University of Toronto
Department of Electrical and Computer Engineering
ECE410F Control Systems
Problem Set #2

1. Given the asymptotically stable system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du\,. \end{aligned}$$

Find the response of the system for the case when $x(0) = x_0$ and when $u(t)$ is given by

$$u(t) = \begin{cases} \overline{u} \in \mathbb{R}^m & 0 \leq t \leq T \\ 0 & T < t < \infty\,. \end{cases}$$

2. Consider the state equation $\dot{x} = Ax$. Let

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$$

and $x_0 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$. Determine the modal decomposition of $x(t)$.

3. Given the state space model

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 2 & -1 & 0 \\ 5 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u\,, \qquad y = [1\ 0\ 0]x\,.$$

Find the modal representation.

4. Find a modal representation of the following system:

$$\dot{x} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -2 & 0 \\ 1 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u\,.$$

$$y = [1\ 0\ 0]x\,.$$

Is the system controllable?

5. Determine if the following systems are controllable.

$$\dot{x} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 4 \\ 1 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$\dot{x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u$$

6. Consider the second-order system

$$\dot{x} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} x + \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} u$$

$$y = [1\ 0]x\,.$$

For what values of k_1 and k_2 is the system completely controllable?

7. We are given the state space system:

$$\dot{x} = \begin{bmatrix} -0.05 & -6 & 0 & 0 \\ -10^{-3} & -0.15 & 0 & 0 \\ 1 & 0 & 0 & 13 \\ 0 & 1 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} -0.2 \\ 0.03 \\ 0 \\ 0 \end{bmatrix} u\,.$$

First determine if this system is stable. Second, determine if this system can be stabilized using a controller:

$$u = -k_1 x_1 - k_3 x_3\,.$$