University of Toronto
Department of Electrical and Computer Engineering
ECE557F Systems Control
Problem Set #3

1. Consider the system

$$\dot{x} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u$$

Is the vector $\begin{bmatrix} 4 \\ 1 \\ 4 \end{bmatrix}$ reachable from the origin?

2. Is the following (A, B) pair controllable?

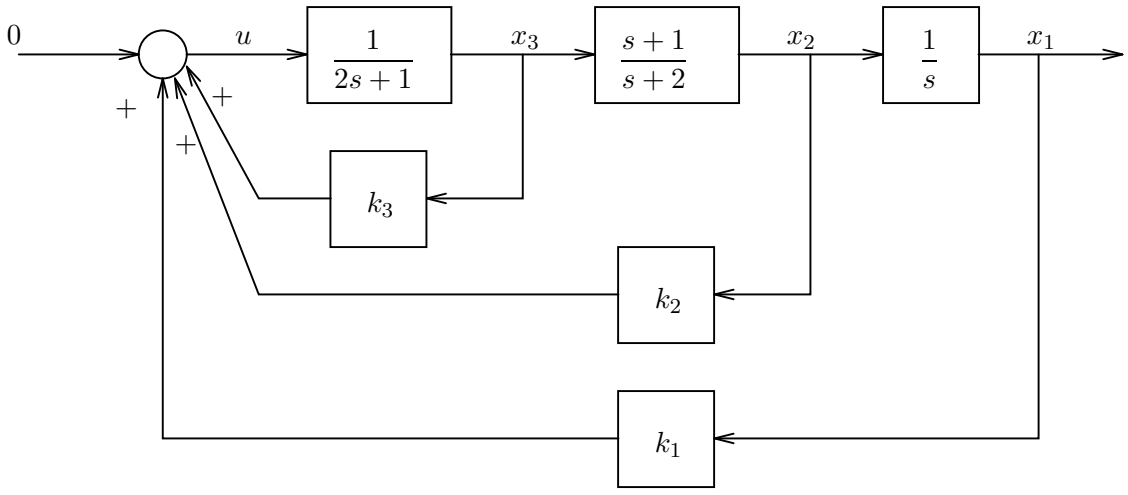
$$A = \begin{bmatrix} 0 & -6 & 0 & 4 \\ 1 & 4 & -1 & 0 \\ 1 & 8 & 0 & 0 \\ 1 & 11 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

3. Let $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

   (i) Check that (A, B) is controllable.

   (ii) Find a feedback law $u = kx$ such that the closed-loop poles are all at -1.

4. Find constants k_1, k_2, k_3 such that the closed-loop poles of the following system are assigned to $\{-1, -3 \pm j\}$.



5. Prove that (A, B) is controllable if and only if $(A + BK, B)$ is controllable for all K.

6. A standard simulation package for analysis and design of control systems is Simulink from MATLAB. You will use it in your labs. In this problem, you are asked to use Simulink to analyze a second order system.

   Consider the following system:

$$\begin{aligned} \dot{x}_1 &= -x_2 \\ \dot{x}_2 &= x_1 - 3x_2 \end{aligned}$$

   (a) Build a Simulink model of this system using standard building blocks of Simulink such as integrators, gains, etc. Output x_1 and x_2 to outport blocks as well as to Simulink scopes. Save your model as a Simulink file, say ss_sys.mdl.

   (b) Run several simulations, starting with different initial conditions. View the two state trajectories on the Simulink scopes.

   (c) You can save your simulation data directly to MATLAB using the commands simset and sim. Consult on-line help about these commands. Do a plot of x_1 versus x_2. This is the phase portrait, showing the dynamic behaviour of the trajectories. What observations can you make about the phase portrait? Interpret the phase portrait in terms of the modal decomposition of the system.