University of Toronto
Department of Electrical and Computer Engineering
ECE557F Systems Control
Problem Set #4

1. Given

$$A = \begin{bmatrix} 5 & 2 & 1 \\ -2 & 1 & -1 \\ -4 & 0 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} -2 & 2 \\ 2 & -1 \\ 2 & -2 \end{bmatrix}$$

   (i) Verify that the system is controllable, but not from a single column of B.

   (ii) Find a feedback gain K such that the closed-loop poles are assigned to $\{-10, -2 \pm i\}$.

2. For the system $\dot{x} = Ax + Bu$ with

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$$

   (i) Verify that the system is not controllable.

   (ii) Change the basis to decompose the system into a controllable part and an uncontrollable part. Is this system stabilizable?

3. Consider the PBH test for stabilizability and detectability. Show that

   (i) (A, B) is stabilizable if and only if

$$Rank[A - \lambda I \quad B] = n \quad \text{for all } \mathrm{Re}\,\lambda \geq 0$$

   (ii) (C, A) is detectable if and only if

$$Rank \begin{bmatrix} C \\ A - \lambda I \end{bmatrix} = n \quad \text{for all } \mathrm{Re}\,\lambda \geq 0$$

4. In this problem, we illustrate the practical algorithm of doing pole placement control design for multi-input systems using Matlab. We shall use a 6th order example.

   (a) First we generate A and B so that the resulting system is not controllable with respect to a single column of B. This can be done as follows. Let

$$F = \begin{bmatrix} 1 & 0 & -8 & 0 & 0 & 0 \\ 3 & -2 & -1 & 0 & 0 & 0 \\ 4 & 1 & -5 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5 & 2 & 6 \\ 0 & 0 & 0 & -4 & -2 & 1 \\ 0 & 0 & 0 & 3 & 7 & 9 \end{bmatrix}$$

and

$$G = \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

(F, G) is not controllable with respect to a single column of G. Create A and B using

$$\begin{aligned} A &= V^{-1} F V \\ B &= V^{-1} G \end{aligned}$$

for some nonsingular V, say

$$V = \begin{bmatrix} 1 & 1 & 1 & 0 & 4 & 0 \\ 2 & 0 & 1 & 5 & 0 & 3 \\ 1 & 0 & 5 & 2 & 1 & 1 \\ 0 & 0 & 8 & 2 & 1 & 4 \\ 1 & 0 & 0 & 1 & 2 & 7 \\ 1 & 8 & 2 & 0 & 0 & 1 \end{bmatrix}$$

Verify that (A, B) is controllable but not using a single column of B. The Matlab commands `ctrb` and `rank` can be used.

(b) Next we use a random matrix K_1 to produce a closed loop system that is controllable with respect to a single column of B. For example, take

$$K_1 = \text{rand}(2, 6) * 1.5$$

Now check to see if $(A + BK_1, B)$ is controllable with respect to a single column of B, say the first column b_1.

(c) Determine, using the command place or acker, a feedback gain K_2 such that the eigenvalues of $A + BK_1 + b_1 K_2$ are located at $-1, -1 \pm i, -2, -2 \pm i$.

(d) Combine K_1 and K_2 to give the overall gain K to place the poles of $A + BK$ at the desired locations. Check that the eigenvalues of $A + BK$ are indeed the ones desired.