Power Systems Operations and Control: An Overview

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DISC Summer School on Control in Power and Energy Systems

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September 15, 2023

Prof. J. W. Simpson-Porco: Control Theory

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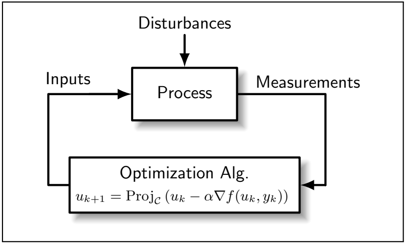
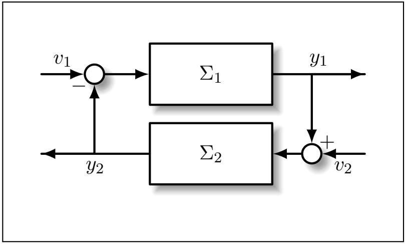
1. optimization and data-driven control
2. nonlinear systems
3. smart grid and energy systems

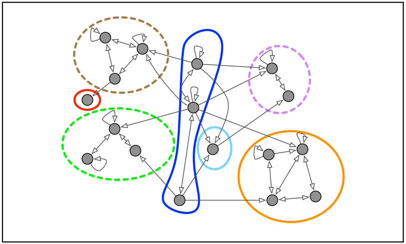
Feedback-Based Optimization



Nonlinear Systems



Network Dynamics & Control



Prof. J. W. Simpson-Porco: Energy Systems

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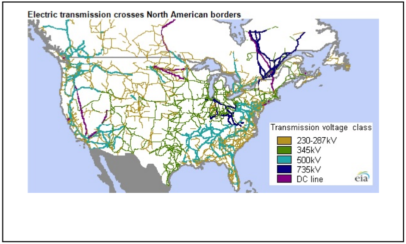
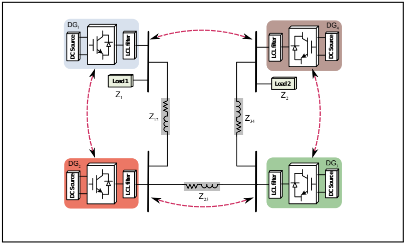
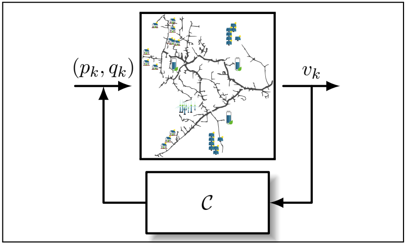
Power Flow Analysis & Algorithms



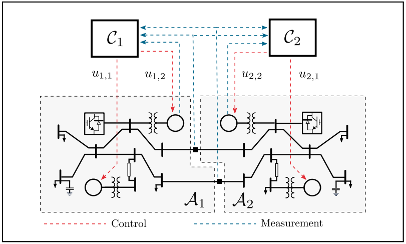
Microgrid Control & Optimization



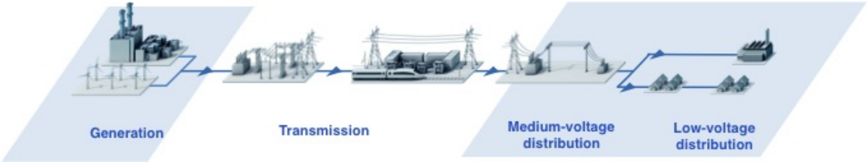
Renewable Energy Integration



Next-Generation Hierarchical Control



Overview of the Bulk Power System



	Classical paradigm	Modern trend
Generation	Bulk, centralized	Small-scale, distrib.
Energy interface	Sync. generators	Power electronics
Net load uncertainty	Low	Renewable-driven
Information	Centralized	Distributed
Sensors/Actuators	Low-bandwidth	High-bandwidth

The time is **now** for advanced control to have **real impact**.

Grid Modernization Design Spec's for Control Engineers

1. Coordinated Control of Many (Heterogeneous) Resources
   - **Real-time system optimization** w/ performance guarantees
   - **Scalability** to thousands of sensors/actuators
2. Grid Architecture (sensors/actuators/IT/algorithms/CPS)
   - Hierarchical **layering** across spatial and temporal scales
   - Prefer **localized** use of measurements (min. latency)
3. Practical Constraints in Power Engineering
   - Seamless integration with **legacy systems**
   - **Simple**, and congruent w/ *established power eng. principles*

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The Power System Control Zoo

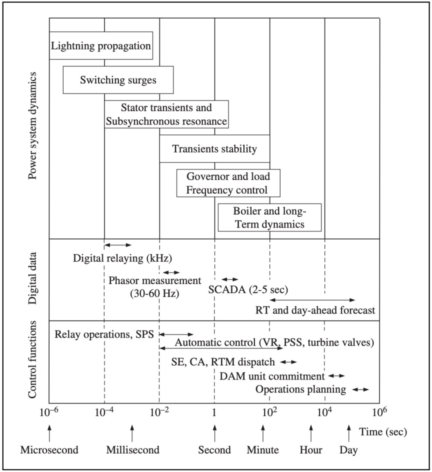
Figure: J. Chow and J.J. Sanchez-Gasca. *Power System Modeling, Computation, and Control*

Purpose of control is to main

1. power quality
2. power security
3. efficiency of operation

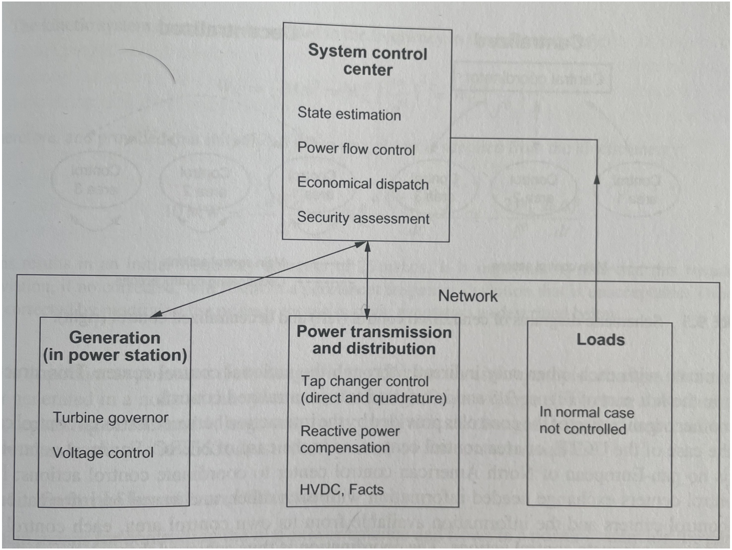
<u>Types of control</u>

- component-level loop designs
- frequency / voltage control
- wide-area damping control
- HVDC control
- economic dispatch / OPF
- energy and service markets
- unit commitment
- ...



Hierarchical Architecture of Power Systems Controls

Figure: G. Andersson, C. A. Bel, C. Cañizares. *Frequency and Voltage Control*



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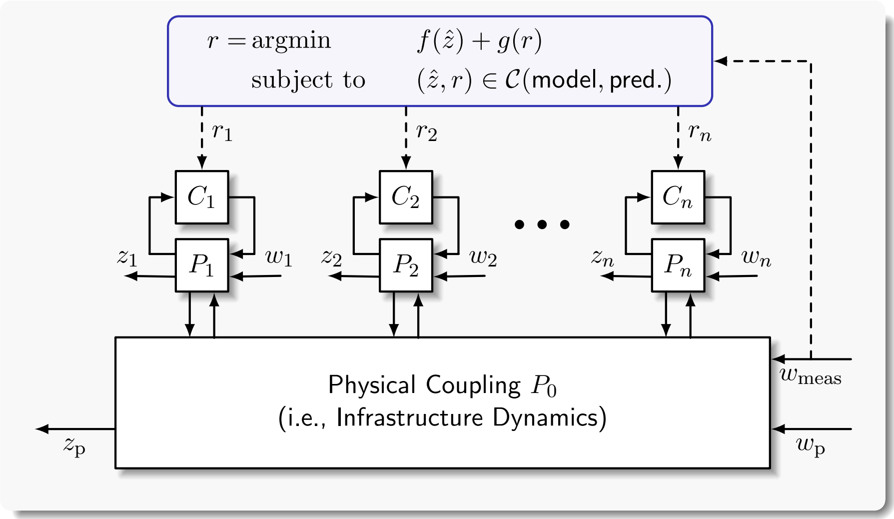
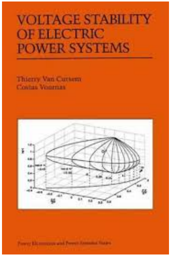
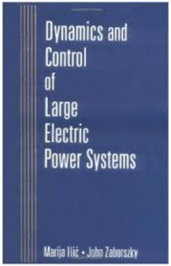
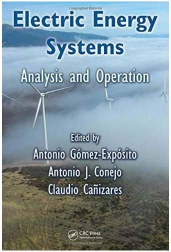
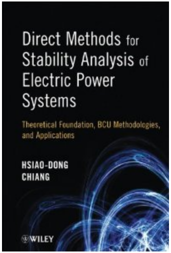


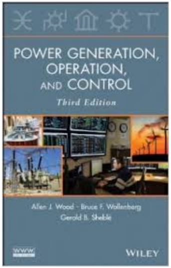
Many Excellent (And Recently Updated) Textbooks

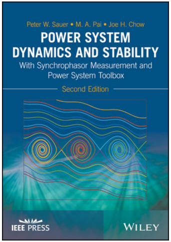


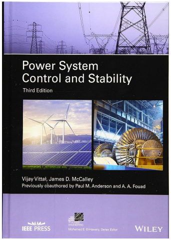


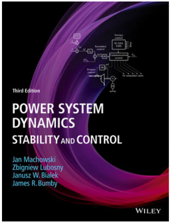


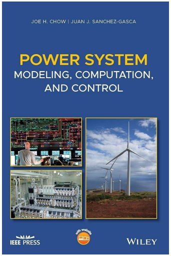


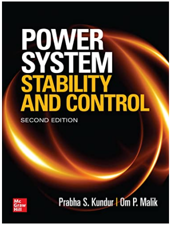












Topics, Disclaimers, Excuses, Etc.

This is a huge, diverse set of topics. What will we cover?

- Coverage biased by my own interests and knowledge
- Mix of theory and practice, key control insights
- Trying to present a viewpoint you can't find in textbooks

Power Flow and Dispatch

- power flow equations
- load flow problem
- the power flow Jacobian
- dispatch / optimal power flow
- contingency analysis

Stability & Control

- power system stability (brief)
- primary frequency control
- automatic generation control
- fast frequency control

Core Ideas in Power Systems Operations/Control

- **Active power** P
  (i) is used as a control variable to regulate **frequency**
  (ii) can be transmitted **long distances** with little loss
  (iii) is the primary variable of economic importance

- **Reactive power** Q
  (i) is used as a control variable to regulate **voltage magnitude**
  (ii) is absorbed by inductance; can be transmitted only **short distances**
  (iii) is important for maintaining efficient transport of active power

- **Frequency** Δf
  (i) is **spatially homogeneous** in steady-state
  (ii) is maintained close to 50/60Hz through a **hierarchy** of control systems

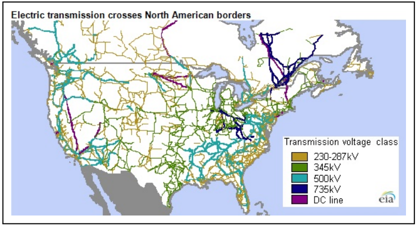
- **Voltage magnitude** V
  (i) is **spatially heterogeneous** in steady-state
  (ii) generally allowed to float between operational bounds
  (iii) primarily governed by local controllers

Steady-State AC Power Flow, Economic Dispatch, and Optimal Power Flow

Power Flow in the Transmission Grid

Figure: U.S. Energy Information Administration, based on Energy Velocity.

The transmission grid is effectively a **giant electrical circuit**, through which power is routed from generation to load.



(i) **alternating current**: (roughly) constant 50Hz or 60Hz

$$v(t) = \mathrm{Re}(Ve^{\mathbf{j}\theta}e^{\mathbf{j}\omega t}), \qquad V \geq 0, \quad \theta \in [0, 2\pi]$$

(ii) **three-phase**: each transmission line is really three lines (a, b, c)

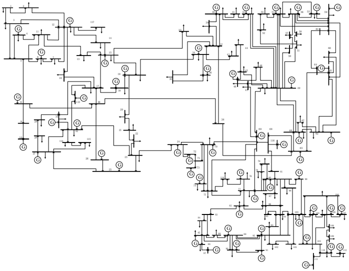
$$v_a(t) = \mathrm{Re}(V_a e^{\mathbf{j}\theta_a}e^{\mathbf{j}\omega t}), \qquad v_b(t) = \cdots ,$$

(iii) **balanced:** $V_a = V_b = V_c, \quad \theta_b = \theta_a - \frac{2\pi}{3}, \quad \theta_c = \theta_a + \frac{2\pi}{3}$

Power Flow in the Transmission Grid

- Under these (and some other mild[1]) conditions
  (i) inductance and capacitance become **impedance**/**admittance**
  (ii) all phases are **decoupled**; no interactions

We can use **single-phase phasor AC circuit analysis**.



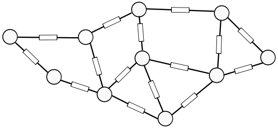
We must describe

1. bus-branch interconnections
2. transmission line models
3. physics (KCL, KVL, Ohm)
4. generation model
5. load model

[1] Sources and loads wye-connected, no mutual inductances between phases.

Power Flow in the Transmission Grid

- Circuit described by a **graph** $\mathcal{G} = (\mathcal{N}, \mathcal{E})$



(i) node/buses $\mathcal{N} = \{1, \ldots, n+m\}$

(ii) edges/branches $\mathcal{E} \subset \mathcal{N} \times \mathcal{N}$

- Edge $(i, j) \in \mathcal{E}$ models a transmission line with **series admittance**[2]

$$y_{ij} = g_{ij} + \mathbf{j} b_{ij}$$

- For each bus $i \in \mathcal{N}$ we have
  (i) a (complex) potential $\tilde{V}_i$
  (ii) a (complex) external current injection I_i
  (iii) a shunt admittance $y_{\mathrm{s},i}$ (typically, capacitive)

[2] Extends fairly easily to more complex line models.

Power Flow in the Transmission System

- **Ohm's Law:** $I_{i\to j} = y_{ij}(\tilde{V}_i - \tilde{V}_j), \quad I_{\mathrm{s},i} = y_{\mathrm{s},i}\tilde{V}_i$
- Current balance using KCL

$$\begin{aligned} I_i &= \sum_{j\neq i} I_{i\to \mathbf{j}} + I_{\mathrm{s},i} \\ &= \sum_{j\neq i} y_{ij}(\tilde{V}_i - \tilde{V}_j) + y_{\mathrm{s},i}\tilde{V}_i \\ &\triangleq \sum_j Y_{ij}\tilde{V}_j \end{aligned}$$

The matrix $Y \in \mathbb{C}^{N\times N}$ is known as the **admittance matrix**

$$Y_{ij} = \begin{cases} y_{\mathrm{s},i} + \sum_{j\neq i} y_{ij} & \text{if } i = j \\ -y_{ij} & \text{if } i \neq j \end{cases}$$

- **conductance** matrix $G = \mathrm{Re}(Y)$
- **susceptance** matrix $B = \mathrm{Im}(Y)$

- The **complex power** $S_i = P_i + \mathbf{j}Q_i$ is given by

$$S_i = \tilde{V}_i I_i^* = \tilde{V}_i \sum_j Y_{ij}^* \tilde{V}_j^* \quad \Longleftrightarrow \quad S = \mathrm{diag}(\tilde{V})(Y\tilde{V})^*$$

The Power Flow Equations

These equations can be written in **many** equivalent ways.

- **Rectangular form:** $S_i = \tilde{V}_i \sum_j Y_{ij}^* \tilde{V}_j^*$
  - nonlinear quadratic equations, useful for analysis and optimization
- **"SDP" form:** $S_i = \sum_j Y_{ij}^* W_{ij}$ with $W_{ij} = V_i V_j^*$
  - useful for semidefinite programming representation of OPF
- **Fixed-point form:** $\tilde{V} = F(\tilde{V})$ for some function F
  - useful for analysis (existence/uniqueness of solns)
- **Polar form:** $\tilde{V}_i = V_i e^{\mathbf{j}\theta_i}$ and $S_i = P_i + \mathbf{j}Q_i$

$$P_i = \sum_j V_i V_j B_{ij} \sin(\theta_i - \theta_j) + \sum_j V_i V_j G_{ij} \cos(\theta_i - \theta_j)$$
$$Q_i = -\sum_j V_i V_j B_{ij} \cos(\theta_i - \theta_j) + \sum_j V_i V_j G_{ij} \sin(\theta_i - \theta_j)$$

The AC Power Flow Problem

- We now incorporate generation and load models into the picture
- n **Loads,** $m-1$ **Generators, 1 Slack** $\mathcal{N} = \mathcal{N}_L \cup \mathcal{N}_G \cup \mathcal{N}_{\mathrm{s}}$

Bus Type	**Fixed Vars.**	**Free Vars.**
Load (PQ) Bus	P_i, Q_i	θ_i, V_i
Generator (PV) Bus	P_i, V_i	θ_i, Q_i
Slack Bus	$\theta_i = 0, V_i$	P_i, Q_i

$$P_i = \sum\nolimits_j V_i V_j B_{ij} \sin(\theta_i - \theta_j) + \sum\nolimits_j V_i V_j G_{ij} \cos(\theta_i - \theta_j)\,, \qquad i \in \mathcal{N}_L \cup \mathcal{N}_G$$

$$Q_i = -\sum\nolimits_j V_i V_j B_{ij} \cos(\theta_i - \theta_j) + \sum\nolimits_j V_i V_j G_{ij} \sin(\theta_i - \theta_j)\,, \qquad i \in \mathcal{N}_L$$

Power Flow Problem: Solve, if possible, the above $2n+m-1$ equations for the $n+m-1$ unknowns $\{\theta_i\}_{i\in\mathcal{N}_L\cup\mathcal{N}_G}$ and the n unknowns $\{V_i\}_{i\in\mathcal{N}_L}$.

Comments on the AC Power Flow Problem

$$P_i = \sum_j V_i V_j B_{ij} \sin(\theta_i - \theta_j) + \sum_j V_i V_j G_{ij} \cos(\theta_i - \theta_j)\,, \qquad i \in \mathcal{N}_L \cup \mathcal{N}_G$$

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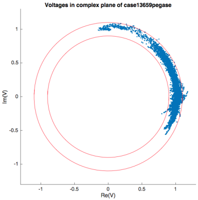
- The most ubiquitous problem in power system operations
- Solution **approximates** the operating equilibrium voltages/angles of the real dynamic grid
- The slack bus is a **mathematical simplification**; provides or extracts real power to balance out the system and enable feasibility of the nonlinear equations.

Many, Many Extensions

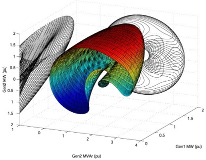
- Voltage-dependent loads
- "Distributed slack bus" (models real generator response)
- Transfer constraints between areas
- Remote regulation of PQ bus voltages
- Multiple generators per bus
- Generator Q limit switching

Intuition on Transmission Grid Power Flow Solutions

- *Normally* there is a unique **high-voltage** solution with the following nice properties
  (i) If $V_i \approx 1$ p.u. for generators $i \in \mathcal{N}_G \cup \mathcal{N}_\mathrm{s}$, then $V_i \approx 1 - \epsilon$ p.u. for loads $i \in \mathcal{N}_L$
  (ii) $|\theta_i - \theta_j| \ll 1$ for all $(i, j) \in \mathcal{E}$
- Just like in undergrad, AC circuits are subject to **maximum power transfer limits**; you can only send so much power from point A to B
- Lightly loaded systems have **many** solutions
- Heavily loaded systems may have **no** solutions; solutions will coalesce and disappear in saddle-node bifurcations as maximum power transfer is reached.



[Josz et al.]



[Hiskens & Davy]

Solution of ACPF Problem via Newton's Method

- With $x = (\theta, V_L)$ the ACPF equations can be expressed as $0 = f(x)$

$$\text{Newton's Method:} \quad x_{k+1} = x_k - \left(\frac{\partial f}{\partial x}(x_k)\right)^{-1} f(x_k)$$

- If **convergent**, may converge to "wrong" solution
- If **non-convergent**, several possibilities:

(a) No power flow solution exists

(b) Numerical instability (conditioning)

(c) x^0 not in any **region of convergence**

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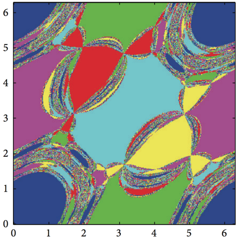

[Deng et al.]

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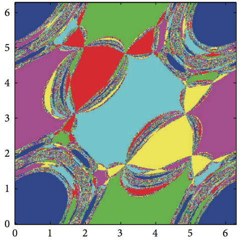

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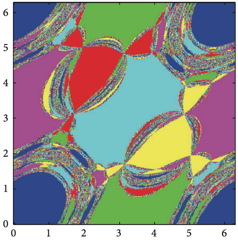

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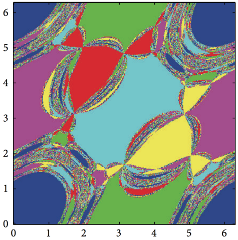

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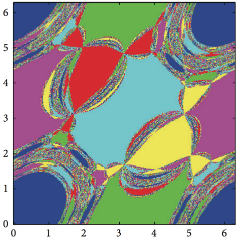


[Deng et al.]

A Closer Look at the Power Flow Jacobian

- High-voltage transmission lines have little resistance; and dropping the conductance terms from the PFE is a very common approximation

$$P_i = \sum_j V_i V_j B_{ij} \sin(\theta_i - \theta_j)\,, i \in \mathcal{N}_L \cup \mathcal{N}_G$$
$$Q_i = -\sum_j V_i V_j B_{ij} \cos(\theta_i - \theta_j)\,, i \in \mathcal{N}_L$$

- Jacobian matrix

$$\begin{bmatrix} \Delta P \\ \Delta Q_L \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial V_L} \\ \frac{\partial Q_L}{\partial \theta} & \frac{\partial Q_L}{\partial V_L} \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V_L \end{bmatrix}$$

Near high-voltage solution

$$|V_i| \approx 1,\ |\theta_i - \theta_j| \quad \ll 1$$

- **Decoupled** Jacobian matrix

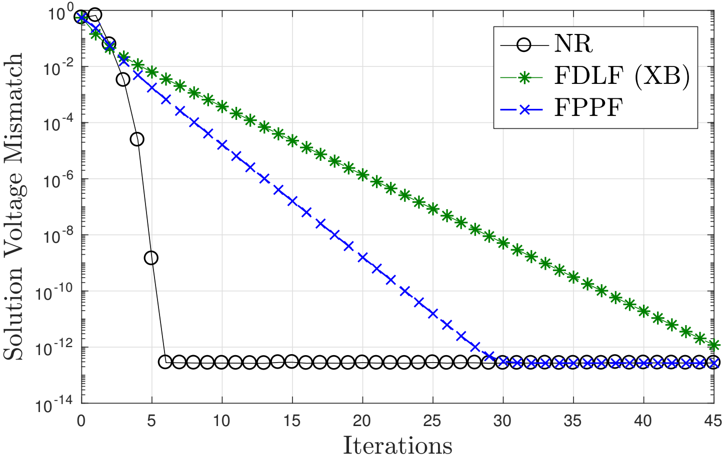
$$\begin{bmatrix} \Delta P \\ \Delta Q_L \end{bmatrix} \approx \begin{bmatrix} \frac{\partial P}{\partial \theta} & \mathbf{0} \\ \mathbf{0} & \frac{\partial Q_L}{\partial V_L} \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V_L \end{bmatrix}$$

$$\frac{\partial P_i}{\partial V_k} \propto \sin(\theta_i - \theta_k) \approx 0$$
$$\frac{\partial Q_i}{\partial \theta_k} \propto \sin(\theta_i - \theta_k) \approx 0$$

Using an approximated Jacobian in Newton = **fast decoupled load flow**

Typical Convergence of Newton and FDLF

JWSP, "A Theory of Solvability for Lossless Power Flow Equations Part I," in IEEE Trans. on Control of Network Syst., 2018.

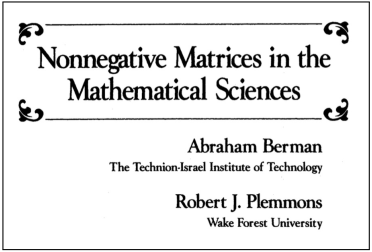


Doing all these computations efficiently is **very practically important**; lots of sparse linear algebra and matrix decompositions used in practice.

Primer: Some Matrix Theory

A matrix $A = [a_{ij}] \in \mathbb{R}^{n \times n}$ is

- a **Z-matrix** if $a_{ij} \leq 0$ for all $i \neq j$
- a **nonsingular M-matrix** if A is a Z-matrix and $A = sI - B$ where $b_{ij} \geq 0$ and $s \geq \rho(B)$
- **weakly diagonally dominant** if $|a_{ii}| \geq \sum_{j \neq i} |a_{ij}|$ for all i
- **irreducible** if the directed graph induced by A is strongly connected
- **irreducibly diagonally dominant** if it is irreducible and weakly diagonally dominant but with strict inequality for at least one $i \in \{1, \ldots, n\}$


Nonnegative Matrices in the Mathematical Sciences

Abraham Berman
The Technion-Israel Institute of Technology

Robert J. Plemmons
Wake Forest University

1. A irreducibly diagonally dominant Z-Matrix $\Longrightarrow$ A irreducible nonsingular M-matrix
2. A (irreducible) nonsingular M-matrix $\Longleftrightarrow$ A^{-1} nonnegative (strictly positive)

Primer: Some Matrix Theory

- A Z-Matrix

$$\begin{bmatrix} -5 & -3 & 0 \\ 0 & 0 & -1 \\ 0 & -2 & 2 \end{bmatrix}$$

- A **reducible** weakly diagonally dominant Z-matrix

$$\begin{bmatrix} 3 & -3 & 0 \\ 0 & 1 & -1 \\ 0 & -2 & 2 \end{bmatrix}$$

- An **irreducible** weakly diagonally dominant Z-matrix

$$\begin{bmatrix} 3 & -3 & 0 \\ 0 & 1 & -1 \\ -1 & -2 & 3 \end{bmatrix}$$

- An irreducibly diagonally dominant Z-matrix

$$\begin{bmatrix} 4 & -3 & 0 \\ 0 & 1 & -1 \\ -1 & -2 & 3 \end{bmatrix}$$

with strictly positive inverse

$$\begin{bmatrix} 1 & 9 & 3 \\ 1 & 12 & 4 \\ 1 & 11 & 4 \end{bmatrix}$$

- An irreducible nonsingular M-matrix

$$\begin{bmatrix} 4 & -3 & 0 \\ 0 & 1 & -1 \\ -1 & -2 & 2.9 \end{bmatrix}$$

which is **not** diagonally dominant

A Closer Look: The Active Power Flow Jacobian

- **Decoupled** Jacobian matrix

$$\begin{bmatrix} \Delta P \\ \Delta Q_L \end{bmatrix} \approx \begin{bmatrix} \frac{\partial P}{\partial \theta} & \mathbf{0} \\ \mathbf{0} & \frac{\partial Q_L}{\partial V_L} \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V_L \end{bmatrix}$$

$$P_i = \sum\nolimits_j V_i V_j B_{ij} \sin(\theta_i - \theta_j)\,, i \in \mathcal{N}_L \cup \mathcal{N}_G$$

$$Q_i = -\sum\nolimits_j V_i V_j B_{ij} \cos(\theta_i - \theta_j)\,, i \in \mathcal{N}_L$$

- Set $w_{ij} = V_i V_j B_{ij} \cos(\theta_i - \theta_j)$ for $i \neq j$. Note $w_{ij} = w_{ji}$. Then

$$\left(\frac{\partial P}{\partial \theta}\right)_{ij} = \begin{cases} -w_{ij} & \text{if } i \neq j \\ \sum_{j \neq i} w_{ij} & \text{if } i = j \end{cases} \quad \Longrightarrow \quad \Delta P_i = \sum_{j \neq i} w_{ij}(\Delta\theta_i - \Delta\theta_j)$$

- If $|\theta_i - \theta_j| < \frac{\pi}{2}$, then $w_{ij} \geq 0$, and strictly if (i,j) or $(j,i) \in \mathcal{E}$; $\frac{\partial P}{\partial \theta}$ is a Z-Matrix
- By network connectivity, $\frac{\partial P}{\partial \theta}$ is irreducible*, weakly diagonally dominant
- Slack bus $\Longrightarrow$ strict diagonal dominance in (at least) one row
- Under normal conditions, $\frac{\partial P}{\partial \theta}$ is a (symmetric) **irreducible non-singular M-matrix!** It is pos. def., D-stable, $\left(\frac{\partial P}{\partial \theta}\right)^{-1}$ is entry-wise positive.

Main Point: Angle controls active power (or active power controls angle!)

The "DC Power Flow"

- A crude but very useful power flow approximation.
- If $V_i \approx 1$ for all buses, and $|\theta_i - \theta_j| \ll 1$, then

$$P_i = \sum_{j=1}^{n+m} V_i V_j B_{ij} \sin(\theta_i - \theta_j) \approx \sum_{j=1}^{n+m} \underbrace{B_{ij}(\theta_i - \theta_j)}_{\text{flow from } i \text{ to } j}, \quad i \in \{1, \ldots, n+m\}$$

Laplacian Matrix:

$$L_{ij} = \begin{cases} -B_{ij} & i \neq j \\ \sum_{j \neq i} B_{ij} & i = j \end{cases}$$

DC Power Flow

$$\begin{bmatrix} P \\ P_s \end{bmatrix} = \begin{bmatrix} \mathsf{L} & L_s \\ L_s^\mathsf{T} & L_{ss} \end{bmatrix} \begin{bmatrix} \theta \\ \theta_s \end{bmatrix}$$

- Since $\theta_s \equiv 0$ by definition, $P = \mathsf{L}\theta$; a simple linear relationship
- With $p_{ij} = B_{ij}(\theta_i - \theta_j)$ the line power flows, can also be expressed as

$$P = A_r p, \qquad p = \text{diag}(B_{ij})_{(i,j)\in\mathcal{E}} A_r^\mathsf{T} \theta, \qquad \mathsf{L} = A_r \text{diag}(B_{ij})_{(i,j)\in\mathcal{E}} A_r^\mathsf{T}.$$

where A_r is the **reduced incidence matrix** of the graph.

A Closer Look: The Reactive Power Flow Jacobian

The Q/V Jacobian is more subtle to understand.

- **Decoupled** Jacobian matrix

$$\begin{bmatrix} \Delta P \\ \Delta Q_L \end{bmatrix} \approx \begin{bmatrix} \frac{\partial P}{\partial \theta} & \mathbf{0} \\ \mathbf{0} & \frac{\partial Q_L}{\partial V_L} \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V_L \end{bmatrix}$$

$$P_i = \sum_j V_i V_j B_{ij} \sin(\theta_i - \theta_j)\,, i \in \mathcal{N}_L \cup \mathcal{N}_G$$

$$Q_i = -V_i \sum_j V_j B_{ij} \cos(\theta_i - \theta_j)\,, i \in \mathcal{N}_L$$

- Let $\tilde{B}_{ij} = B_{ij} cos(\theta_i - \theta_j)$ for $i \in \mathcal{N}_L$ and $j \in \mathcal{N}$. Then

$$\left(\frac{\partial Q_L}{\partial V_L}\right)_{ij} = \begin{cases} -V_i \tilde{B}_{ij} & \text{if } i \in \mathcal{N}_L,\ j \in \mathcal{N} \setminus \{i\} \\ -V_i \tilde{B}_{ii} - \sum_{j=1}^{n+m} \tilde{B}_{ij} V_j & \text{if } i \in \mathcal{N}_L,\ i = j \end{cases}$$

- Under normal conditions[3] the elements of B satisfy
  - (i) $B_{ij} \geq 0$ for $i \neq j$, with $B_{ij} > 0$ if (i, j) or $(j, i) \in \mathcal{E}$;
  - (ii) $B_{ii} = -\sum_{j \neq i} B_{ij} + B_{\mathrm{s},i} < 0$ with $B_{\mathrm{s},i} \geq 0$ if shunt capacitance.

[3] No significant series capacitance.

A Closer Look: The Reactive Power Flow Jacobian

- Under normal operating conditions $V_i \approx 1$ and $|\theta_i - \theta_j| \approx 0$, thus

$$\left(\frac{\partial Q_L}{\partial V_L}\right)_{ij} \approx \begin{cases} -B_{ij} & \text{if } i \in \mathcal{N}_L,\ j \in \mathcal{N} \setminus \{i\} \\ \sum_{j \neq i} B_{ij} - 2B_{\mathrm{s},i} & \text{if } i \in \mathcal{N}_L,\ i = j \end{cases}$$

For simplicity only: assume that $\mathcal{N}_L$ induces a connected subgraph.

Inductive shunts $B_{\mathrm{s},i} \leq 0$

- $\frac{\partial Q_L}{\partial V_L}$ is symmetric, irreducible*
- Z-mat, weakly diag. dominant
- strict d.d. in at least 1 row

$\frac{\partial Q_L}{\partial V_L}$ is an M-matrix!

$$\frac{\partial V_i}{\partial Q_i} \geq \frac{\partial V_i}{\partial Q_j} > 0$$

Capacitive shunts $B_{\mathrm{s},i} \geq 0$

- $\frac{\partial Q_L}{\partial V_L}$ is symmetric, irreducible*
- Z-mat, **not** weakly diag. dominant!
- But **might** still be an M-matrix!

If $\frac{\partial Q_L}{\partial V_L}$ is an M-matrix

$$\frac{\partial V_i}{\partial Q_i} \not\geq \frac{\partial V_i}{\partial Q_j} > 0$$

Main Point: Voltage controls reactive power (or reactive power controls voltage!)

Other Power Flow-Related Sensitivities

JWSP and F. Bullo, "Distributed Monitoring of Voltage Collapse Sensitivity Indices," in IEEE Trans. on Smart Grid, 2016.

- Can also look at other sensitivities coming from the full Jacobian matrix

$$\begin{pmatrix} \Delta P \\ \Delta Q_L \\ \Delta Q_G \end{pmatrix} = \left(\begin{array}{c|c|c} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial V_L} & \frac{\partial P}{\partial V_G} \\ \hline \frac{\partial Q_L}{\partial \theta} & \frac{\partial Q_L}{\partial V_L} & \frac{\partial Q_L}{\partial V_G} \\ \hline \frac{\partial Q_G}{\partial \theta} & \frac{\partial Q_G}{\partial V_L} & \frac{\partial Q_G}{\partial V_G} \end{array}\right) \begin{pmatrix} \Delta \theta \\ \Delta V_L \\ \Delta V_G \end{pmatrix} .$$

- **For example**: If I as the grid operator adjust the generator voltages, what will the effect be on voltages at load buses? Just set $\Delta P = \Delta Q_L = 0$, and eliminate to obtain

$$\Delta V_L = \underbrace{\left[\frac{\partial Q_L}{\partial V_L} - \frac{\partial Q_L}{\partial \theta}\left(\frac{\partial P}{\partial \theta}\right)^{-1}\frac{\partial P}{\partial V_L}\right]^{-1}\left[\frac{\partial Q_L}{\partial V_G} - \frac{\partial Q_L}{\partial \theta}\left(\frac{\partial P}{\partial \theta}\right)^{-1}\frac{\partial P}{\partial V_G}\right]}_{\triangleq \frac{\partial V_L}{\partial V_G}} \Delta V_G$$

- Intuitively, raising all generator voltages 1% should raise all load voltages close to 1% as well. So we expect $\left(\frac{\partial V_L}{\partial V_G}\right)_{ij} \geq 0$ and $\sum_j \left(\frac{\partial V_L}{\partial V_G}\right)_{ij} \approx 1$ for all i.

This is the basis for various classical power system monitoring indices.

Literature on Power Flow Solvability

JWSP, "A Theory of Solvability for Lossless Power Flow Equations Part II," in IEEE Trans. on Control of Network Syst., 2018.

Given data: network topology, impedances, generation & loads

Q: $\exists$ "stable high-voltage" solution? unique? properties?

Many approaches over **45+ years** of literature:

- [Weedy '67]: Jacobian singularity
- [Korsak '72]: Multiple "stable" solutions
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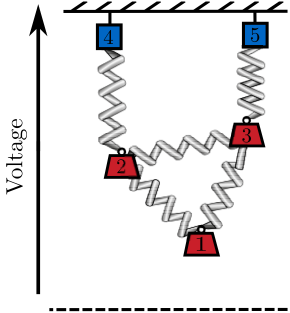
Main insight: stiffness vs. loading

1. Stiff network + light loading ⇒ feasible
2. Weak network + heavy loading ⇒ infeasible

This intuition can be built upon into a partial theory of solvability for **lossless** systems. We will just look at a simple example.



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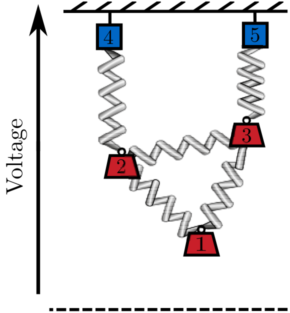
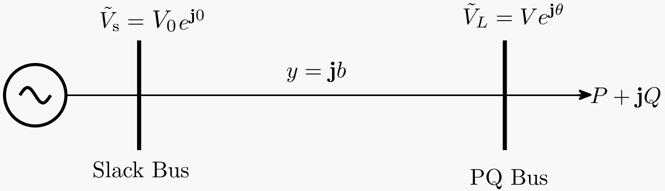


The Two-Bus Power Flow Problem

Simplest model of a **perfect generator** feeding a **voltage-independent load** through a **lossless** transmission line.



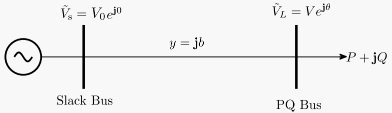
Active Power at PQ Bus: $-P = V V_0 b \sin(\theta - 0)$

Reactive Power at PQ Bus: $-Q = bV^2 - bV V_0 \cos(\theta - 0)$

Even the simplest case is a nasty trionometric/quadratic nonlinear equation! Remarkably, it is analytically solvable.

The Two-Bus Power Flow Problem

$$-P = VV_0 b \sin(\theta)$$
$$-Q = bV^2 - bVV_0 \cos(\theta)$$



1. **Change Variables**

$$v := \frac{V}{V_0} \qquad \Gamma := \frac{P}{bV_0^2} \qquad \Delta := \frac{Q}{\frac{1}{4}bV_0^2}$$

2. **Square equations, add, and solve quadratic in v^2**

$$v_\pm = \sqrt{\frac{1}{2}\left(1 - \frac{\Delta}{2} \pm \sqrt{1 - (4\Gamma^2 + \Delta)}\right)}$$

3. **Nec. & Suff. Condition**

$$4\Gamma^2 + \Delta < 1$$

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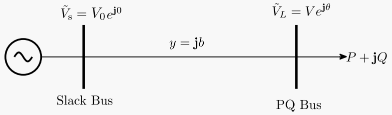

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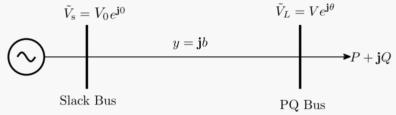

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$$4\Gamma^2 + \Delta < 1$$

1. **High-voltage** solution
   $v_+ \in [\frac{1}{2}, 1)$
2. **Low-voltage** solution
   $v_- \in [0, \frac{1}{\sqrt{2}})$

Angle: $\sin(\eta_\mp) = \Gamma/v_\pm$

1. **Small-angle** solution
   $-\theta_- \in [0, \pi/4)$
2. **Large-angle** solution
   $-\theta_+ \in [0, \pi/2)$

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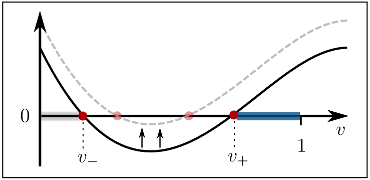

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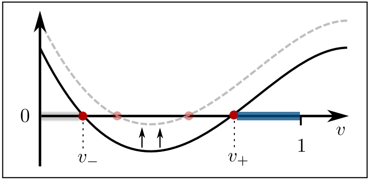

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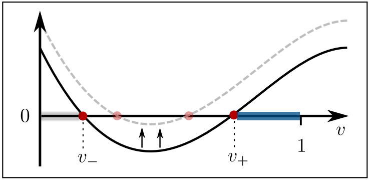

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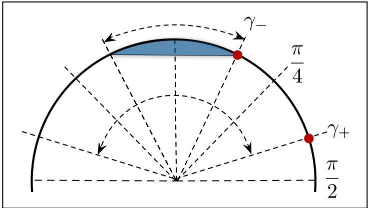


Summary and Open Questions

Summary:

- ACPF (roughly) determines the grid operating point
- Solved numerically using Newton's method
- "Usually" one unique high-voltage small-angle solution
- Jacobian matrix provides insights into static grid behaviour

Open Problems:

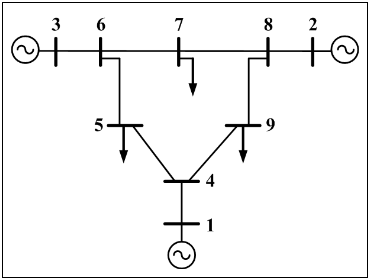
- Incomplete theory of ACPF solution space
- Incomplete matrix theory of ACPF Jacobian
- Implications of theory for behaviour of numerical methods
- Lack of *provably* robust high-performance numerical algorithms

Exercise: Solve ACPF in MATPOWER

1. Download MATPOWER v7.1 `https://matpower.org/download/`
2. Run `install_matpower.m`; choose option 2

```
define_constants;              %useful acronyms
mpc = loadcase('case9'); %load the 9-bus test case
runpf(mpc)                     %run power flow and print summary
results = runpf(mpc);      %run power flow and store results
plot(results.bus(:,BUS_I),results.bus(:,VM));
```

Exercise: Modify the case9 file to answer the following security analysis question: if the line $(4,9)$ is tripped, will V_9 remain above 0.95 p.u.?



From Power Flow to Dispatch and OPF

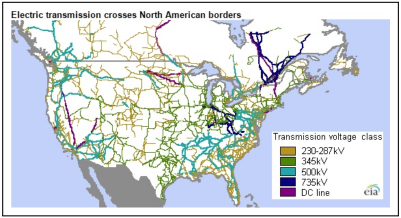
- In the ACPF problem, generator powers and voltages are **givens**
- In reality, given the installed generation, operators must decide
  (i) which generators will be used (**unit commitment**)
  (ii) the power and voltage set-points for those generators (**dispatch**)
- These problems could be considered for a single block of time, or could be multi-period with inter-period constraints taken into account over a rolling horizon
- For example: unit commitment is solved roughly 24 hours in advance, while dispatch is recomputed every 5 to 15 minutes.
- Once set-points are computed, they are sent as feedforward commands to the generation units, and local controllers are responsible for ensuring tracking

We won't focus heavily here on optimization, but getting a sense for dispatch is important, so we will consider some of the simplest instances. We consider the centralized dispatch case; market mechanisms discussed by other speakers.

Classical Economic Dispatch

Figure: U.S. Energy Information Administration, based on Energy Velocity.

Goal: Find the cheapest selection of generation such that power flows through the network to satisfy the load.



- There are some potential challenges in achieving this:
  (i) The network is described by AC power flow, which is hard.
  (ii) We may encounter various operational limits (voltage, current, power, . . .)
  (iii) We don't actually know the load
- The simplest possible way to proceed is to
  (i) Ignore the network; assume everything is **lumped** (the "copper plate" grid)
  (ii) Ignore the limits
  (iii) Use our best guess of what the load will be

Classical Economic Dispatch

- Assign to each generator $i \in \mathcal{N}_G$ a cost $C_i : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$; for simplicity here, assume this is convex and twice continuously differentiable

- The classical E.D. problem, ignoring limits, is

$$\underset{\{P_i^{\rm set}\}}{\text{minimize}} \sum_{i \in \mathcal{N}_G} C_i(P_i^{\rm set}) \quad \text{subject to} \quad \sum_{i \in \mathcal{N}_G} P_i^{\rm set} = P_{\rm load}$$

- Analysis is via Lagrange duality. Introduce the **Lagrangian** function

$$\mathcal{L}(P^{\rm set}, \lambda) = \sum\nolimits_{i \in \mathcal{N}_G} C_i(P_i^{\rm set}) - \lambda \left(\sum\nolimits_{i \in \mathcal{N}_G} P_i^{\rm set} - P_{\rm load} \right), \quad \lambda \in \mathbb{R},$$

- Optimal points are characterized by the KKT conditions

$$0 = \sum\nolimits_{i \in \mathcal{N}_G} P_i^{\rm set} - P_{\rm load}, \qquad \frac{{\rm d}C_i}{{\rm d}P_i^{\rm set}}(P_i^{\rm set}) = \lambda$$

- The second condition says that for optimality, the **individual marginal cost** $\frac{{\rm d}C_i}{{\rm d}P_i^{\rm set}}$ should be **equal** for all generators!

Classical Economic Dispatch

- Solving, we find that

$$\underbrace{P_{\text{load}} = \sum_{i \in \mathcal{N}_G} \left(\frac{\mathrm{d}C_i}{\mathrm{d}P_i^{\text{set}}} \right)^{-1} (\lambda),}_{\text{demand-supply matching}} \qquad \underbrace{P_i^{\text{set}} = \left(\frac{\mathrm{d}C_i}{\mathrm{d}P_i^{\text{set}}} \right)^{-1} (\lambda)}_{\text{price determines dispatch}}$$

- λ can also be interpreted as a **system-wide** marginal cost

- If $C(P^{\text{set}}) = \sum_{i \in \mathcal{N}_G} C_i(P_i^{\text{set}})$, then

$$\frac{\mathrm{d}C(P_{\text{set}})}{\mathrm{d}P_{\text{load}}} = \sum_{i \in \mathcal{N}_G} \frac{\mathrm{d}C_i}{\mathrm{d}P_i^{\text{set}}} \frac{\mathrm{d}P_i^{\text{set}}}{\mathrm{d}P_{\text{load}}} = \sum_{i \in \mathcal{N}_G} \lambda \frac{\mathrm{d}P_i^{\text{set}}}{\mathrm{d}P_{\text{load}}} = \lambda.$$

- **Extensions:** generator limits, power losses, less restrictive cost functions, etc.

Optimal Power Flow

- Optimal Power Flow (OPF) extends E.D. by including the network, e.g.,

$$
\begin{aligned}
\underset{P_i, V_i}{\text{minimize}} \quad & \sum\nolimits_{i \in \mathcal{N}_G} C_i(P_i) + \tilde{C}_i(Q_i) \\
\text{subject to} \quad & P_i = \sum\nolimits_j V_i V_j B_{ij} \sin(\theta_i - \theta_j) && i \in \mathcal{N}_L \cup \mathcal{N}_G \,, \\
& Q_i = -\sum\nolimits_j V_i V_j B_{ij} \cos(\theta_i - \theta_j) && i \in \mathcal{N}_L \cup \mathcal{N}_G \,, \\
& V_i^{\min} \leq V_i \leq V_i^{\max} && i \in \mathcal{N}_L \cup \mathcal{N}_G \,, \\
& S_i^{\min} \leq |P_i + \mathbf{j} Q_i| \leq S_i^{\max} && i \in \mathcal{N}_G \,, \\
& s_{ij}^{\min} \leq |p_{i \to j} + \mathbf{j} q_{i \to j}| \leq s_{ij}^{\max} && (i,j) \in \mathcal{E} \,,
\end{aligned}
$$

- An inherently **non-convex** optimization problem
- **Extensions:** lossess, binary decisions, multi-period, ...
- **Convex relaxations:** DC OPF, SOCP, SDP/moment hierarchy, ...
- **Uncertainty management:** robust versions, stochastic versions, ...

DC Optimal Power Flow

- In practice, most (but not all) transmission grid operators clear their markets using an approximate OPF model based on the DC Power Flow

$$
\begin{aligned}
\underset{\{P_i\}_{i\in\mathcal{N}_G}}{\text{minimize}} \quad & \sum\nolimits_{i\in\mathcal{N}_G} C_i(P_i) \\
\text{subject to} \quad & P_i - P_i^{\text{load}} = \sum\nolimits_j B_{ij}(\theta_i - \theta_j) \\
& P_i^{\min} \leq P_i \leq P_i^{\max} \\
& p_{ij}^{\min} \leq |B_{ij}(\theta_i - \theta_j)| \leq p_{ij}^{\max}
\end{aligned}
$$

- This is often a **linear** program; can be solved very quickly. Operators
  (i) Solve the DC OPF to obtain generation profile
  (ii) Plug generation into ACPF and solve to verify constraint satisfaction (possibly with outer loops to adjust generator voltage setpoints, etc.)
  (iii) Adjust DC OPF constraints and repeat as necessary

- This is does not have much theoretical sex-appeal, but it definitely "works".

Why Does Power Systems Optimization "Work"?

- ACPF, Economic Dispatch, OPF are based on many assumptions, such as
  (i) the grid operates in balanced synchronous steady-state
  (i) generators will do what you want them to
  (ii) you accurately know grid parameters, load forecasts, ...
- By itself, is a recipe for "garbage in, garbage out"!
- In reality
  (i) a variety of feedback mechanisms maintain grid stability
  (ii) local feedback controllers make generators follow commands
  (iii) system-level feedback controllers correct for OPF's mistakes

> Power system optimization is effective because it sits on top of an elaborate set of control mechanisms based on (1) traditional control engineering principles, and (2) deep insight into component and grid behaviour.

Overview of Power System Stability

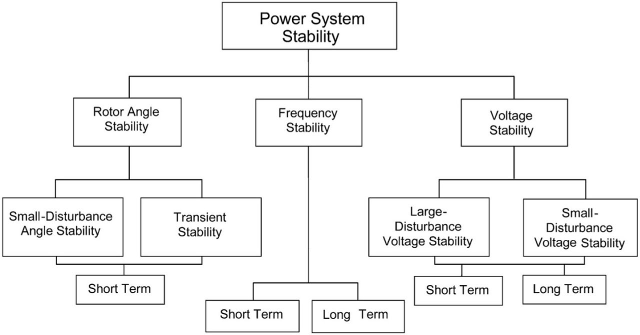
Classification of Bulk Power System Stability (2004)

IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 19, NO. 2, MAY 2004 1387

Definition and Classification of Power System Stability

IEEE/CIGRE Joint Task Force on Stability Terms and Definitions

Prabha Kundur (Canada, Convener), John Paserba (USA, Secretary), Venkat Ajjarapu (USA), Göran Andersson (Switzerland), Anjan Bose (USA) , Claudio Canizares (Canada), Nikos Hatziargyriou (Greece), David Hill (Australia), Alex Stankovic (USA), Carson Taylor (USA), Thierry Van Cutsem (Belgium), and *Vijay Vittal (USA)*

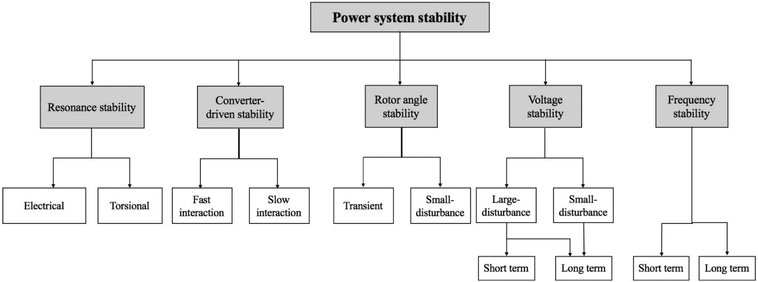


Classification of Bulk Power System Stability (2021)

IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 36, NO. 4, JULY 2021 3271

Definition and Classification of Power System Stability – Revisited & Extended

Nikos Hatziargyriou, *Fellow, IEEE*, Jovica Milanovic, *Fellow, IEEE*, Claudia Rahmann, *Senior Member, IEEE*, Venkataramana Ajjarapu, *Fellow, IEEE*, Claudio Canizares, *Fellow, IEEE*, Istvan Erlich, *Senior Member, IEEE*, David Hill, *Fellow, IEEE*, Ian Hiskens, *Fellow, IEEE*, Innocent Kamwa, *Fellow, IEEE*, Bikash Pal, *Fellow, IEEE*, Pouyan Pourbeik, *Fellow, IEEE*, Juan Sanchez-Gasca, *Fellow, IEEE*, Aleksandar Stankovic, *Fellow, IEEE*, Thierry Van Cutsem, *Fellow, IEEE*, Vijay Vittal, *Fellow, IEEE*, and Costas Vournas, *Fellow, IEEE*



Transient Stability

Constrained Swing Dynamics

$$\text{Gen}: \begin{cases} \dot{\theta}_i = \omega_i \\ M_i\dot{\omega}_i = -D_i\omega_i + P_i - \sum_j V_iV_jB_{ij}\sin(\theta_i - \theta_j) \end{cases}$$

$$\text{Load}: \begin{cases} 0 = P_i - \sum_j V_iV_jB_{ij}\sin(\theta_i - \theta_j) \\ 0 = Q_i + \sum_j V_iV_jB_{ij}\cos(\theta_i - \theta_j) \end{cases}$$

Challenge: Characterize equilibria, stability, basin of attraction

Approaches: Energy functions, nearest unstable eq. point, S.O.S., ...

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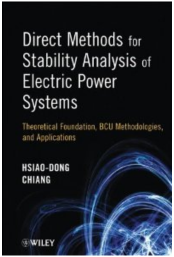

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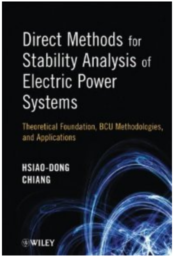


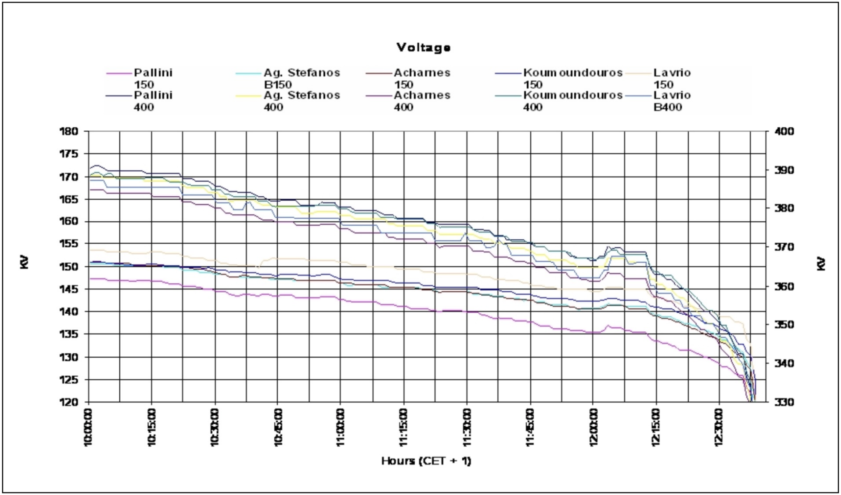
Challenge: Characterize equilibria, stability, basin of attraction

Approaches: Energy functions, nearest unstable eq. point, S.O.S., . . .

{Equilibria} = {Power Flow Solutions}

Voltage Instability

- Complex instability involving multiple components and time-scales
- 2004 blackout in Greece (Figure from Van Cutsem)



Power System Frequency Control

The Power System Control Zoo

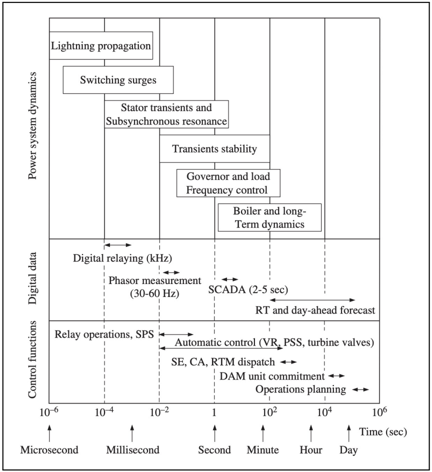
Figure: J. Chow and J.J. Sanchez-Gasca. *Power System Modeling, Computation, and Control*

Purpose of control is to main

1. power quality
2. power security
3. efficiency of operation

Types of control

- component-level loop designs
- frequency / voltage control
- wide-area damping control
- HVDC control
- economic dispatch / OPF
- energy and service markets
- unit commitment



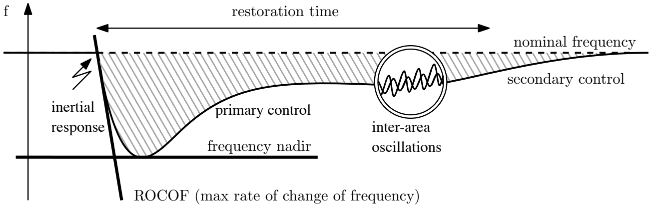
Time-scale separation the essential idea for managing complexity.

Key General Ideas in Power Systems Control

- The control architecture is **hierarchical**, meaning the control loops are nested.
- Higher-level controllers provide commands to lower-level controllers
- Lower-level control loops are faster than higher-level control loops; this allows higher-level loops to be designed based on **equilibrium models** of lower-level control loops (i.e., minor loop design, or singular perturbation theory)
- These control loops may be
  1. **Local:** local measurement and actuation, no communication
  2. **Wide-Area:** coordinated control using geographically dispersed measurements
  3. **Centralized:** communication to, and calculations performed at, one point
  4. **Distributed:** communication and computation dispersed
- Frequency control is an **ancillary service**, and is provisioned through a market.
- Voltage control has poor stand-alone economics, and is instead tacked on as requirements in generation contracts.

Frequency Control in Bulk Interconnected Power Systems

Figure: F. Dörfler



Three stages of frequency control:

1. **Inertial** response: fast response of rotating machines

   Time scale: immediate/seconds

2. **Primary** control: turbine-governor control for *stabilization*

   Time scale: seconds

3. **Automatic Generation Control (AGC)**: multi-area control which eliminates *generation-load mismatch* within each area

   Time scale: minutes

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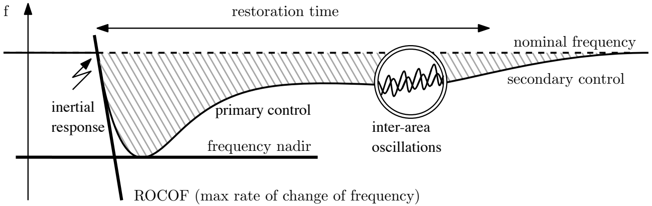

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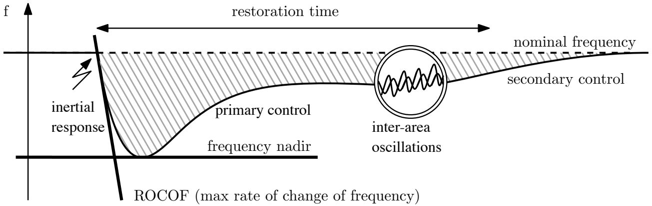

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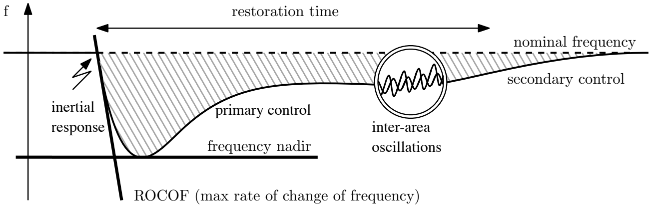

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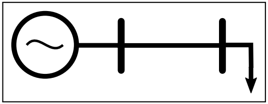
Fundamentals of Frequency Control

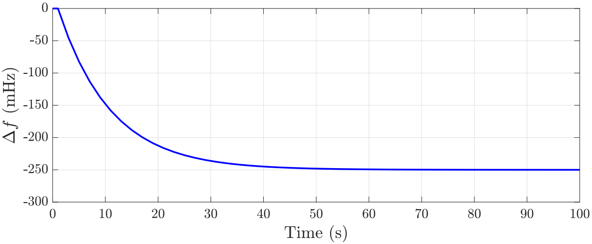
- Let's return to the linearized generator model from Prof. Schiffer's lecture

$$\frac{\mathrm{d}}{\mathrm{d}t}\Delta f = \frac{1}{2H}(\Delta P_{\mathrm{m}} - \Delta P_{\mathrm{e}})$$



- **Lossless transmission**: electrical power change ΔP_{e} must equal load change ΔP_{L}
- Model ΔP_{L} as constant + frequency-dependent, i.e., $\Delta P_{\mathrm{L}} = \Delta d + D\Delta f$
- If the mechanical power provided is constant, then $\Delta P_{\mathrm{m}} = 0$, and

$$\frac{\mathrm{d}}{\mathrm{d}t}\Delta f = -\frac{D}{2H}\Delta f - \frac{1}{2H}\Delta d \qquad \Longleftrightarrow \qquad \frac{\Delta f(s)}{\Delta d(s)} = \frac{-(1/D)}{2Hs+1}$$

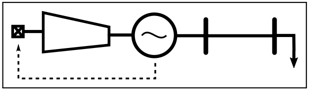


Fundamentals of Frequency Control

Figure: G. Andersson, C. A. Bel, C. Cañizares. *Frequency and Voltage Control*

- We need to control the resulting frequency deviation, so we will use feedback.
- Must increase mechanical power ΔP_{m}

$$\frac{\mathrm{d}}{\mathrm{d}t}\Delta f = \frac{1}{2H}(\Delta P_{\mathrm{m}} - \Delta P_{\mathrm{e}})$$



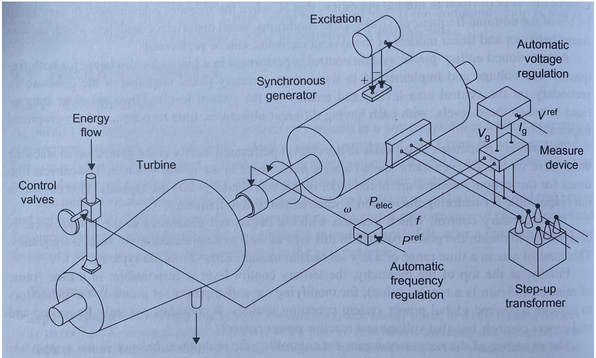


FIGURE 9.6 Generator controls.

Turbines

Figure: J. Chow and J.J. Sanchez-Gasca. *Power System Modeling, Computation, and Control*

- Traditional fossil fuel-fired or nuclear plants boil water to produce steam which drives a **steam turbine**, and this provides torque to the generator.
- Steam turbines typically have multiple stages to increase efficiency

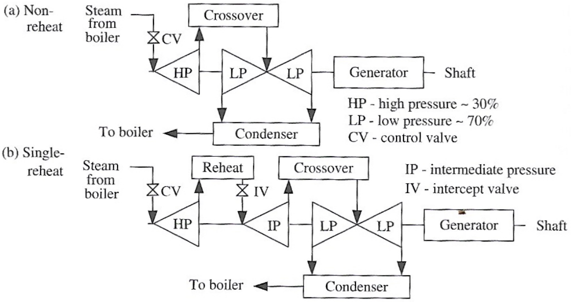


Figure 12.1 Steam turbine configurations: (a) a non-reheat unit and (b) a single-reheat unit.

- Turbines for hydro-electric facilities and gas generators have different models

Simplest Steam Turbine Model

Figure: J. Chow and J.J. Sanchez-Gasca. *Power System Modeling, Computation, and Control*

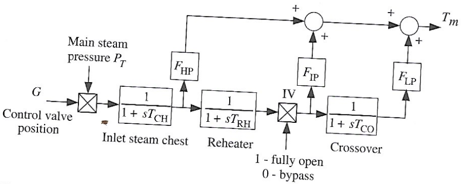


Figure 12.4 Block diagram of a reheat steam turbine.

$$T_{\mathrm{CH}} = 0.3\text{s}$$
$$T_{\mathrm{RH}} = 5.0\text{s}$$
$$T_{\mathrm{CO}} = 0.5\text{s}$$
$$F_{\mathrm{HP}} = 0.3$$
$$F_{\mathrm{IP}} = 0.3$$
$$F_{\mathrm{LP}} = 0.4$$

- An approximate model is therefore

$$\frac{\Delta P_{\mathrm{m}}(s)}{\Delta G(s)} \approx \frac{F_{\mathrm{HP}} T_{\mathrm{RH}} s + 1}{T_{\mathrm{RH}} s + 1}$$

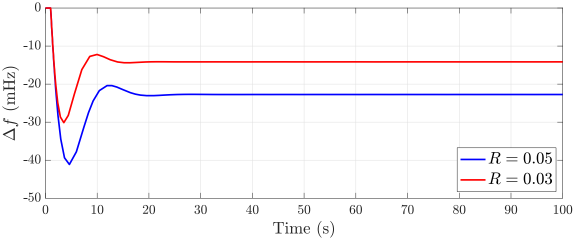
- Think of the turbine as your **actuator**, and the actuator is, well ... kinda slow.
- For typical parameters, this is a lag-type filter; the (stable) zero at $s = -\frac{1}{F_{\mathrm{HP}} T_{\mathrm{RH}}}$ can have a major impact on the dynamics.

Primary Control (Speed Governor)

- We now adjust the control valve position based on frequency deviation feedback
- The simplest **local** control loop, called **primary control**, is just proportional frequency deviation feedback

$$\Delta G = -\frac{1}{R}\Delta f, \qquad R = \text{“droop”}$$

- Typically $R = 0.05$ in p.u. Smaller R means larger feedback gain.

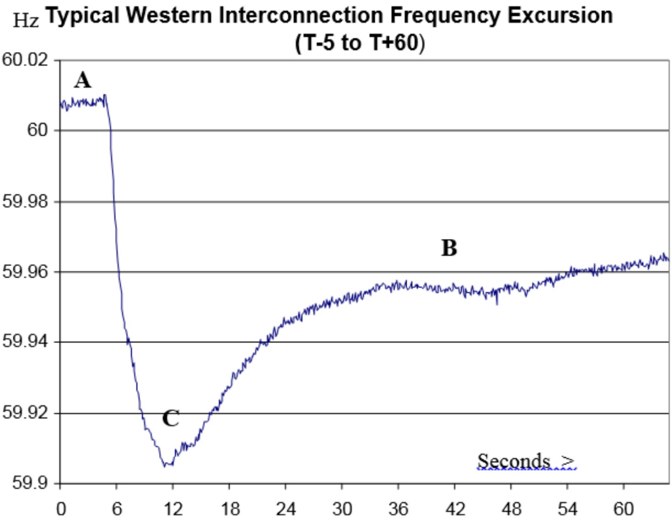


Note: ROCOF, Nadir, steady-state all are important.

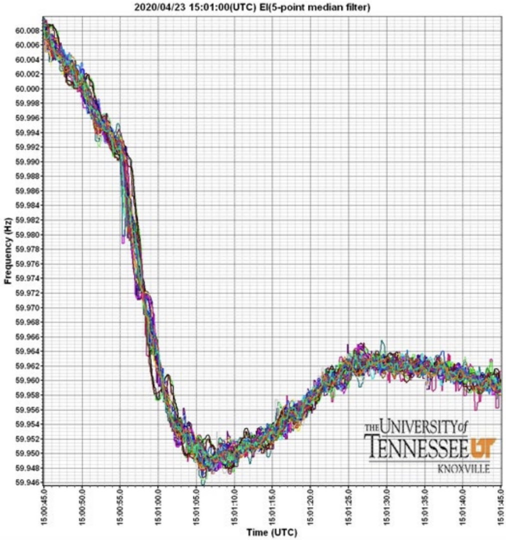
- Note the initial slope of decline is **independent** of R, depends only on H!

Frequency Excursion in WECC after Generator Trip

Figure: NERC *Balancing and Frequency Control*

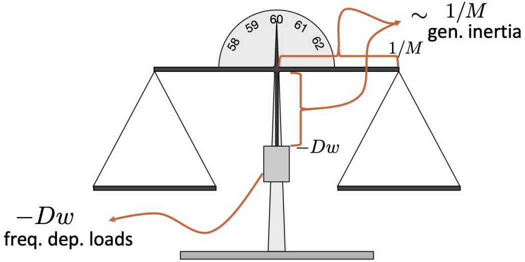


Frequency Excursion in EI after Generator Trip



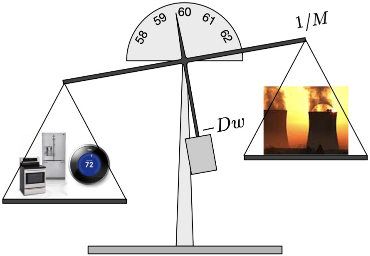
Supply-Demand Balance . . . Literally

Figure: E. Mallada



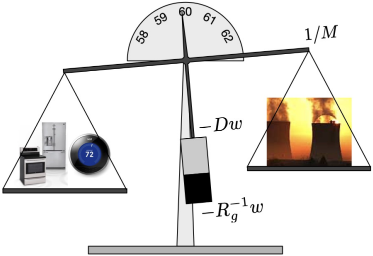
Supply-Demand Balance . . . Literally

Figure: E. Mallada



Supply-Demand Balance . . . Literally

Figure: E. Mallada



Steady-State Analysis: Frequency Deviation

- From our simple dynamic model so far, the sensitivity function is

$$\frac{\Delta f(s)}{\Delta d(s)} = S(s) = \frac{T_{\mathrm{RH}}s + 1}{(2Hs + D)(T_{\mathrm{RH}}s + 1) + \frac{1}{R}(F_{\mathrm{HP}}T_{\mathrm{RH}}s + 1)}$$

- It's second-order, so this *particular* model is closed-loop stable for all values of parameters; this is **definitely not** true in general.

- If Δd is a step load change, then the final value theorem gives that

$$\Delta f_{\mathrm{ss}} = S(0)\Delta d_{\mathrm{ss}} = \frac{1}{D + \frac{1}{R}}\Delta d_{\mathrm{ss}} = \frac{1}{\beta}\Delta d_{\mathrm{ss}}$$

- The quantity $\beta = D + \frac{1}{R}$ is known as the *frequency response characteristic* of the system, and has units of p.u. power / p.u. frequency.

- A "stiff" system has a large β, and its frequency is insensitive to load changes.

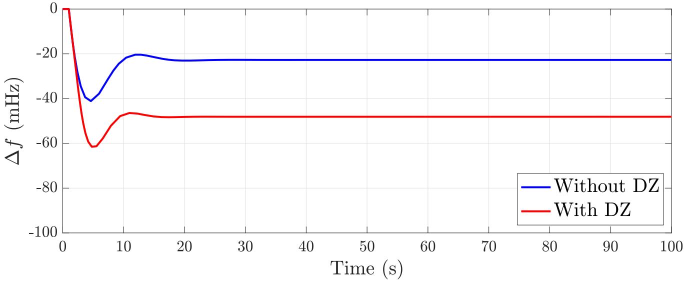
> The droop gain R primarily determines the steady-state frequency deviation after a disturbance. (Duh, it's the proportional gain).

Governor Deadband

- The purpose of primary control is to maintain the grid frequency within a (ENTSO-E/NSERC prescribed) operating band.
- However, there are constant small load variations in the system which cause the frequency to bounce around. Implementation of the feedback with deadzone

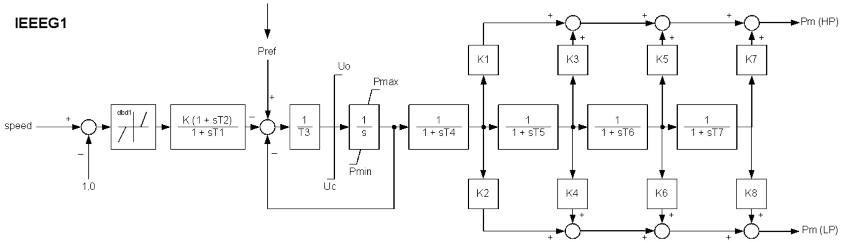
$$\Delta G = -\frac{1}{R}\text{deadzone}(\Delta f), \qquad \text{e.g., } \pm \text{ 36 mHz}$$

stops the governor from chasing small deviations



Comments on Turbine-Governor Models

- The previous steam turbine + governor model is known as TGOV1
- There are several more accurate models available, e.g., IEEEG1



- **Note:** Deadband and saturation elements within these models
- See *IEEE PES-TR1: Dynamic Models for Turbine-Governors in Power System Studies* for much, much more.

The Case of Multiple Generators: Frequency Deviation

Figure: P. Kundur *Power System Stability and Control*

- Let's now consider a system where there are multiple generators feeding a load
- Ignoring interactions temporarily, the generators will be coherent, and can be modelled as an equivalent inertia driven by the sum of all mechanical powers

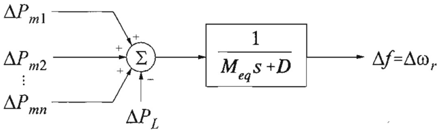


Figure 11.16 System equivalent for LFC analysis

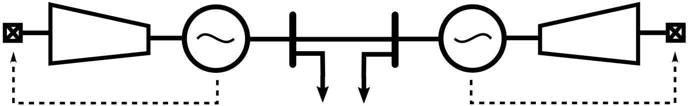
- The effect of all generators will combine, yielding a steady-state response

$$\Delta f_{\text{ss}} = -\frac{1}{\sum_{i \in \mathcal{N}_G} \frac{1}{R_i} + D} \Delta d_{\text{ss}}, \qquad R_{\text{eq}} = \frac{1}{\sum_{i \in \mathcal{N}_G} \frac{1}{R_i}}$$

More proportional feedback leads to tighter frequency control. Duh.

Basic Dynamics of Two-Generator System

- Let's now look in a bit more detail at the dynamics of two generators with governors



- The individual dynamics of each generator $i \in \{1, 2\}$ are

$$\Delta\dot{\theta}_i = f_0 \cdot \Delta f_i$$

$$2H_i \Delta\dot{f}_i = \Delta P_{\mathrm{m},i} - \Delta P_{\mathrm{e},i\to j} - (D_i \Delta f_i + \Delta d_i)$$

$$T_i \Delta\dot{P}_{\mathrm{m},i} = -\Delta P_{\mathrm{m},i} - \frac{1}{R_i}(\Delta f_i + F_i T_i \Delta\dot{f}_i)$$

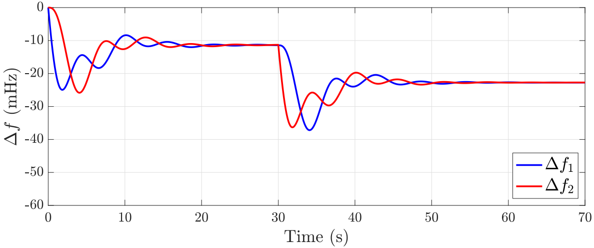
- **Lossless interconnection** with susceptance $-B$, we use the DCPF:

$$\Delta P_{\mathrm{e},1\to 2} = -\Delta P_{\mathrm{e},2\to 1} \approx B(\Delta\theta_1 - \Delta\theta_2)$$

- Let's see how this responds to disturbances

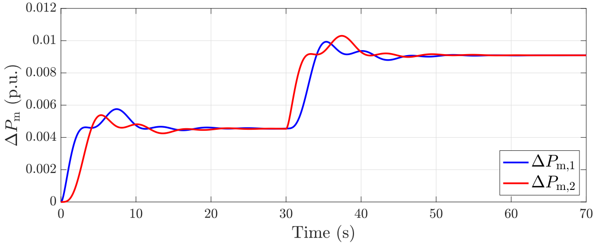
Simulation: Two Identical Generators

- $\Delta d_1 = 0.01$ p.u. at $t = 0$, then $\Delta d_2 = 0.01$ p.u. at $t = 30$s
- strong coupling $B = 0.2$ p.u.



- Frequency drops faster **near** the disturbance
- **Electromechanical oscillations** occur during the transient period; the two inertias are oscillating against one another, mediated by the electrical power transfer
- **Synchronization** of frequencies after the transient

Simulation: Two Identical Generators



- Power ramps up first **near** the disturbance
- Other generator helps out shortly thereafter
- **Why doesn't generated power match total disturbance of 0.02 p.u.?**
- **How is the power allocation between generators determined?**

This model can be used to analytically study some interesting cases, e.g., (i) strongly coupled generators, (ii) weakly coupled generators, (iii) one very small inertia, one very large inertia, ...

Steady-State Analysis: Power Sharing

- For each generator, our turbine-governor model is

$$\Delta P_{\mathrm{m},i,\mathrm{ss}}(s) = -\frac{1}{R_i}\frac{F_i T_i s + 1}{T_i s + 1}\Delta f_i$$

- After a disturbance, in **steady-state** it therefore holds that

$$\Delta P_{\mathrm{m},i,\mathrm{ss}} = -\frac{1}{R_i}\Delta f_{\mathrm{ss}} = \frac{1}{R_i}\frac{1}{\sum_{k\in\mathcal{N}_G}\frac{1}{R_k} + D}\Delta d_{\mathrm{ss}}$$

$$\approx \underbrace{\frac{R_i^{-1}}{\sum_{k\in\mathcal{N}_G} R_k^{-1}}}_{\triangleq c_i}\Delta d_{\mathrm{ss}}$$

- **Supply = Demand**: $\sum_i \Delta P_{\mathrm{m},i,\mathrm{ss}} = \sum_i c_i \Delta d_{\mathrm{ss}} = \Delta d_{\mathrm{ss}}(\sum_i c_i) = \Delta d_{\mathrm{ss}}$

- The generators **share the load proportionally with their (inverse) droop gains**

$$\frac{\Delta P_{\mathrm{m},i,\mathrm{ss}}}{\Delta P_{\mathrm{m},j,\mathrm{ss}}} = \frac{R_i^{-1}}{R_j^{-1}}$$

Multi-Machine Generalization of Previous Model

- We can generalize to an arbitrary network, and add a generator set-point change

$$\begin{aligned}
\Delta\dot{\theta}_i &= f_0 \cdot \Delta f_i \\
2H_i \Delta\dot{f}_i &= \Delta P_{\mathrm{m},i} - \sum_j \Delta P_{\mathrm{e},i\to j} - (D_i \Delta f_i + \Delta d_i) \\
T_i \Delta\dot{P}_{\mathrm{m},i} &= -\Delta P_{\mathrm{m},i} - \frac{1}{R_i}(\Delta f_i + F_i T_i \Delta\dot{f}_i) + \Delta P_i^{\mathrm{set}} \\
\Delta P_{\mathrm{e},i\to j} &= b_{ij}(\Delta\theta_i - \Delta\theta_j)
\end{aligned}$$

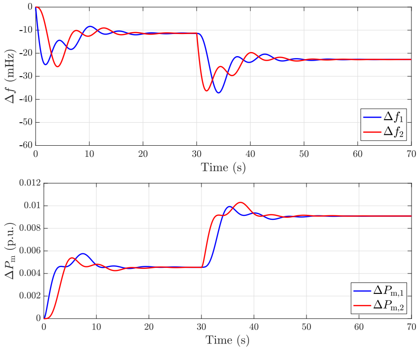
- When vectorized ($F_i = 0$ for simplicity), the model becomes

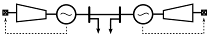
$$\begin{bmatrix} \frac{1}{f_0} I & 0 & 0 \\ 0 & 2\boldsymbol{H} & 0 \\ 0 & 0 & \boldsymbol{T} \end{bmatrix} \begin{bmatrix} \Delta\dot{\boldsymbol{\theta}} \\ \Delta\dot{\boldsymbol{f}} \\ \Delta\dot{\boldsymbol{P}}_{\mathrm{m}} \end{bmatrix} = \begin{bmatrix} 0 & I & 0 \\ -\mathsf{L} & -\boldsymbol{D} & I \\ 0 & \boldsymbol{R}^{-1} & -I \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{\theta} \\ \Delta\boldsymbol{f} \\ \Delta\boldsymbol{P}_{\mathrm{m}} \end{bmatrix} + \begin{bmatrix} 0 \\ \Delta\boldsymbol{d} \\ \Delta\boldsymbol{P}^{\mathrm{set}} \end{bmatrix}$$

Exercise: Prove the steady-state relationship

$$\Delta\boldsymbol{f}_{\mathrm{ss}} = \mathbb{1}\frac{1}{\beta}(\mathbb{1}^{\mathsf{T}}\Delta\boldsymbol{P}^{\mathrm{set}}_{\mathrm{ss}} - \mathbb{1}^{\mathsf{T}}\Delta\boldsymbol{d}_{\mathrm{ss}}) \quad \text{where} \quad \beta = D + \sum_{i\in\mathcal{N}_G} \frac{1}{R_i}.$$

Critical Examination of Primary Control Response





Primary control is **purely local proportional feedback**; measure local frequency, adjust local power production

- **Problem #1:** We have steady-state error in frequency (**Why? Do we care?**)
- **Problem #2:** The response is **global**; all generators will respond to all disturbances, according to their droop gains. **Why? Is this good or bad? Do we care?**

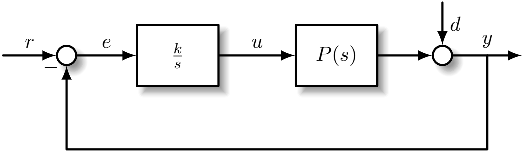
A change in load "will be taken care of, but it may be taken care of by any of the governor-regulated machines then in operation on the system. Therein lies the nub of the load control problem." – N. Cohn, *Power Flow Control – Basic Concepts for Interconnected Systems*, 1950.

Secondary Frequency Control

- If you have steady-state error in a controlled variable in response to a constant disturbance/model error, then you use **integral control** to remove the error
- At the simplest and most naive level, **secondary frequency control** just means "add an integral control loop"
- However, I claim there are a lot of questions without immediate answers!
  1. Should this be a local control? a wide-area control?
  2. If it's wide-area, should the implementation be distributed? centralized?
  3. Should the resulting response be global, local, somewhere inbetween?
  4. What model information can we rely on for tuning?
  5. How much should each generator participate in this process?
  6. Is frequency all that matters?

Our goal is now to pick this problem apart.

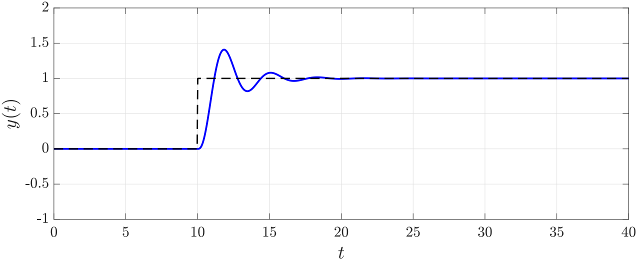
Fundamentals of Integral Control



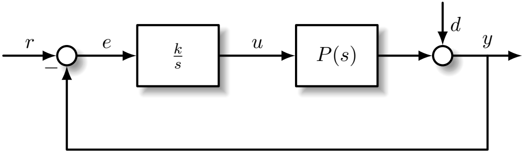
Assume P stable, $P(0) \neq 0$. What are the basic facts and tuning principles?

The Integral Control Dichotomy: Either

(a) the closed-loop is BIBO stable **and** $\lim_{t\to\infty} e(t) = 0$, or



Fundamentals of Integral Control



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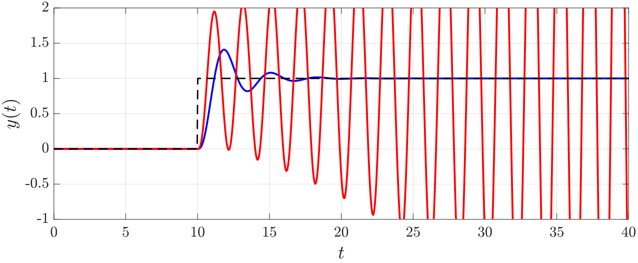
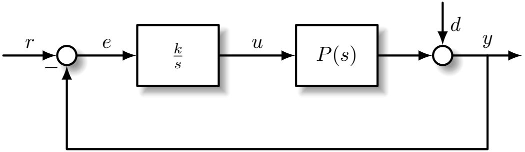
The Integral Control Dichotomy: Either

(a) the closed-loop is BIBO stable **and** $\lim_{t\to\infty} e(t) = 0$, or

(b) the closed-loop is unstable.



Fundamentals of Integral Control



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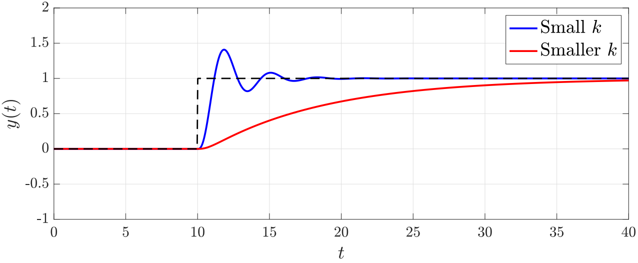
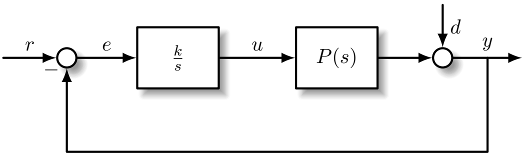


Integral Control and Model Uncertainty



Assume P stable, $P(0) \neq 0$. What are the basic facts and tuning principles?

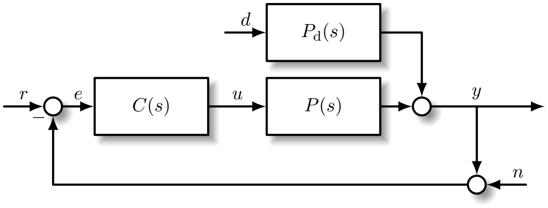
- Integral control forces e to zero; there is no other option (**robust**)
- Integral control tends to **destabilize** stable plants
  (i) 90 degrees of phase lag
  (ii) infinite gain at low frequencies
- Except for special circumstances (e.g., passive systems), there is a maximum gain $k^\star(P)$ above which the loop is unstable
- **Problem:** $k^\star(P)$ depends strongly on ... well ... P. If you don't know P very well, you need to be **conservative** and use **low integral gains**

An Aside on Power System Model Uncertainty

But certainly operators know their own grid models . . . *right*?

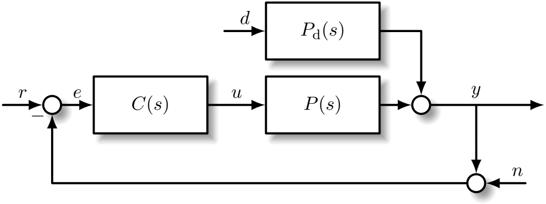
- Well, they certainly *build* high-fidelity dynamic models, but mostly for running security studies. These generally are not used for frequency control design.
- Challenges in actually **maintaining accurate** dynamic models for control are
  (i) Time-variation from unit commitment, dispatch, seasonality, . . .
  (ii) Proliferation of black-box IBRs
  (iii) Lack of governor response and turbine-governor data
  (iv) Generally poor dynamic load models
- Two observations (draw your own conclusions):
  (i) Balancing areas under NERC simply estimate β as 1% of peak load . . .
  (ii) secondary control time constants on the order of 60s-100s (**low gain**)
- From a model-based design standpoint, this is all disappointing. From a data-driven design standpoint, there are huge opportunities for improvement

Low-Gain Integral Control of LTI Systems

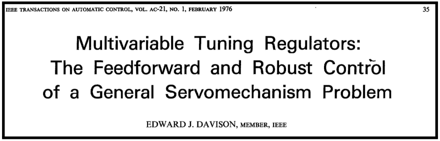


- P, P_d = LTI **exp. stable**
- d, r = **constant**

Low-Gain Integral Control of LTI Systems



- $P, P_{\mathrm{d}} =$ LTI **exp. stable**
- $d, r =$ **constant**



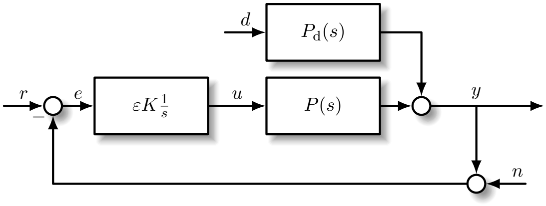
IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. AC-21, NO. 1, FEBRUARY 1976 35

Multivariable Tuning Regulators: The Feedforward and Robust Control of a General Servomechanism Problem

EDWARD J. DAVISON, MEMBER, IEEE

$$\dot{\eta} = e$$
$$u = -\varepsilon K \eta$$
$$K = P(0)^{\dagger}$$

Low-Gain Integral Control of LTI Systems



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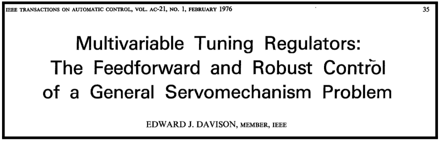
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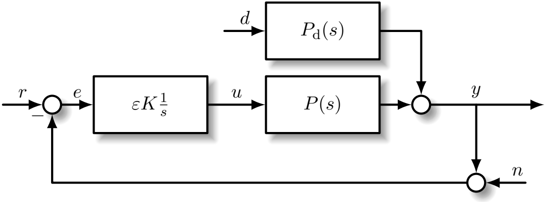

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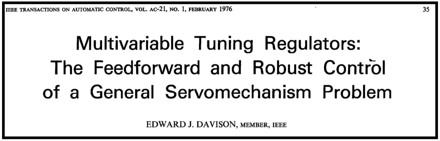
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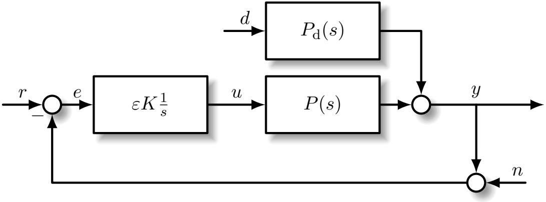
$-P(0)K$ Hurwitz $\iff$ $\exists \varepsilon^\star > 0$ s.t. $\forall \varepsilon \in (0, \varepsilon^\star)$ C.L.S. exp. stable & $e(t) \to 0$

Low-Gain Integral Control of LTI Systems



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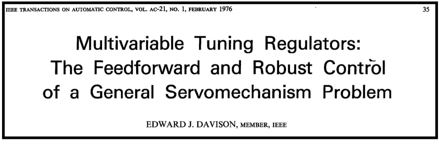
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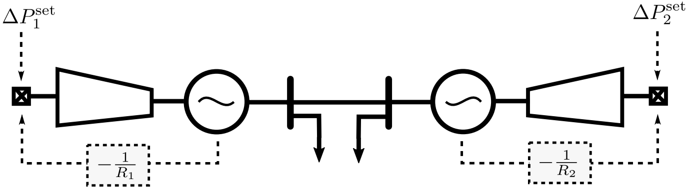
Only required model information is the DC gain!

Key Theoretical Insights into Secondary Frequency Control

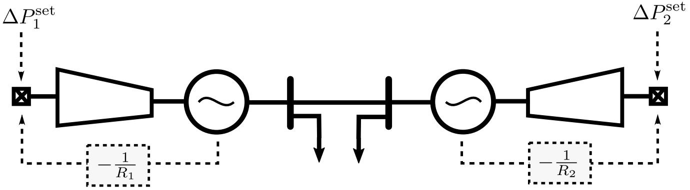


$$\Delta \boldsymbol{f}_{\rm ss} = \frac{1}{\beta}\mathbb{1}\mathbb{1}^{\mathsf{T}}\Delta \boldsymbol{P}_{\rm ss}^{\rm set} - \frac{1}{\beta}\mathbb{1}\mathbb{1}^{\mathsf{T}}\Delta \boldsymbol{d}_{\rm ss} \quad \text{so} \quad P(0) = \frac{1}{\beta}\mathbb{1}\mathbb{1}^{\mathsf{T}}$$

- $\Delta \boldsymbol{f}_{\rm ss} \in \mathrm{Im}(\mathbb{1}_n) \implies$ if you reach steady-state, you synchronize.
- $\mathbb{1}^{\mathsf{T}}\Delta \boldsymbol{P}_{\rm ss}^{\rm set} = \sum_i \Delta P_{i,\rm ss}^{\rm set} \implies$ only *sum* of power set-points impacts steady-state
- $\mathrm{rank}(P(0)) = 1 \implies$ there **does not exist** K such that $-P(0)K$ is Hurwitz.

The last point says you are **not allowed** to individually integrate different frequency measurements and feed them back, it's always unstable.

Centralized Secondary Control in Isolated Systems



- We only get to use one integrator, so let's integrate $\Delta f_{\text{avg}} = \frac{1}{2}(\Delta f_1 + \Delta f_2)$
- **Centralized control:** Send $\Delta f_1, \Delta f_2$ to a central controller, average and integrate

$$\tau \dot{\eta} = -\beta \Delta f_{\text{avg}}, \qquad \begin{aligned} \Delta P_1^{\text{set}} &= \alpha_1 \eta \\ \Delta P_2^{\text{set}} &= \alpha_2 \eta \end{aligned}$$

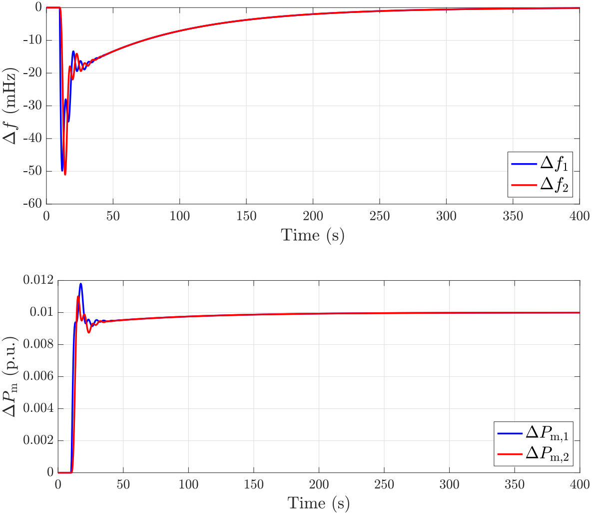
- $\alpha_1, \alpha_2 \geq 0$ are called *participation factors*, $\alpha_1 + \alpha_2 = 1$
- **Note:** By Davison's theory . . .

$$-P(0)K = -\tfrac{1}{2}\begin{bmatrix}1 & 1\end{bmatrix} \cdot \tfrac{1}{\beta}\begin{bmatrix}1 & 1 \\ 1 & 1\end{bmatrix} \cdot \begin{bmatrix}\alpha_1 \\ \alpha_2\end{bmatrix} = -\tfrac{1}{\beta} < 0 \quad \text{(Hurwitz!)}$$

so loop is stable for large τ

Centralized Secondary Control in Isolated Systems

- $\Delta d_1 = 0.02$ disturbance, $\alpha_1 = 0.5$, $\alpha_2 = 0.5$, $\tau = 80$s



Participation Factors from Economic Dispatch

- **Recall:** generation set-points are scheduled via economic dispatch

$$\underset{\{P_i^{\text{set}}\}}{\text{minimize}} \sum_{i \in \mathcal{N}_G} C_i(P_i^{\text{set}}) \quad \text{subject to} \quad \sum_{i \in \mathcal{N}_G} P_i^{\text{set}} = P_{\text{load}}$$

with stationarity conditions

$$\underbrace{P_{\text{load}} = \sum_{i \in \mathcal{N}_G} \left(\frac{\mathrm{d}C_i}{\mathrm{d}P_i^{\text{set}}} \right)^{-1} (\lambda),}_{\text{demand-supply matching}} \qquad \underbrace{P_i^{\text{set}} = \left(\frac{\mathrm{d}C_i}{\mathrm{d}P_i^{\text{set}}} \right)^{-1} (\lambda)}_{\text{price determines dispatch}}$$

- If P_{load} changes a bit, then

$$\Delta P_{\text{load}} \approx \sum_{i \in \mathcal{N}_G} \frac{1}{C_i''} \Delta\lambda, \qquad \Delta P_i^{\text{set}} \approx \frac{1}{C_i''} \Delta\lambda$$

and so

$$\Delta P_i^{\text{set}} \approx \underbrace{\frac{\frac{1}{C_i''}}{\sum_{k \in \mathcal{N}_G} \frac{1}{C_k''}}}_{\triangleq \alpha_i} \Delta P_{\text{load}}$$

Distributed Secondary Control in Isolated Systems

J. W. Simpson-Porco, "On Stability of Distributed-Averaging Proportional-Integral Frequency Control" in IEEE L-CSS, 2021.

- You can also implement single-area secondary control in a **distributed** fashion, where each generator has a controller, and the controllers communicate in a peer-to-peer manner

- Key idea is to combine integral action and **distributed averaging**

$$\tau\dot{\eta}_1 = -\beta\Delta f_1 - k(\eta_1 - \eta_2), \qquad \Delta P_1^{\mathrm{set}} = \alpha_1\eta_1$$
$$\tau\dot{\eta}_2 = -\beta\Delta f_2 - k(\eta_2 - \eta_1), \qquad \Delta P_1^{\mathrm{set}} = \alpha_2\eta_2$$

- **Doesn't this violate the "only one integrator" rule**. Nope! Let

$$\eta = \frac{\eta_1 + \eta_2}{2}, \qquad \delta = \frac{\eta_1 - \eta_2}{2}$$

  leading to

$$\tau\dot{\eta} = -\beta\Delta f_{\mathrm{avg}}, \qquad \tau\dot{\delta} = -k\delta - \frac{\beta}{2}(\Delta f_1 - \Delta f_2)$$

  so we are actually only integrating Δf_{avg}!

- **Is this stable?** We can't directly apply Davison's result, because it's not pure integral control anymore.

Distributed Secondary Control in Isolated Systems

J. W. Simpson-Porco, "On Stability of Distributed-Averaging Proportional-Integral Frequency Control," in IEEE L-CSS, 2021.

- Let's again use **time-scale separation**. If τ is very large, the controller is very slow, so the grid+primary control will settle into a quasi steady-state:

$$\begin{aligned}\Delta f_{\text{avg}} = \Delta f_{\text{ss}} &= \frac{1}{\beta}(\Delta P_1^{\text{set}} + \Delta P_2^{\text{set}}) - \frac{1}{\beta}(\Delta d_1 - \Delta d_2) \\ &= \frac{1}{\beta}(\alpha_1\eta_1 + \alpha_2\eta_2) - \frac{1}{\beta}(\Delta d_1 - \Delta d_2) \\ &= \frac{1}{\beta}(\eta + (\alpha_1 - \alpha_2)\delta) - \frac{1}{\beta}(\Delta d_1 + \Delta d_2)\end{aligned}$$

- Substituting, we have the **slow time-scale dynamics**

$$\tau\dot{\eta} = -\eta - (\alpha_1 - \alpha_2)\delta - (\Delta d_1 + \Delta d_2), \qquad \tau\dot{\delta} = -k\delta$$

which is a cascade of two linear systems, and thus stable.

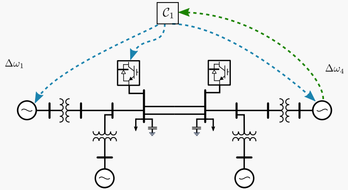
We again conclude that if the integral time constant τ is sufficiently large, the distributed controller will robustly regulate frequency as desired.

Centralized vs. Distributed Secondary Control

Can be extended to nonlinear grid models and can include power set-point limits

1. Centralized control

$$\tau \dot{\eta} = -\beta \Delta f_{\mathrm{avg}}$$
$$\Delta P_i^{\mathrm{set}} = \alpha_i \eta$$



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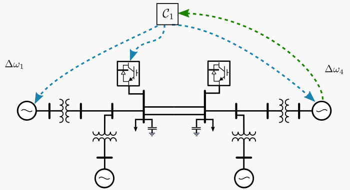
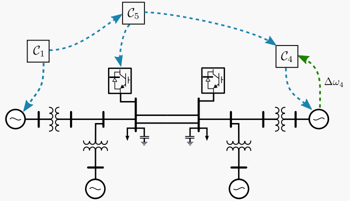


2. Distributed control

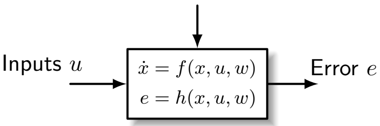
$$\tau \dot{\eta}_i = -\beta \Delta f_i - \sum_j a_{ij}(\eta_i - \eta_j)$$
$$\Delta P_i^{\text{set}} = \alpha_i \eta_i$$

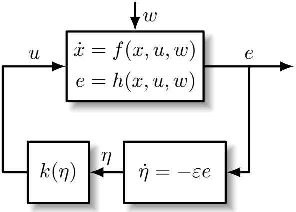


Integral Control of Nonlinear Systems

JWSP, "Analysis and Synthesis of Low-Gain Integral Controllers for Nonlinear Systems," in IEEE TAC, 2021.

Yes, the previous ideas extend to nonlinear systems.





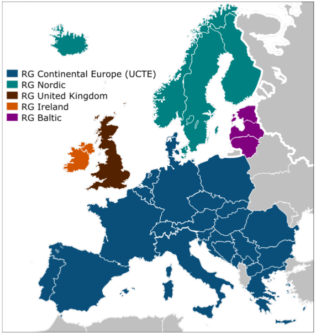
- Plant is "exponentially stable"
- Input-to-output equilibrium mapping $\bar{e} = \pi(\bar{u}, \bar{w})$ generalizes DC gain $P(0)$
- Small ε induces time-scale separation
- Closed-loop stability ensured under *monotonicity* or *contraction* condition on composed mapping

$$\pi(k(\eta), w)$$

From Prof. Schiffer's Presentation . . .

Interconnected power systems - Motivation

- Interconnection of power systems has advantages in reliability and economy
  - Power support in emergencies
  - Cross-border power transfers and trading
  - → Fundamental prerequisite for international electricity market
- Two power systems can be coupled
  - Synchronously = AC connection (e.g., continental Europe)
  - Asynchronously = DC connection (e.g., UK)



Synchronous grids in Europe, ©Kimdime

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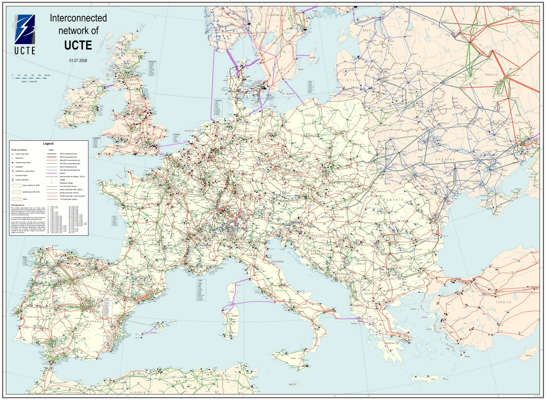
From Prof. Schiffer's Presentation ...

Interconnected power systems - ENTSO-E



Source: ENTSO-E

- European Network of Transmission System Operators for Electricity (ENTSO-E)
- 41 transmission system operators
- 34 countries, 450 mio. people
- 1,000 GW generation capacity

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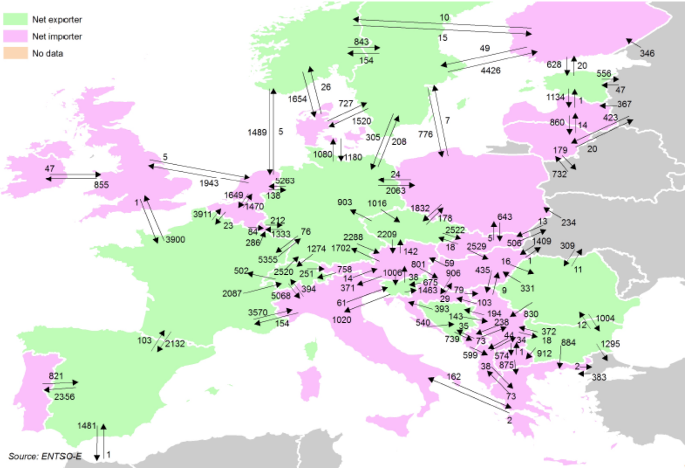
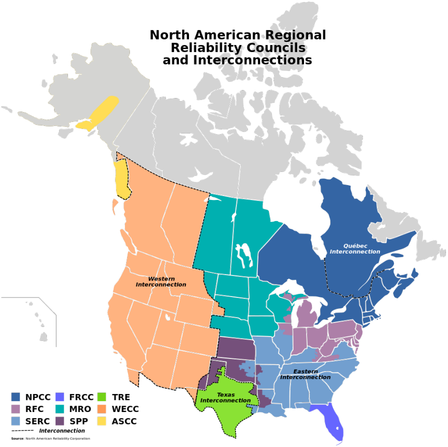
Commercial elect. flows, Europe May-July 2014 [GWh]



Source: European Commission, Quarterly Report on European Electricity Markets

From Prof. Schiffer's Presentation . . .

Interconnected power systems - NERC

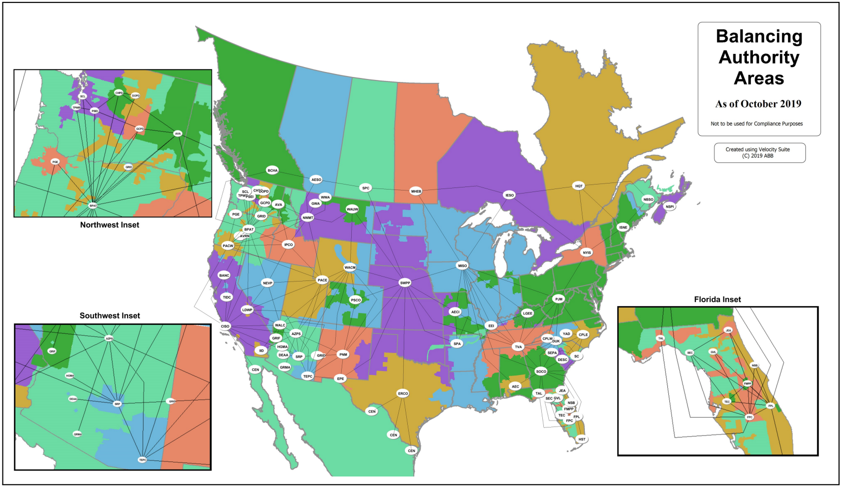


©Bouchecl

- North American Electric Reliability Corporation (NERC)
- 8 regional reliability entities
- $>$ 1,000 GW installed capacity

Balancing Authorities

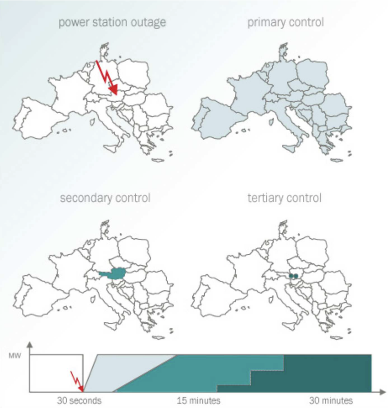
- In North America, so-called *balancing authorities* are the "control areas".



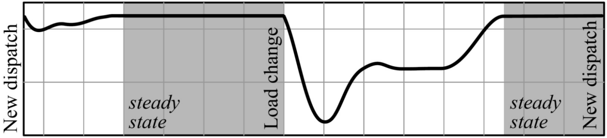
"After transients, you take care of your disturbance, I'll take care of mine"

Secondary Control Localizes and Rejects Disturbances

Figures: ENTSO-E, S. Dhople

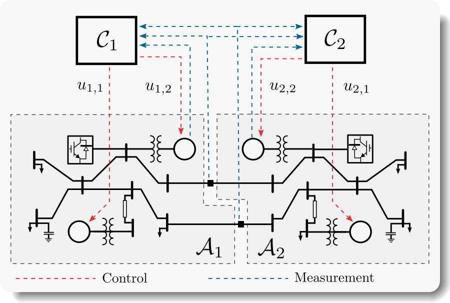


- Primary control causes (i) frequency to stabilize (ii) **power flow from supporting areas to contingent area**
- Secondary control **rebalances** the system so that disturbance is **compensated locally**
- Tertiary control (e.g., OPF) re-optimizes all the generation later to globally minimize cost



Automatic Generation Control

Rebalancing supply and demand in interconnected systems



- BA-wise decentralized control
- Deployed since 1940's
- Eliminates generation-load mismatch within each BA
- **Operates slowly compared to primary control**
- Perhaps the first large-scale distributed control system

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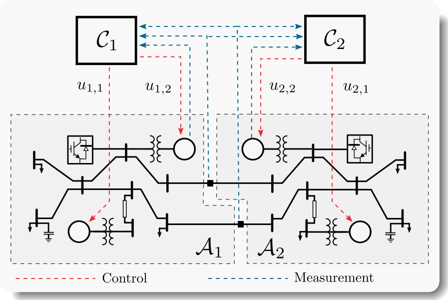

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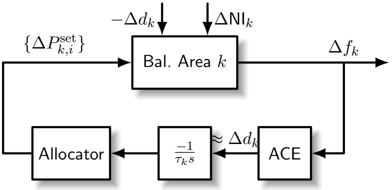

A change in load "will be taken care of, but it may be taken care of by any of the governor-regulated machines then in operation on the system. Therein lies the nub of the load control problem." – N. Cohn, *Power Flow Control – Basic Concepts for Interconnected Systems*, 1950.

Simple Model for Analysis of AGC

- Areas $k \in \mathcal{A} = \{1, \ldots, N\}$, generators $\mathcal{G}_k$ with set-points $\Delta P_{ki}^{\text{set}}$ for $i \in \mathcal{G}_k$
- Model each area $k \in \mathcal{A}$ as **coherent**; lumped inertia with several turbine/gov's

$$2H_k \Delta \dot{f}_k = \sum_{i \in \mathcal{G}_k} \Delta P_{\text{m},ki} - \underbrace{\sum_{j \in \mathcal{A}} \Delta P_{\text{e},k \to j}}_{\text{Net Interchange} \triangleq \Delta \text{NI}_k} - (D_k \Delta f_k + \Delta d_k)$$

$$T_{ki} \Delta \dot{P}_{\text{m},ki} = -\Delta P_{\text{m},ki} - \frac{1}{R_{ki}} \Delta f_k + \Delta P_{ki}^{\text{set}}$$

- Define the **area control error** with bias $b_k > 0$

$$\text{ACE}_k = \underbrace{\Delta \text{NI}_k}_{\text{local flow error}} + \underbrace{b_k \Delta f_k}_{\text{global imbalance}}, \qquad i \in \mathcal{A},$$

- AGC is now simply area-wise decentralized integral control on the ACE:

$$\begin{aligned} \text{Integrate error:} \quad & \tau_k \dot{\eta}_k = -\text{ACE}_k, & k \in \mathcal{A} \\ \text{Dispatch generators:} \quad & \Delta P_{ki}^{\text{set}} = \alpha_{ki} \eta_k, & i \in \mathcal{G}_k \end{aligned}$$

with $\sum_i \alpha_{ki} = 1$ for each $k \in \mathcal{A}$

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Simple Model for Analysis of AGC

- **Assuming stability**, for constant inputs $(\Delta d_k, \Delta P_{ki}^{\text{set}})$, at equilibrium we have

Synchronization:	$\Delta f_1 = \Delta f_2 = \cdots = \Delta f_N = \Delta f_{\text{ss}}$	
Area Balance:	$\sum_{i \in \mathcal{G}_k} \Delta P_{\text{m},ki} = D_k \Delta f_k + \Delta d_k + \Delta \text{NI}_k,$	$k \in \mathcal{A}$
Governors:	$\Delta P_{\text{m},ki} = \Delta P_{ki}^{\text{set}} - \frac{1}{R_{ki}} \Delta f_k,$	$k \in \mathcal{A}$
Global Balance:	$0 = \sum_{k \in \mathcal{A}} \Delta \text{NI}_k$	

- Let $\Delta P_k^{\text{set}} = \sum_{i \in \mathcal{G}_k} \Delta P_{ki}^{\text{set}}$ be total set-point change for area k
- Easy algebra to find that

$$\Delta f_{\text{ss}} = \frac{1}{\beta} \left(\sum_{k \in \mathcal{A}} \Delta P_k^{\text{set}} - \sum_{k \in \mathcal{A}} \Delta d_k \right), \qquad \beta_k = D_k + \sum_{i \in \mathcal{G}_k} \frac{1}{R_{ki}}$$

$$\Delta \text{NI}_{k,\text{ss}} = (\Delta P_k^{\text{set}} - \Delta d_k) - \sum_{j \in \mathcal{A}} \frac{\beta_k}{\beta} (\Delta P_j^{\text{set}} - \Delta d_j), \qquad \beta = \sum_{k \in \mathcal{A}} \beta_k$$

Let's do a steady-state and a dynamic analysis.

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Let's do a steady-state and a dynamic analysis.

Steady-State Analysis of AGC

JWSP and N. Monshizadeh "Diagonal Stability of Systems With . . .", IEEE TCONS, 2022.

- Cohn's insight was that each area can independently measure the ACE and drive it to zero, and that this will achieve system-wide rebalancing

Theorem: For any bias constants $b_k > 0$

(i) $\Delta P_k^{\mathrm{set}} = \Delta d_k$ for all areas $k \in \mathcal{A}$ iff

(ii) $\Delta f_k = 0$ and $\Delta \mathrm{NI}_k = 0$ for all areas $k \in \mathcal{A}$ iff

(iii) $\mathrm{ACE}_k = 0$ for all areas $k \in \mathcal{A}$.

- This result was known roughly by 1950
- Statement (i) is the objective; you want each area to match its disturbance
- Statement (iii) is how you can do it; you can drive the ACE to zero
- Does this mean the bias b_k doesn't matter? **No**. More soon.

Dynamic Analysis of AGC via Time-Scale Separation

JWSP and N. Monshizadeh "Diagonal Stability of Systems With ...", IEEE TCONS, 2022.

- Substitute steady-state grid quantities into AGC equations

$$\begin{bmatrix} \tau_1 \dot{\eta}_1 \\ \tau_2 \dot{\eta}_2 \\ \vdots \\ \tau_N \dot{\eta}_N \end{bmatrix} = -\frac{1}{\beta} \begin{bmatrix} \beta + b_1 - \beta_1 & b_1 - \beta_1 & \cdots & b_1 - \beta_1 \\ b_2 - \beta_2 & \beta + b_2 - \beta_2 & \cdots & \cdots \\ \vdots & \cdots & \ddots & b_{N-1} - \beta_{N-1} \\ b_N - \beta_N & \cdots & b_N - \beta_N & \beta + b_N - \beta_N \end{bmatrix} \begin{bmatrix} \eta_1 - \Delta d_1 \\ \eta_2 - \Delta d_2 \\ \vdots \\ \eta_N - \Delta d_N \end{bmatrix}$$

or simply

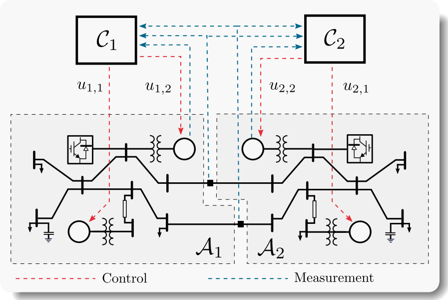
$$\boldsymbol{\tau} \dot{\boldsymbol{\eta}} = -\boldsymbol{\mathcal{B}}(\boldsymbol{\eta} - \Delta \boldsymbol{d}), \qquad \boldsymbol{\mathcal{B}} = -I_N + \frac{1}{\beta}(\boldsymbol{\beta} - \boldsymbol{b})\mathbb{1}_N^{\mathsf{T}}.$$

- Slow time-scale dynamics of AGC governed by **simple LTI system**

- $\boldsymbol{\mathcal{B}}$ is Hurwitz! It has
  - (i) $N-1$ eigenvalues at -1
  - (ii) one eigenvalue at $-1 + \mathbb{1}_N^{\mathsf{T}} \frac{1}{\beta}(\boldsymbol{\beta} - \boldsymbol{b}) = -\frac{\sum_k b_k}{\sum_k \beta_k} < 0$

- Even stronger, can prove that $\boldsymbol{\mathcal{B}}$ is **diagonally stable**: there exists $\boldsymbol{C} = \mathrm{diag}(c_1, \ldots, c_N)$ such that $\boldsymbol{\mathcal{B}}^{\mathsf{T}}\boldsymbol{C} + \boldsymbol{C}\boldsymbol{\mathcal{B}} \prec 0$.

Automatic Generation Control



- BA-wise decentralized control
- Deployed since 1940's
- Eliminates generation-load mismatch within each BA
- **Operates slowly compared to primary control**
- Perhaps the first large-scale distributed control system

Theorem: For all sufficiently large AGC time constants $\tau_k > 0$, closed-loop system is internally stable and $\lim_{t\to\infty} \text{ACE}_k(t) = 0$ for all areas $k \in \mathcal{A}$.

- Result is **independent** of bias tunings $b_k > 0$; any biasing works.
- Consistent with **practice**; no coordination required for stable tuning
- Rigorous justification for engineering practice.

Insights into Dynamic Performance of AGC

JWSP and N. Monshizadeh "Diagonal Stability of Systems With ...", IEEE TCONS, 2022.

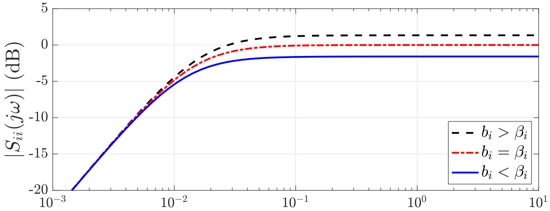
- The previous analysis provides a **reduced LTI model** of AGC dynamics

$$\boldsymbol{\tau}\dot{\boldsymbol{\eta}} = -\boldsymbol{\mathcal{B}}(\boldsymbol{\eta} - \Delta\boldsymbol{d}) \qquad \boldsymbol{\mathcal{B}} = -I_N + \frac{1}{\beta}(\boldsymbol{\beta} - \boldsymbol{b})\mathbb{1}_N^{\mathsf{T}}$$

$$\textbf{ACE} = \boldsymbol{\mathcal{B}}(\boldsymbol{\eta} - \Delta\boldsymbol{d})$$

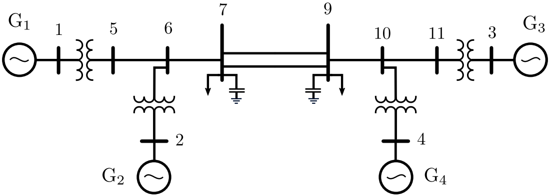
- If $b_k < \beta_k$ the tuning is called **underbiased**, $b_k > \beta_k$ is **overbiased**
  - (i) Underbiased tunings degrade the stability margin of $-\boldsymbol{\mathcal{B}}$
  - (ii) If $\tau_k = \tau > 0$, then the sensitivity function is

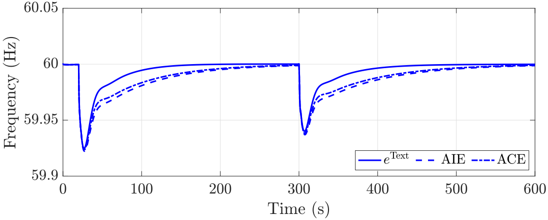
$$S_{ij}(s) = \frac{\mathsf{ACE}_i(s)}{\Delta d_j(s)} = -\frac{\tau s}{\tau s + 1}\left[\delta_{ij} - \frac{1}{\beta}(\beta_i - b_i)\frac{\tau s}{\tau s + \frac{1}{\beta}\sum_k b_k}\right]$$



Simulation on Two-Area "Kundur" System

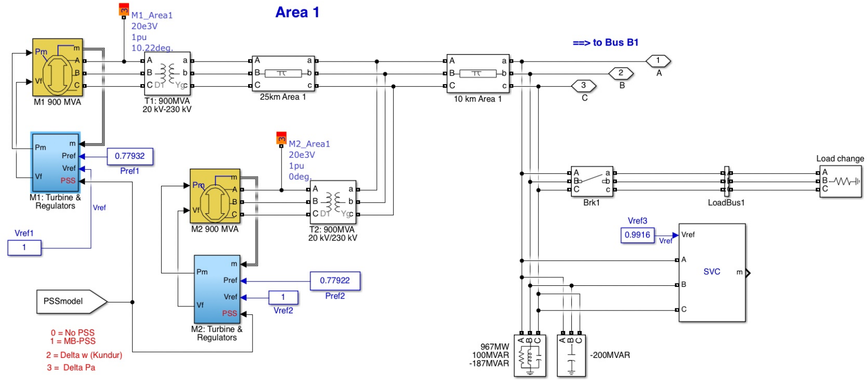
- A classic system for various power system stability tests
- Three-phase **Simscape Electrical** model with high-order machine models, turbine/governors, exciters, limiters, SVCs, ...



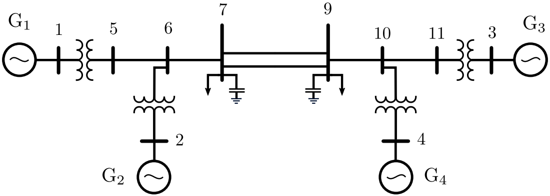


Simscape Electrical (SimPowerSystems)

- **My Opinion:** Beyond toy models, you should not code your own power system simulations. People do their whole PhDs building power system simulation software.
- MATLAB has decent tools for doing simulations on small to medium-sized systems



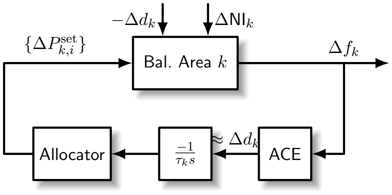
Exercise: AGC in Kundur System



Investigate the following questions:

1. How **small** can the AGC time constants $\tau = \tau_1 = \tau_2$ be before the closed-loop system becomes unstable?
2. How does increasing the bias constant b_2 impact the closed-loop response?
3. How does decreasing the bias constant b_2 impact the closed-loop response?

Why is Frequency Control Slow?



AGC is necessarily slow because

(i) operates over large geographic regions (delay tolerance)

(ii) designed with zero knowledge of primary control dynamics

(iii) slow actuators (turbine/governor systems)

How can the main idea of <u>disturbance estimation</u> be **modernized**?

(i) use fast communication / smaller geographic areas

(ii) integrate some crude model information for improved dynamic performance

(iii) use of fast **inverter-based resources** (IBRs)

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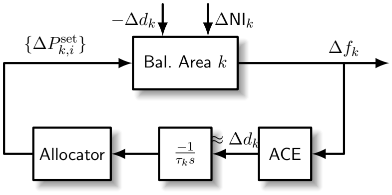

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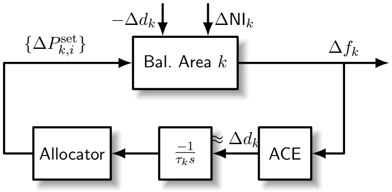

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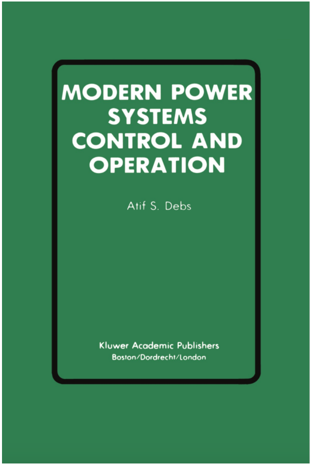
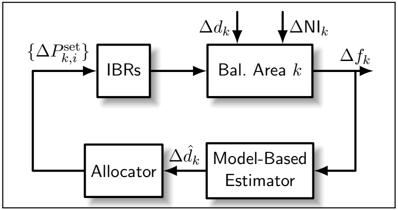
Towards a Rigorous Modernization of AGC



"The advent of modern control theory in the sixties and early seventies did little to change these very successful AGC practices. However, it has provided, and will continue to provide, a more careful understanding of the entire problem. **By doing so, a possible new generation AGC may emerge. Such an AGC will have to retain the simplicity of classical AGC but with improved overall performance.**"

Model-Based Fast Frequency Control

E. Ekomwenrenren et al., "Hierarchical Coordinated Fast Frequency Control . . . ," in IEEE TPWRS, 2021.



- IBRs have local droop curve

$$T_{k,i}\Delta \dot{P}_{k,i} = -\Delta P_{k,i} - \frac{\Delta f_{k,i}}{R_{k,i}} + \Delta P_{k,i}^{\mathrm{set}}$$

- Inverter controls ensure $T_{k,i}$ is small (e.g., 200ms)

- Assume simple dynamic model for area dynamics

$$\Delta \dot{x}_k = A_k \Delta x_k + B_k(\Delta P_k^{\mathrm{set}} - \Delta d_k + \Delta \mathrm{NI}_k)$$

- Fictitious disturbance model $\Delta \dot{d}_k = 0$

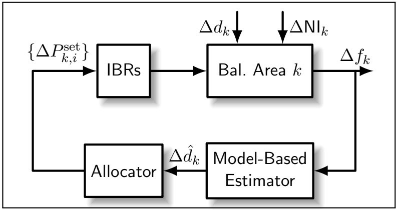
- Extended-state Luenberger observer

$$\begin{bmatrix} \Delta \dot{\hat{x}}_k \\ \Delta \dot{\hat{d}}_k \end{bmatrix} = \begin{bmatrix} A_k & -B_k \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \hat{x}_k \\ \Delta \hat{d}_k \end{bmatrix} + \begin{bmatrix} B_k \\ 0 \end{bmatrix} (\Delta P_k^{\mathrm{set}} + \Delta \mathrm{NI}_k) + L_k(\Delta f_k - \Delta \hat{f}_k)$$

$$\Delta \hat{f}_k = C_k \Delta \hat{x}_k$$

Model-Based Fast Frequency Control

E. Ekomwenrenren et al., "Hierarchical Coordinated Fast Frequency Control . . . ," in IEEE TPWRS, 2021.



- IBRs have local droop curve

$$T_{k,i}\Delta \dot{P}_{k,i} = -\Delta P_{k,i} - \frac{\Delta f_{k,i}}{R_{k,i}} + \Delta P^{\mathrm{set}}_{k,i}$$

- Inverter controls ensure $T_{k,i}$ is small (e.g., 200ms)

- Assume simple dynamic model for area dynamics

$$\Delta \dot{x}_k = A_k \Delta x_k + B_k(\Delta P^{\mathrm{set}}_k - \Delta d_k + \Delta \mathrm{NI}_k)$$

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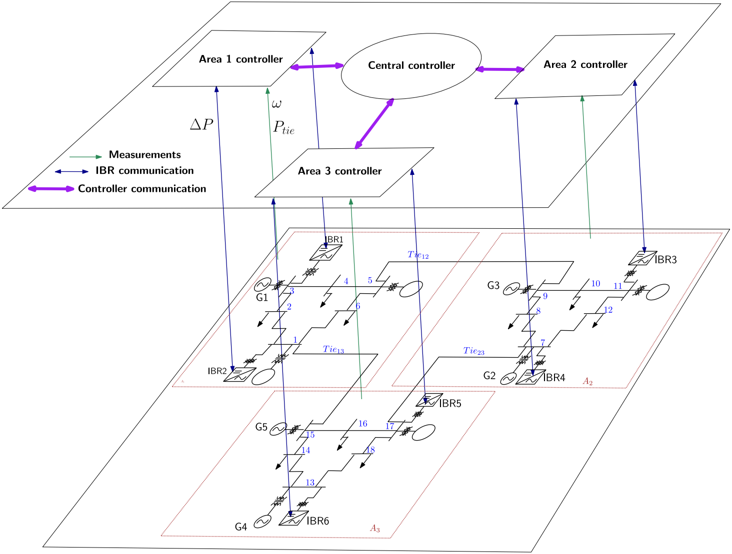
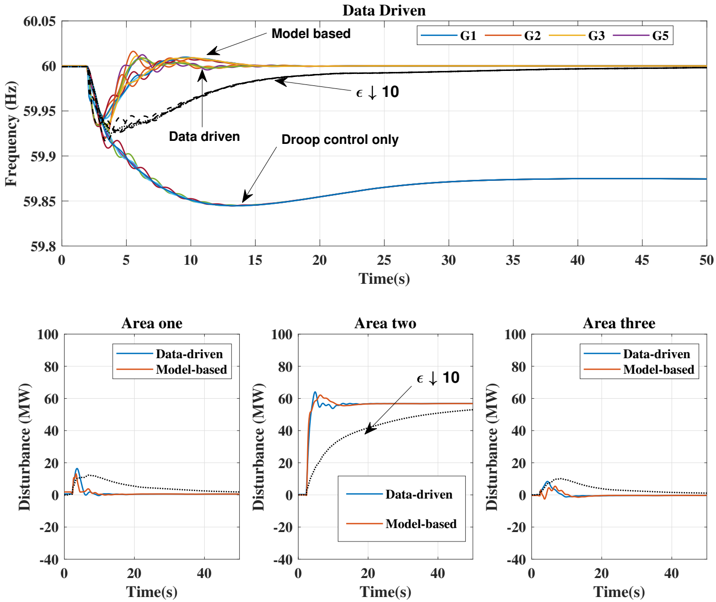
Case Study: Three-LCA System

E. Ekomwenrenren et al., "Hierarchical Coordinated Fast Frequency Control . . . ," in IEEE TPWRS, 2021.



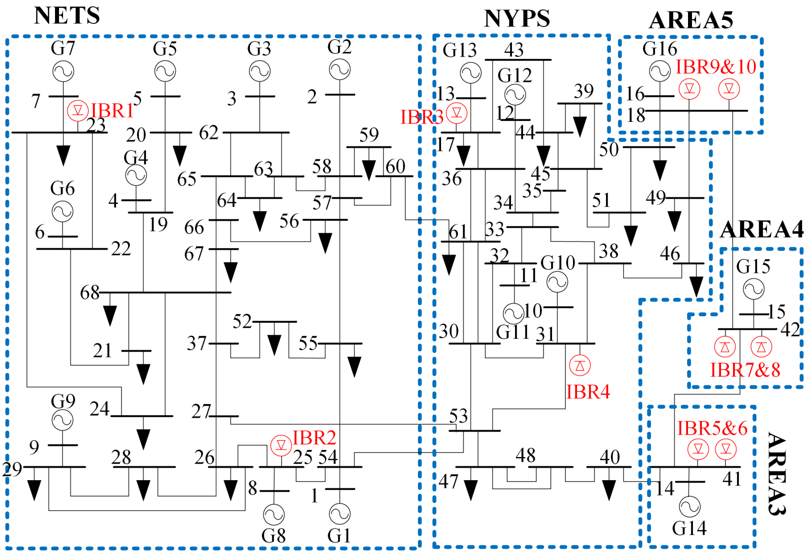
Scenario: 63 MW Disturbance, Area 2

E. Ekomwenrenren et al., "Hierarchical Coordinated Fast Frequency Control . . . ," in IEEE TPWRS, 2021.



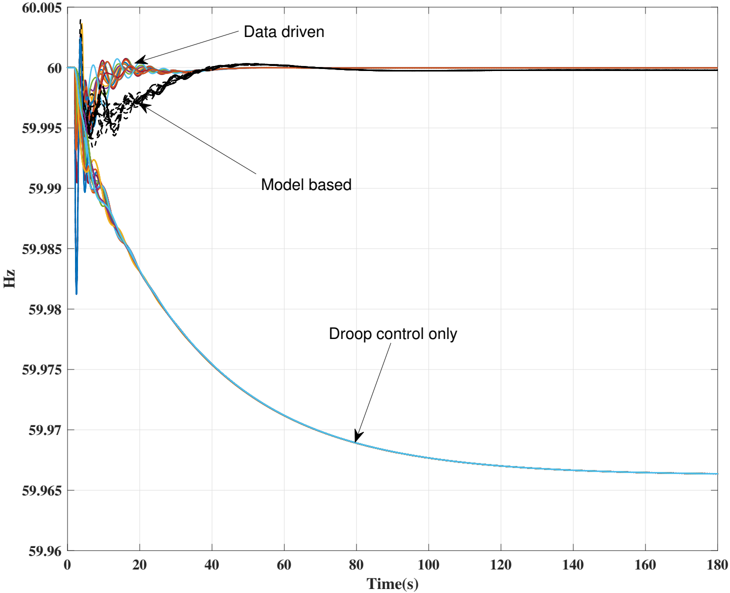
Five-Area IEEE68 Bus Test System

E. Ekomwenrenren et al., "Hierarchical Coordinated Fast Frequency Control . . . ," in IEEE TPWRS, 2021.



Scenario: 450MW Load Change in NYPS Area

E. Ekomwenrenren et al., "Hierarchical Coordinated Fast Frequency Control . . . ," in IEEE TPWRS, 2021.



Conclusions

A narrow and biased look at some grid operation/control basics

- Power flow / dispatch sets the grid operating point
- Frequency control maintains operating point between redispatch
- **Neglected:** Voltage control, stability enhancement, FACTS, ...

Opportunities at $\{$**control**$\} \cap \{$**power systems**$\} \cap \cdots$

- Hierarchical feedback design
- Data-driven and learning-based control w/ guarantees ...

Parting thoughts:

- Nothing more practical than a good theory
- Run semi-serious simulations (e.g., in Simscape Electrical)
- Make a power engineer / power electronics friend

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Questions

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Questions

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appendix