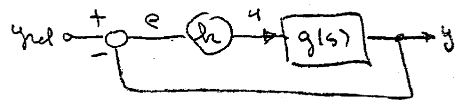
SOLUTIONS TO MID-TERM TEST ECE311S : SPRING 2001

① (a) Given the system 

, then by inspection

$$g(s) = \frac{1}{(s+1)}\left(\frac{k_3}{s+2}\right) + \frac{1}{(s+10)}\left(\frac{k_1}{s-1} + \frac{k_2}{s+3}\right)$$

$$= \frac{k_3(s+10)(s-1)(s+3) + (s+1)(s+2)\left[k_1(s+3) + k_2(s-1)\right]}{(s+1)(s+2)(s+10)(s-1)(s+3)}$$

$$\Rightarrow \quad G(s) = \frac{k\,g(s)}{1+k\,g(s)} \quad \text{where } y(s) = G(s)\,y_{ref}(s)$$

or

$$\boxed{G(s) = \frac{k\left\{k_3(s+10)(s-1)(s+3) + (s+1)(s+2)\left[k_1(s+3)+k_2(s-1)\right]\right\}}{(s+1)(s+2)(s+10)(s-1)(s+3) + k\left\{k_3(s+10)(s-1)(s+3) + (s+1)(s+2)\left[k_1(s+3)+k_2(s-1)\right]\right\}}}$$

(b) Since $g(s)$ has no pole $= 0$ $\Rightarrow$ $\boxed{\text{system type} = \phi}$

(c) We have that $e(\infty) = \dfrac{1}{1+k\,g(0)}$ for the case when $y_{ref}(s) = \frac{1}{s}$ for a type ϕ system, where

$$g(0) = \frac{-30k_3 + 2(3k_1 - k_2)}{-60} = \frac{15k_3 - (3k_1 - k_2)}{30}$$

$$\therefore \quad \boxed{e(\infty) = \frac{30}{30 + k\left[15k_3 - (3k_1 - k_2)\right]}}$$

PROBLEM 2

(a) Closed-loop transfer function:

$$T(s) = \frac{k(T_I s + 1)}{T_I s^2 + (k-1) T_I s + k}$$

- System is stable iff roots of $T_I s^2 + (k-1) T_I s + k = 0$ are in the LHP.

Must have $\left.\begin{array}{l}(k-1) T_I > 0 \\ k > 0\end{array}\right\} \boxed{k > 1,\ T_I > 0}$

(b) Characteristic polynomial : $s^2 + \underbrace{(k-1)}_{2\zeta\omega_n} s + \underbrace{k/T_I}_{\omega_n^2} = 0$

Neglecting the zero in $T(s)$, the system may be approximated by a standard second-order system with no zeros with the following identifications:

$$2\zeta\omega_n = k-1 \qquad \zeta\omega_n = \frac{k-1}{2}$$

$$\omega_n^2 = \frac{k}{T_I}$$

$$t_s \le 2s \iff \frac{4.6}{\omega_n \zeta} \le 2 \iff \omega_n \zeta \geq \frac{4.6}{2} \iff \boxed{k \geq 5.6}$$

Overshoot: $M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \leqslant 0.01$

$\Longleftrightarrow \zeta \gtrsim 0.883$

Using the fact that $\zeta = \dfrac{k-1}{2\omega_n} = \dfrac{k-1}{2\sqrt{k/T_I}}$

we have $\boxed{T_I = k\dfrac{4\zeta^2}{(k-1)^2} = 0.73}$

Solution

3 a) KVL $\Rightarrow$ $v = \dfrac{d\lambda}{dt} + v_c$

$$\boxed{\frac{d\lambda}{dt} = v - v_c}$$

KCL $\Rightarrow$ $i_l = i_c + i_R$

$$A\lambda + B\lambda^3 = C\frac{dv_c}{dt} + \frac{1}{R}v_c$$

$$\boxed{\frac{dv_c}{dt} = -\frac{1}{RC}v_c + \frac{A}{C}\lambda + \frac{B}{C}\lambda^3}$$

$$\boxed{y = e = v - v_c}$$

let $x_1 = \lambda$ $\qquad u = v$

$x_2 = v_c$

$$\boxed{\begin{aligned} \dot{x}_1 &= -x_2 + u \\ \dot{x}_2 &= -\frac{1}{RC}x_2 + \frac{A}{C}x_1 + \frac{B}{C}x_1^3 \end{aligned}}$$

$$\boxed{y = -x_2 + u}$$

3 b) Steady State

If the given point is a steady state operating point then $\dot{x}_1 = \dot{x}_2 = 0$

$$\dot{x}_1 = -x_2 + u$$
$$\dot{x}_2 = -x_2 + 5x_1 + 40x_1^3$$

Plug in $x_1 = \frac{1}{2}$ $x_2 = 7.5$ $u = 7.5$
and see if $\dot{x}_1 = \dot{x}_2 = 0$

$$\dot{x}_1 = -7.5 + 7.5 = 0$$
$$\dot{x}_2 = -7.5 + 5\left(\tfrac{1}{2}\right) + 40\left(\tfrac{1}{2}\right)^3 = 0$$

$\therefore$ the op. pt. is valid.

3 c) $A = 1$ $B = 8$ $C = 0.2$ $R = 5$

$$\mathring{x}_1 = -x_2 + u$$

$$\mathring{x}_2 = -\frac{1}{RC}x_2 + \frac{A}{C}x_1 + \frac{B}{C}x_1^3$$

$$y = -x_2 + u$$

$$\mathring{\Delta x} = \left.\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}\right|_{\substack{x = x_0 \\ u = u_0}} \Delta x + \left.\begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \end{bmatrix}\right|_{\substack{x = x_0 \\ u = u_0}} \Delta u$$

$$\begin{bmatrix} \Delta\mathring{x}_1 \\ \Delta\mathring{x}_2 \end{bmatrix} = \left.\begin{bmatrix} 0 & -1 \\ \frac{A}{C} + 3\frac{B}{C}x_1^2 & -\frac{1}{RC} \end{bmatrix}\right|_{\substack{x_1 = 0.5 \\ x_2 = 7.5 \\ u = 7.5}} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Delta u$$

$$5 + 3\left(\frac{8}{.2}\right)\left(\frac{1}{2}\right)^2$$

$$\begin{bmatrix} \Delta\mathring{x}_1 \\ \Delta x_2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 35 & -1 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Delta u$$

$$y = \begin{bmatrix} 0 & -1 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} + \begin{bmatrix} 1 \end{bmatrix} u$$