**ECE557F – Systems Control**

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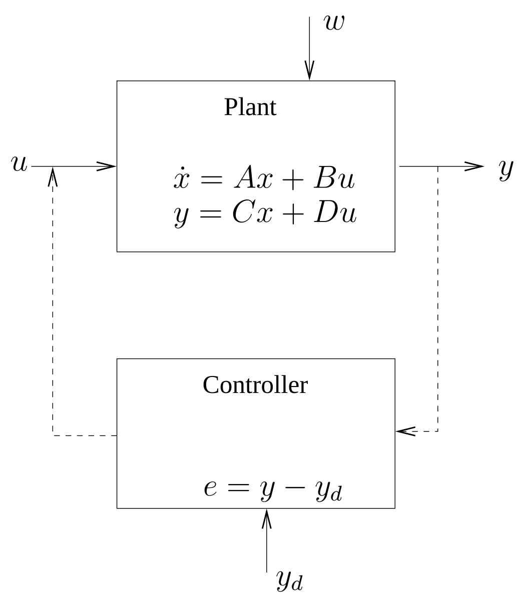
This course concerns *state space* design of linear time-invariant control systems. The model we consider is

$$\dot{x} = Ax + Bu \tag{1}$$
$$y = Cx + Du\,, \tag{2}$$

where $x \in \mathbb{R}^n$ is the *state*, $u \in \mathbb{R}^m$ is the *input*, and $y \in \mathbb{R}^p$ is the *output*.

This mathematical model captures a wide array of physical, economic, biological, and management (to name a few) systems, including nonlinear systems that are linearized about an equilibrium point. The basic problem to be considered is the following:

Given a plant (the open loop system) modelled according to (1)–(2) (which may have some disturbance inputs $w$), find a controller $u$ as a function of $y$ such that the error $e$ between the actual output $y$ and the desired output $y_d$ tends to zero. See the Figure below. We will not always include the disturbance in our designs, but we will at times add an additional requirement that the problem be solved in some optimal sense.




Linear control systems received a great deal of attention by control researchers in the 1960's and 1970's for several reasons:

- There has been a high level of success in developing useful design techniques for this model; for instance, pole placement, optimal control, and tracking control techniques.

- This model best exposes the fundamental properties of controllability and observability.
- The model eases the analysis of multi-input multi-output systems (more difficult using transfer functions).
- Many industrial applications can be effectively modelled using (1)–(2).
- Since the 1980's, one can handily design state-space based controllers using the control design toolboxes of `Matlab`.

Some successful industrial applications of linear system theory include:

- Control of a distillation column
- Control of large flexible structures
- Traffic light control
- Control of a steel-rolling mill
- Automotive systems: Cruise control, carburator control, and anti-lock braking systems
- Control of heat flow and temperature control
- Control of disk drives
- Space robot control
- Autonomous vehicle control
- Aircraft autopilots
- High precision control of machine tools
- Magnetically levitated trains
- Control for paper manufacturing
- Anesthesia control

# Course Outline

The following topics will be covered in the course.

1. **Introduction.**
   Modelling, state equations, examples.
2. **Linear Algebra Background.**
   Vector spaces, subspaces, linear transformations, change of basis, Jordan form, Cayley-Hamilton theorem, invariant subspaces.

3. **Solution of Linear ODE's.**
   Existence and uniqueness, solution of $\dot{x} = Ax$, state transition matrix, modal decomposition, phase portraits of 2D linear ODE's, stability

4. **Controllability.**
   Controllability matrix, main theorem on controllability, invariance under change of basis, invariance under state feedback, decomposition into controllable and uncontrollable parts, PBH test, controllable canonical form.

5. **Pole Assigment.**
   Single and multi- input, stabilizability.

6. **Observability.**
   Observability matrix, Kalman decomposition, detectability.

7. **Observers.**
   Design of observers and observer-based control, Kalman filter.

8. **Linear quadratic optimal control.**

9. **Tracking.**
   Regulation problem, observer-based output regulation.