

# CONTROL SYSTEMS LABORATORY

## ECE311S

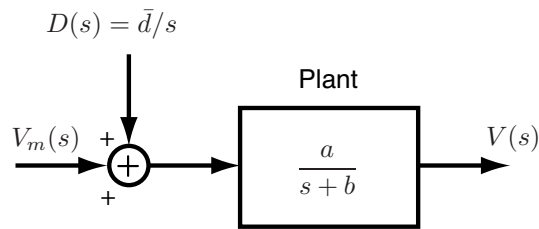
### LAB 2: Control Design Using the Root Locus

## 1 Purpose

The purpose of this laboratory is to design a cruise control system for a car using the root locus.

## 2 Introduction

Recall the mathematical model of a car moving on a straight road with unknown slope  $\theta$  derived in the introduction to laboratory 1.



In the block diagram above,  $\bar{d} = \frac{g}{a} \sin \theta$ , where  $g$  is the gravitational constant.

**Problem Statement:** Consider the feedback control system in Figure 1.

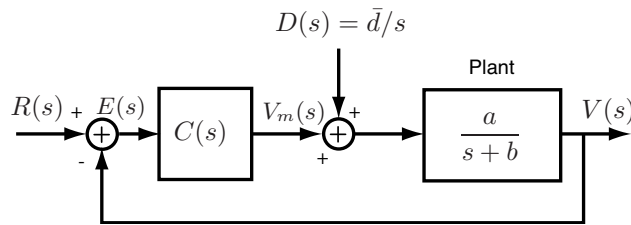


Figure 1: Block diagram of the closed-loop system.

Design the controller  $C(s)$  to meet the specifications below.

- (SPEC 1) The output  $v(t)$  of the closed-loop system should asymptotically track reference step signals  $r(t)$  despite the presence of the unknown disturbance  $d(t) = \bar{d} \cdot \mathbf{1}(t)$ .
- (SPEC 2) The closed-loop system (input  $R(s)$ , output  $V(s)$ , and no disturbance,  $D(s) = 0$ ) should be BIBO stable.
- (SPEC 3) All poles of the closed-loop system should lie on the real axis, so that the output  $v(t)$  does not have oscillatory behavior.
- (SPEC 4) When  $D(s) = 0$  (i.e., when the cart track is horizontal), the settling time  $T_s$  should be less than 0.2sec.

(SPEC 5) The magnitude voltage  $v_m(t)$  imparted by the control system to the DC motor should be less than 13V.

Concretely, you are required to design a control system making the IP02 cart track constant reference speeds even when the IP02 track is not horizontal, and meeting certain transient performance specifications.

### 3 Preparation

1. Referring to Figure 1, verify that the plant has type 0. Hence, in order to meet SPEC 1, you need a controller with a pole at zero. Letting  $E(s) = R(s) - V(s)$ , define the PI controller

$$\frac{V_m(s)}{E(s)} = K \left( 1 + \frac{1}{T_I s} \right), \quad K, T_I > 0.$$

Write the transfer functions from  $R(s)$  to  $E(s)$ , and from  $D(s)$  to  $E(s)$ .

2. Let  $R(s) = \bar{v}/s$  and recall that  $D(s) = \bar{d}/s$ . Using superposition, write the expression for  $E(s)$  in the closed-loop system.
3. Applying the final value theorem to  $E(s)$ , show that if  $\lim_{t \rightarrow \infty} e(t)$  exists, then it must be zero. You have thus shown that a PI controller is capable of meeting SPEC 1.

Submit your derivations for points 1-3 to your lab TA at the beginning of the lab. Thoroughly read the next section prior to the lab.

### 4 Experiment

**CAUTION!** Before running the experiment:

- Please ensure that the cart is positioned at the middle of the track!
- If for any reason your model goes unstable, turn off the experiment right away by clicking the STOP button on the Wincon Client!

After the lab you'll need to fill in the Word file `report_lab2.doc` posted on the course web site. You'll have one week to do it.

#### Identification of model parameters $a$ and $b$

If you are using a different IP02 unit than the one used in lab 1, you need to re-estimate the plant parameters  $a$  and  $b$  following the procedure outlined in lab 1. If instead you are working with the same IP02 unit, use the values of  $a$  and  $b$  you estimated in lab 1.

#### Control design using Matlab

1. In your preparation you've shown that a PI controller is capable of meeting SPEC1. We now focus on SPEC2-SPEC4. Let  $\theta = 0$  or, equivalently,  $D(s) = 0$ . Recall that an approximate expression for the settling time in a second-order system with complex conjugate poles is  $T_s = \frac{4}{\sigma}$ , where  $-\sigma$  is the *real part of the poles of the closed-loop system*. In order to satisfy SPEC4, you thus need  $\frac{4}{\sigma} \leq 0.2$ , or  $\sigma \geq 20$ . You'll achieve this by an appropriate choice of  $K$  and  $T_I$ .

Pick a guess for  $T_I$ ,  $T_I = 1$ . Using Matlab, you will now draw the root locus of the system, i.e., the locus of the poles of the closed-loop system as the gain  $K$  is varied between 0 and  $+\infty$ . First, define

the transfer function  $G(s)$  of the open-loop system without the gain  $K$ . This is given by

$$G(s) = \frac{T_I s + 1}{T_I s} \frac{a}{s + b}.$$

In Matlab, you can define  $G(s)$  as follows

```
>> TI=1; a=(your value); b=(your value);
>> G=tf([TI 1],[TI 0])*tf([a],[1 b])
```

The root locus is the locus of the poles of  $KG(s)/(1 + KG(s))$  as  $K$  is varied. Plot the root locus by issuing the command.

```
>> rlocus(G)
```

Verify, using the plot, that for all  $K > 0$  the poles of the closed-loop system have negative real part, and hence SPEC2 is met. However, for the present value of  $T_I$ , SPEC4 cannot be met by any choice of  $K > 0$ . Specifically, there doesn't exist  $K > 0$  such that the closed-loop system has two poles on the real axis with real part  $\leq -20$ . Prove the truth of the claim by using the root locus plot.

Evidently, we need to choose a different value for  $T_I$ . Try different (positive) values of  $T_I$ . For each choice of  $T_I$ , plot the corresponding root locus. By trial and error, find the value of  $T_I$  compatible with this requirement: *there exists  $K > 0$  such that the closed-loop system has two poles close to  $s = -20$  on the negative real axis.* Save the root locus plot.

Once you found  $T_I > 0$  satisfying the requirement above, you need to find  $K > 0$  for which the closed-loop system has two poles close to  $s = -20$ . First, plot the root locus corresponding to the value of  $T_I$  you just selected. Next, issue the command

```
>> rlocfind(G);
```

Move the mouse cursor over the root locus and click on the desired location of the closed-loop poles on the real axis at  $s = -20$ . This action will return the value of  $K$  you were looking for. Notice that the PI controller with the values of  $K$  and  $T_I$  you have just found should meet SPEC2-SPEC4.

2. Next, you'll double-check that using the values of  $K$  and  $T_I$  you just found, SPEC2-SPEC4 are met in simulation. Download the file `lab2.mdl` from the course web site and open it. Begin by double-checking that your controller indeed meets SPEC4. Enter the values of  $\theta$ ,  $T_I$ , and  $K$  into the Matlab workspace

```
>> a=(your value); b=(your value); theta = 0; TI=(your value); K=(your value);
```

Run the Simulink block by clicking on **Simulation > Start**. The scopes depict (i) the output  $v(t)$  vs. the reference signal  $r(t)$ , (ii) the tracking error  $e(t)$ , and (iii) the voltage  $v_m(t)$ . The reference signal is a square wave of frequency 0.5 Hertz and amplitude 0.2 m/s.

Recall that the settling time  $T_s$  of  $v(t)$  is the time  $v(t)$  takes to reach and stay within the range

$$\left[ v(\infty) - 0.02(v(\infty) - v(0)), v(\infty) + 0.02(v(\infty) - v(0)) \right],$$

where  $v(\infty)$  is the value  $v(t)$  asymptotically settles to.

By zooming in on one period of the simulation output, graphically estimate the settling time  $T_s$ . Save the plot you used to derive your estimate. Is it true that  $T_s \simeq 0.2\text{sec}$ ?

Verify that SPEC5 is approximately met.

**Show your TA the completion of this step before proceeding to the next step.**

3. Now you'll try to design a more "aggressive" PI controller. Similarly to what you did in step 1, use the root locus and trial and error to find the value of  $T_I > 0$  such that there exists  $K > 0$  such that the closed-loop system has two poles close to  $s = -40$ . Use the command `rlocfind` to find the value of  $K$  for which the closed-loop system has two poles close to  $-40$ .

Enter the values of  $K$  and  $T_I$  you just found in the Matlab workspace and run the Simulink diagram `lab2.mdl`. Similarly to what you did in step 2, evaluate the settling time  $T_s$  by zooming in on one period of the simulation output. Save the plot. Compare the performance of this “aggressive” controller to that of the controller you evaluated earlier.

How do the settling times and overshoots compare?

How do the maximum values of  $V_m(t)$  compare?

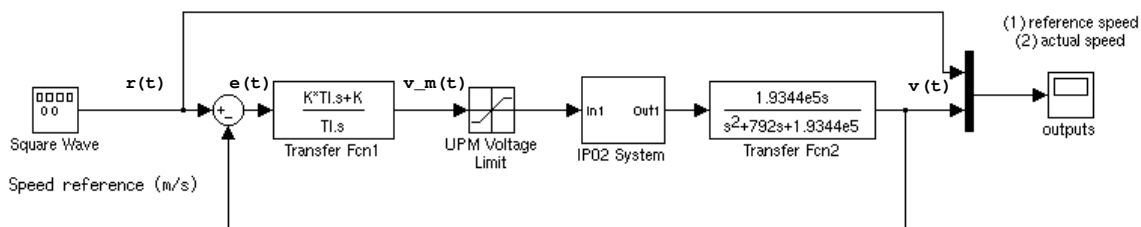
Which controller is best suited to meet the specs?

What is the cause of the differences you observe?

**Show your TA the completion of this step before proceeding to the next step.**

## Controller implementation

1. Recycle from lab 1 the Simulink block depicted below. If you haven’t saved the block, create it following the instructions outlined in lab 1. Make sure that the saturation limits are set at  $\pm 13V$  (using the parameter  $VMAXUPM$ ) and that the reference signal is a square wave with amplitude 0.2 m/s and frequency 0.5 Hz.



**Show your TA the completion of this step before proceeding to the next step.**

2. Turn on the UPM amplifier and place the cart at the middle of the track. Enter in the Matlab workspace the values of  $K$  and  $T_I$  associated with the less “aggressive” controller. Compile the Simulink diagram and run the experiment. Record the signals on the scope in a data file. Your TA will help you doing that.
3. Load the experimental data into Matlab; plot  $v(t)$  and  $r(t)$  versus time. Does your controller satisfy the design specifications? Run the experiment in front of the TA and show him your Matlab plot and save your plot.
4. Now you’ll test the disturbance of your controller against the unknown disturbance  $d(t)$ . Using one or more books, raise one end of the track. This corresponds to setting  $\theta \neq 0$ . Ensure (by possibly bribing the student) that one student in your team holds the track firmly in place and run the experiment, recording all signals in the scope in a new data file.
5. Repeat step 3 with the new data. Explain any differences between this experiment and the previous one.

**Show your TA the completion of this step before proceeding to the next step.**

6. Now enter in the Matlab workspace the values of  $K$  and  $T_I$  associated with the more “aggressive” controller and repeat steps 2-5.