University of Toronto
Department of Electrical and Computer Engineering
ECE557F Systems Control
Problem Set #7

1. Consider the linear system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} x \end{aligned}$$

   (i) Suppose the control design objective is closed loop stability and that the output y asymptotically tracks a constant reference set point y_d. Determine, using feedforward control, a state feedback controller which will place the closed loop poles at -1 and -2 and achieves asymptotic output tracking.

   (ii) Now assume that the state is not measured. Design a minimal order observer to estimate the state that is not directly measured, placing the observer pole at -4.

   (iii) Using the separation principle, combine the results of (i) and (ii) to design an output feedback controller which, together with the feedforward of the reference input, ensures that the design objectives are satisfied. Determine the closed loop transfer function from y_d to y, and verify that indeed asymptotic tracking is achieved.

2. Again consider the linear system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} x \end{aligned}$$

   (i) Augment the system dynamics with the equation

$$\dot{\xi} = y - y_d$$

   Design a state feedback law of the form

$$u = -Kx - K_I \xi$$

   to place the poles of the closed loop system at -1,-1,-2. Determine the closed transfer function from y_d to y and verify that asymptotic tracking is achieved.

   (ii) Now suppose that system matrix is perturbed to

$$A_p = \begin{bmatrix} 0 & 1 \\ 2.1 & 1 \end{bmatrix}$$

   Assume that the same control law as in (i) is used. Verify that the closed loop system is still stable and that asymptotic tracking is maintained despite the perturbation.

(iii) Now use the minimal order observer contructed in problem 1 to design an output feedback law to achieve the design objectives. Determine the closed loop transfer function from y_d to y, and verify that asymptotic tracking is achieved.

3. We have studied 2 methods to achieve asymptotic set point tracking. In this problem we consider combining both methods to see if there are any additional advantages to be gained.

   (i) Consider the simple scalar system

   $$\begin{aligned} \dot{x} &= 2x + u \\ y &= x \end{aligned}$$

   Again the objective is stabilization and for y to track y_d. First we add an integrator

   $$\dot{\xi} = y - y_d$$

   Now determine a feedback law $u = -Kx - K_I\xi$ such that the closed loop poles are located at -1 and -2.

   (ii) Now add also the feedforward term Ny_d to the controller so that $u = -Kx - K_I\xi + Ny_d$. Determine the closed loop transfer function from y_d to y as a function of the gain N. What role can N play?

4. The Canadian Transportation Agency has contracted you to design a longitudinal controller for an automated snowplow. Let x_1 be position, x_2 velocity, u force input, m mass, and k viscous friction. A simplified model of the longitudinal dynamics of the snowplow is

   $$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= \frac{1}{m}(-kx_2 + u)\,. \end{aligned}$$

   Suppose $m = k = 1$. The control objective is, starting from rest, bring the snowplow to a velocity of $20m/s$ by tracking a reference position of $p(t) = t^2$, $0 \leq t \leq 10$.

   Design a tracking controller (exact matching and asymptotic parts) to track $p(t)$, $0 \leq t \leq 10$, assuming full state information, so that the error between the plant state and exosystem state decays as $\mathrm{e}^{-\mathrm{t}}$ and $\mathrm{e}^{-2\mathrm{t}}$.

5. Consider the previous problem. Suppose only the position is available for measurement. Design an observer such that the observer error decays as $\mathrm{e}^{-10\mathrm{t}}$. Write your final compensator transfer function (observer + tracking controller) in terms of the system matrices and the observer and controller parameters.

6. Suppose that after the snowplow reaches a speed of $20m/s$ it must move at a constant speed thereafter. Design a tracking controller so that the snowplow tracks a constant reference speed of $20m/s$ starting from $t = 10$ sec, with exponential convergence of the tracking error of e^{-5t}.