Optimal control problems on Lie groups
by
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Abstract. Stricly speaking my lecture is about the optimal control problem of minimizing the
integral $\frac{1}{2}\int_0^T \langle u(t), Pu(t)\rangle\, dt$ over the trajectories of a control system

$$\frac{dg}{dt}(t) = g(t)(A_0 + \sum_{i=1}^{m} u_i(t)A_i)$$

that satisfy some initial and terminal boundary conditions in a linear group G, where $A_0, \ldots, A_m$ are matrices in the Lie algebra $\mathfrak{g}$ of G and P is a positive definite $m \times m$ matrix. But because of a large variety of problems from mechanics, elasticity and geometry that are covered by this class of systems, the lecture will necessarily touch upon more general and pertinent questions, namely, the contributions of non-linear control to geomery and mechanics, and vice-versa, the contributions of mechanics and geometry to the problems of non-linear control.

I will use the Maximum Principle to show that the above optimal problems reduce to Hamiltonian systems on the cotangent bundle $G \times \mathfrak{g}^*$,where $\mathfrak{g}^*$ denotes the dual of the Lie algebra $\mathfrak{g}$. generated by the Hamiltonian $\mathcal{H} = \frac{1}{2}\langle (H_{1,\ldots,}H_m), P^{-1}(H_{1,\ldots,}H_m)\rangle + H_0$. The solutions of such systems then depend on the underlying symmetries and related integrals of motion, a part of large topic known as the integrability theory of left- invariant Hamiltonian systems on a Lie algebra $\mathfrak{g}$.

The lecture will also touch upon the highlights of my recent monograph " Integrable Hamiltonian Systems on Complex Lie groups" , Memoirs AMS, Number 838.