University of Toronto
Department of Electrical and Computer Engineering
ECE410F Control Systems
Problem Set #6

1. The Canadian Transportation Agency has contracted you to design a longitudinal controller for an automated snowplow. Let x_1 be position, x_2 velocity, u force input, m mass, and k viscous friction. A simplified model of the longitudinal dynamics of the snowplow is

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= \frac{1}{m}(-kx_2 + u)\,. \end{aligned}$$

   Suppose $m = k = 1$. The control objective is, starting from rest, bring the snowplow to a velocity of $20m/s$ by tracking a reference position of $p(t) = t^2$, $0 \leq t \leq 10$.

   Design a tracking controller (exact matching and asymptotic parts) to track $p(t)$, $0 \leq t \leq 10$, assuming full state information, so that the error between the plant state and exosystem state decays as $\mathrm{e}^{-\mathrm{t}}$ and $\mathrm{e}^{-2\mathrm{t}}$.

2. Consider the previous problem. Suppose only the position is available for measurement. Design an observer such that the observer error decays as $\mathrm{e}^{-10\mathrm{t}}$. Write your final compensator transfer function (observer + tracking controller) in terms of the system matrices and the observer and controller parameters.

3. Suppose that after the snowplow reaches a speed of $20m/s$ it must move at a constant speed thereafter. Design a tracking controller so that the snowplow tracks a constant reference speed of $20m/s$ starting from $t = 10$ sec, with exponential convergence of the tracking error of e^{-5t}.