

# CONTROL SYSTEMS LABORATORY

## ECE311S

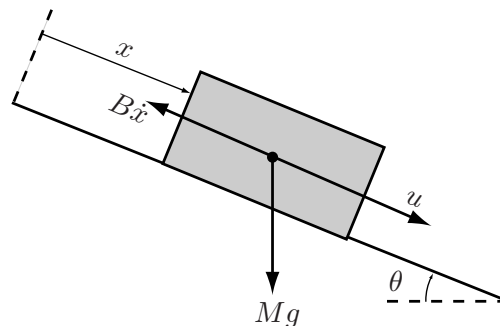
### LAB 1: Familiarization with Equipment and Basic Cruise Control Design

## 1 Purpose

The purpose of this experiment is to introduce you to the lab setup (the IP02 cart system by Quanser Inc.) and the associated control problem. Namely, the design of a cruise control system for a car.

## 2 Introduction

A car moves on a straight road with an unknown slope. It is assumed that the inertia of the wheels is negligible, that the friction force is proportional to the speed of the car, and that the engine imparts a force  $u$ . The schematic representation of the system is depicted below.



Applying Newton's law we obtain the following mathematical model of the system

$$M\ddot{x} = -B\dot{x} + u + Mg \sin \theta.$$

Since we are interested in controlling the speed  $\dot{x}$  of the car, we rewrite the model using  $v = \dot{x}$ ,

$$M\dot{v} = -Bv + u + Mg \sin \theta.$$

The IP02 cart system by Quanser is an idealized car. In the IP02 experiment the force  $u$ , imparted by a DC motor, is approximately proportional to the voltage  $v_m$  applied to the motor,

$$u = K_m v_m.$$

The voltage  $v_m$  is the control input to the plant. The mathematical model of the cart system becomes

$$\dot{v} = -\frac{B}{M}v + \frac{K_m}{M}\left(v_m + \frac{M}{K_m}g \sin \theta\right). \quad (1)$$

Since the slope  $\theta$  is unknown, the constant  $\frac{M}{K_m}g \sin \theta$  is a disturbance acting on the plant. Letting<sup>1</sup>

$$a = \frac{K_m}{M}, \quad b = \frac{B}{M}, \quad d(t) = \bar{d} \cdot \mathbf{1}(t) := \frac{M}{K_m}g \sin \theta \cdot \mathbf{1}(t),$$

the plant block diagram is depicted in Figure 1.

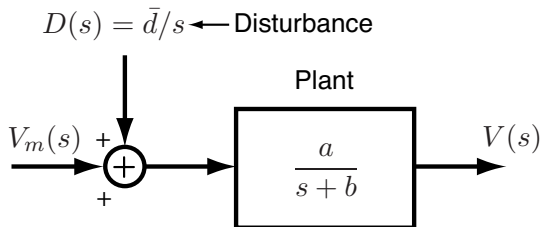


Figure 1: Block diagram of the plant. The control input is the voltage  $V_M$  (Volts). The measured output is the speed  $V$  (m/s).

In this lab, you will familiarize yourself with the experimental setup, you will experimentally determine the constants  $a$  and  $b$ , and you'll implement basic proportional and proportional-integral controllers to regulate the car speed.

### 3 Preparation

1. Download the file `experiment_familiarization.pdf` and read Sections 4, 5.1, 5.4, and 5.5. Skip Sections 5.2 and 5.3. The experimental setup you'll work on is labelled IP02.
2. Consider the block diagram in Figure 1 and assume that the road is flat, i.e.,  $\theta = 0$  and hence  $D(s) = 0$ . Suppose that a step voltage  $v_m(t) = V_0 \cdot \mathbf{1}(t)$  ( $V_0 > 0$ ) is applied to the DC motor and that at time  $t = 0$  the cart is still (i.e.,  $v(0) = 0$ ). Using the Final Value Theorem, determine  $v(+\infty) = \lim_{t \rightarrow \infty} v(t)$  in terms of  $V_0$ ,  $a$ , and  $b$ .

Submit this expression and its clear derivation to your lab TA at the beginning of the lab.

### 4 Experiment

**CAUTION!** Before running the experiment:

- Please ensure that the cart is positioned at the middle of the track!
- If for any reason your model goes unstable, turn off the experiment right away by clicking the STOP button on the Wincon Client!

After the lab you'll need to fill in the Word file `report_lab1.doc` posted on the course web site

<sup>1</sup>Here,  $\mathbf{1}(t)$  indicates the unit step function.

(one report per group) and submit it according to the instructions on the course webpage (typically a drop-off box). You'll have one week from the date of your lab to do it.

## Familiarization with the equipment and WinCon

1. Following the steps described in Section 5.1 of the document `experiment_familiarization.pdf`, apply a sinusoidal voltage  $v_m(t)$  to the DC motor and plot the real-time voltage data on the Wincon scope.

**Show your TA the completion of this step before proceeding to the next step.**

2. Following the steps described in Section 5.4 of the document `experiment_familiarization.pdf`, measure the real-time position of the cart as you move it manually back and forth. **Note:** rather than converting the encoder count to millimeters, make sure to convert the count to meters. In other words, the calibration factor in your Simulink diagram should be  $22.75e - 6$ , rather than  $22.75e - 3$ .

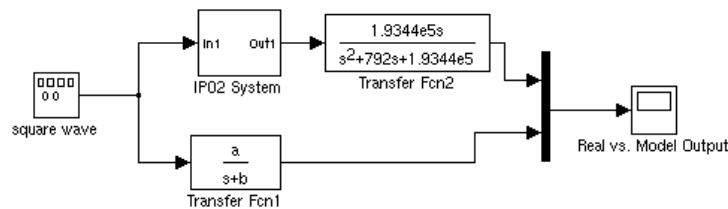
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3. Following the steps described in Section 5.5 of the document `experiment_familiarization.pdf`, create a Simulink block which you'll name IP02 combining the blocks you developed in parts 1 and 2; run the experiment and monitor the voltage  $v_m(t)$  and the cart position  $x(t)$ .

**Show your TA the completion of this step before proceeding to the next step.**

## Identification of model parameters $a$ and $b$

4. Create a Simulink model as in the figure below. The detailed steps to construct it are listed below.



- Open a blank Simulink model and the Library browser.
- Place your IP02 block from step 3 on the diagram. Since the optical encoder in the cart measures the displacement  $x(t)$ , and since we need to measure the velocity  $v(t)$ , we use a filter to approximately differentiate the output of the IP02 block. Using the Continuous/Transfer Function block, connect the output of the IP02 block to the transfer function

$$\frac{1.9344 \cdot 10^5 s}{s^2 + 792s + 1.9344 \cdot 10^5}$$

The output of this transfer function is (the estimate of) the cart velocity in m/s. We'll refer to this signal as the actual plant output.

- Using the Continuous/Transfer Function block, create a system with transfer function  $\frac{a}{s+b}$ . This block represents the mathematical model of the plant assuming that the disturbance is zero, that is, assuming that the cart track is horizontal. The objective here is to experimentally determine the values  $a$  and  $b$ .
- Using the Sources/Signal Generator block in Simulink, create a block that generates a square wave for  $v_m(t)$  with amplitude 1.5 V and frequency 0.5Hz.
- Connect the signal  $v_m(t)$  to both the input of the transfer function block and to the input of the IP02 block.
- Create a Signal Routing/Mux block with 2 inputs. Connect the actual plant output to input 1, and the model output to input 2 of the Mux.
- Connect the output of the Mux block to a Scope (found in the Sinks library). Name the scope “Real vs. Model Output.”

Look at the diagram you’ve just created: a square wave signal feeds both the model of the plant and the actual plant. The actual plant output and model output are fed to a scope. The idea now is to look at the discrepancy between model output and actual plant output and tune the parameters  $a$  and  $b$  to minimize it. A fundamental ingredient here will be the expression for  $v(+\infty)$  you found in your lab preparation.

**Show your TA the completion of this step before proceeding to the next step.**

5. We choose initial guesses for  $a$  and  $b$ . Define  $a = 0.5$  and  $b = 5$  in the Matlab workspace.
6. Build the diagram you’ve created. Using the Open Plot button in the WinCon Server toolbar, select the “Real vs. Model Output” signal. This action will open a WinCon scope. Now run your code using the Start button in the WinCon Server toolbar. After a few seconds, hit the Stop button.
7. The WinCon scope displays the actual velocity of the cart and the velocity predicted by the model. Since we’ve guessed the parameters  $a$  and  $b$ , it is not surprising that there is a discrepancy between the two measurements. Look at one  $1/2$  period of the square wave signal (don’t use the first period). Look at the actual plant output  $v(t)$  and determine as accurately as you can its total variation over the half period. We’ll denote such variation by  $\Delta v$ .
8. Notice that over the  $1/2$  period under consideration, the signal  $v_m(t)$  performs a step of amplitude  $V_0 = 3V$ .

We’ll make the approximation to consider the signal  $v(t)$  to be in steady-state at the end of the  $1/2$  period. Using the formula you found in your lab preparation and the value  $\Delta v$  you just found, find a relationship between  $a$  and  $b$ . Specifically, find an expression of the type  $a = f(b)$ .

9. It should be now clear that for any choice of  $b$ , setting  $a = f(b)$  guarantees that the steady-state value of the model output coincides with that of the actual plant output. Now you’ll tune  $b$  to make sure that the transients coincide as much as possible.

Keep the value of  $b$  you were using earlier, and set  $a = f(b)$  in the Matlab workspace. Run the experiment and verify that the steady-state values of the model and actual plant outputs coincide. Stop the experiment.

- Now try to increase  $b$ . Don't forget, everytime you modify  $b$ , to also set  $a = f(b)$  in the Matlab workspace. You don't need to build the diagram after changing  $b$ . Run the experiment to see if the new value of  $b$  yields better results. Keep tuning  $b$  until you minimize the discrepancy between the actual plant and model outputs.

You'll use the values of  $a$  and  $b$  you just found in Lab 2.

**Show your TA the completion of this step before proceeding to the next step.**

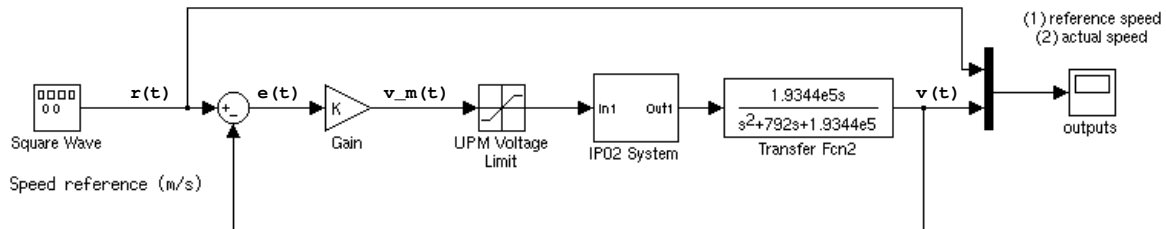
## Proportional control

- Now you'll implement a proportional controller (also called P controller) to regulate the speed of the cart. A proportional controller is a controller of the form

$$v_m(t) = Ke(t),$$

where  $e(t) := r(t) - v(t)$  is called the **tracking error**. This is the difference between the reference signal  $r(t)$  and the actual plant output  $v(t)$ . In the cart experiment,  $r(t)$  represents a desired velocity profile for the cart, while  $v(t)$  represents the actual cart speed.

Create a Simulink model as in the figure below.



Note the presence of the gain  $K$  - this is the proportional controller - and the saturation function which makes sure that the voltage  $v_m(t)$  sent to the amplifier is not too high.

- You'll find the Gain block in the Math Operations library.
- You'll find the Saturation block in the Discontinuities library. Double-click on the block and set the saturation limits to +13 and -13.
- The signal generator block should generate a square wave with amplitude 0.2m/s and frequency 0.5Hz.

**Show your TA the completion of this step before proceeding to the next step.**

- Set  $K = 5$ , then build the Simulink diagram.

13. Run the system. You'll notice that the controller does not succeed in regulating the speed to the desired value, it undershoots it. Stop the system, increase the controller gain, and run the system again (no need to rebuild it). Repeat this operation a few times. **Do not increase  $K$  beyond 20.**

Record your observations: Does the P controller successfully regulate the speed to the desired value? What's the effect of increasing the gain  $K$  on the output response?

14. Save the plot of the output response obtained when  $K = 20$  (your TA will show you how to do it).

### Proportional-integral control

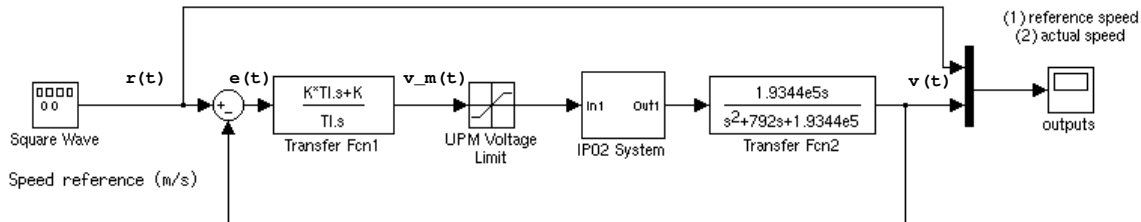
15. You'll now implement a proportional-integral controller, also called a PI controller. A PI controller is an enhancement of a P controller and has the form

$$v_m(t) = K e(t) + \frac{K}{T_I} \int_0^t e(\tau) d\tau,$$

where  $K$  and  $T_I$  are two positive design constants. Your TA will explain to you the rationale for using this controller. Taking the Laplace transform, we find that a PI controller has a transfer function (from  $e$  to  $v_m$ ),

$$K + \frac{K}{T_I s} = K \left( \frac{T_I s + 1}{T_I s} \right).$$

Replace the P controller in your Simulink model by a PI controller. You should obtain a block diagram like the one below.



**Show your TA the completion of this step before proceeding to the next step.**

16. Set  $T_I = 0.07$  and  $K = 2$ . Build the model and run it. Next, keep  $T_I$  constant and start increasing  $K$ . **Do not increase  $K$  beyond 20.**

Record your observations: what's the effect of increasing  $K$ ? How does the performance of the P and PI controllers compare?

Save a plot of the output response when  $T_I = 0.07$  and  $K = 20$ .