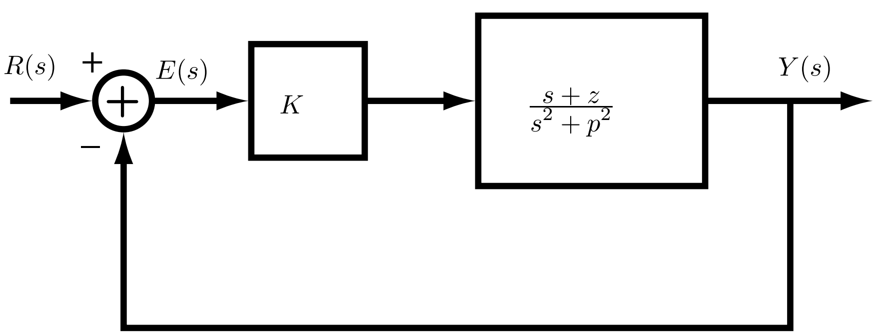
Problem 9

Consider the control system in the figure below.

(i) Let $r(t) = \mathbf{1}(t)$, $p = 1$, and $z = 1$. Pick a value of $K > 0$ such that the overshoot $\%OS$ in the output response of the closed-loop system satisfies $\%OS \leq 1\%$.

(ii) Now let $r(t) = R \sin t \cdot \mathbf{1}(t)$, with R a real number. Find the most general conditions on K, z, and p such that

$$(\forall R \in \Re)\ \lim_{t\to\infty} e(t) = 0.$$



R(s)
+
−
E(s)
K
s + z
s^2 + p^2
Y(s)