1. Consider the linear system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

   Design a state feedback law $u = -Kx$ so that the following cost criterion is minimized:

$$J = \int_0^\infty [x^T(t)Qx(t) + u^2(t)]dt$$

   with $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Determine also the poles of the closed loop system.

2. Consider the linear system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -10 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$$

   Design a state feedback law $u = -Kx$ so that the following cost criterion is minimized:

$$J = \int_0^\infty [x^T(t)Qx(t) + u^2(t)]dt$$

   with $Q = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$. Determine also the poles of the closed loop system.

3. An unstable robot system is described by the state equation

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \,.$$

   Assume that the initial condition is $x(0) = [1 \ 1]^T$. Suppose the control is set to

$$u = Kx \,.$$

   Design the gain K so that the cost function

$$J = \int_0^\infty (x^T x + \epsilon u^T u)dt$$

   is minimized. Plot the magnitude of the control $\|u(0)\|$ at the initial time for $\epsilon \in (0, 100]$.

4. Consider the plant

$$Y(s) = \frac{1}{s - \lambda}U(s) \,,$$

where λ is an arbitrary parameter. Find a controller that stabilizes the system and minimizes the performance index

$$J = \int_0^\infty \left[y^2(\tau) + \epsilon u^2(\tau) \right] d\tau \, .$$

Next, examine the closed-loop eigenvalues of the system as $\epsilon \longrightarrow 0$ and $\epsilon \longrightarrow \infty$ for the case when (i) $\lambda > 0$, i.e. the plant is open-loop unstable, and (ii) $\lambda < 0$, i.e. the plant is open-loop stable.

5. Let $Q \geq 0$. Suppose we can factor Q into $Q = C^T C$. Show that the following conditions are equivalent:

   (i) $(\sqrt{Q}, A)$ is detectable

   (ii) (Q, A) is detectable

   (iii) (C, A) is detectable