ECE311 Control & Systems

Problem Set No. 4

Problem 3.2

f)

$$\begin{aligned}\mathcal{L}\{te^{-at} + 2t\cos t\} &= \mathcal{L}\{te^{-at}\} + \mathcal{L}\{2t\cos t\}, \\ &= \frac{1}{(s+a)^2} + 2\frac{s^2-1}{(s^2+1)^2},\end{aligned}$$

since

$$\begin{aligned}\mathcal{L}\{te^{-at}\} &= \frac{1}{(s+a)^2}, \\ \mathcal{L}\{t\cos at\} &= \frac{s^2-a^2}{(s^2+a^2)^2}.\end{aligned}$$

□

Problem 3.4

c)

$$\begin{aligned}\mathcal{L}\{t^2 + e^{-at}\sin bt\} &= \mathcal{L}\{t^2\} + \mathcal{L}\{e^{-at}\sin bt\}, \\ &= \frac{2}{s^3} + \frac{b}{(s+a)^2+b^2}.\end{aligned}$$

□

Problem 3.6

a)

$$\mathcal{L}^{-1}\{\frac{1}{s^6}\} = \frac{t^5}{5!} = \frac{t^5}{60}.$$

$\square$

Problem 3.7

d) Write

$$\frac{2(s+2)}{(s+1)(s^2+4)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+4}.$$

Therefore,

$$A = \frac{2(s+2)}{s^2+4}\Big|_{s=-1} = \frac{2}{5},$$

and

$$A(s^2+4) + (Bs+1)(s+1) = 2(s+2)$$

or

$$(A+B)s^2 + (C+B)s + (C+4A) = 0s^2 + 2s + 4.$$

Now, equate the coefficients to obtain

$$B = -A = -\frac{2}{5}, \qquad C = 2 - B = 2 + \frac{2}{5} = \frac{12}{5}.$$

(Check: $C + 4A = \frac{12}{5} + \frac{8}{5} = 4.$)

Since,

$$\mathcal{L}\{\cos at\} = \frac{s}{s^2+a^2}, \qquad \mathcal{L}\{\sin at\} = \frac{a}{s^2+a^2},$$

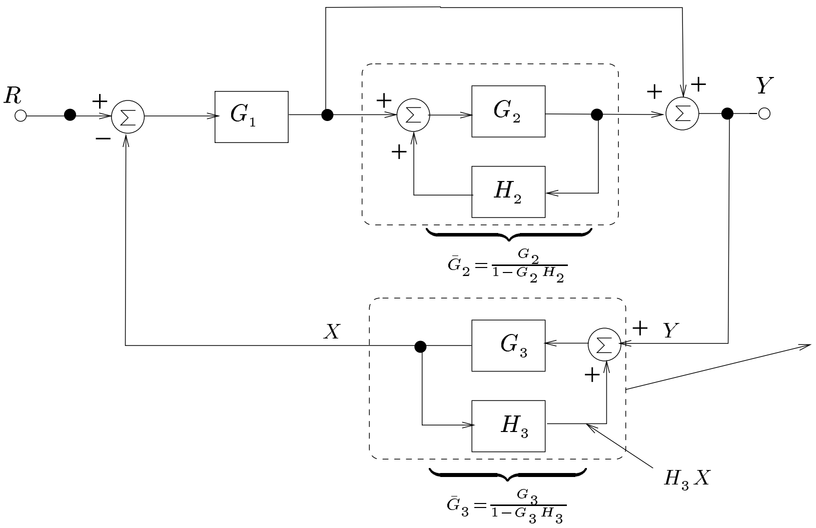
we have

$$\begin{aligned} \mathcal{L}^{-1}\{\frac{A}{s+1} + \frac{Bs+C}{s^2+4}\} &= Ae^{-t} + B\cos 2t + C\sin 2t, \\ \therefore \quad \mathcal{L}^{-1}\{\frac{2(s+2)}{(s+1)(s^2+4)}\} &= \frac{2}{5}e^{-t} - \frac{2}{5}\cos 2t + \frac{12}{5}\sin 2t. \end{aligned}$$

$\square$

Problem 3.22 (Figure 3.39 a)

a) Using elementary block diagrams shown in Figures 3.1 and 3.2

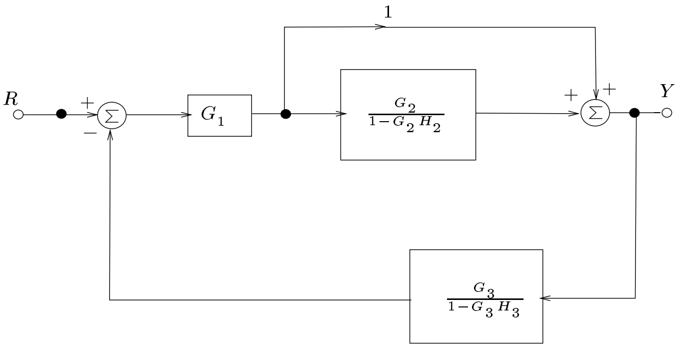


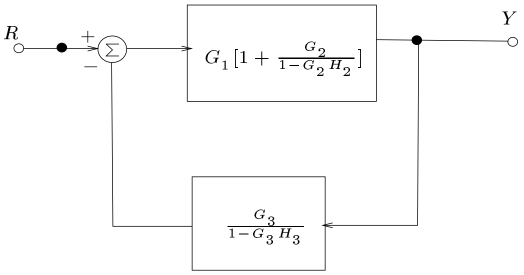
Derive the transfer function of this block

$X = G_3(Y + H_3 X)$

$X = G_3 Y + G_3 H_3 X$

$\frac{X}{Y} = \frac{G_3}{1-G_3 H_3}$





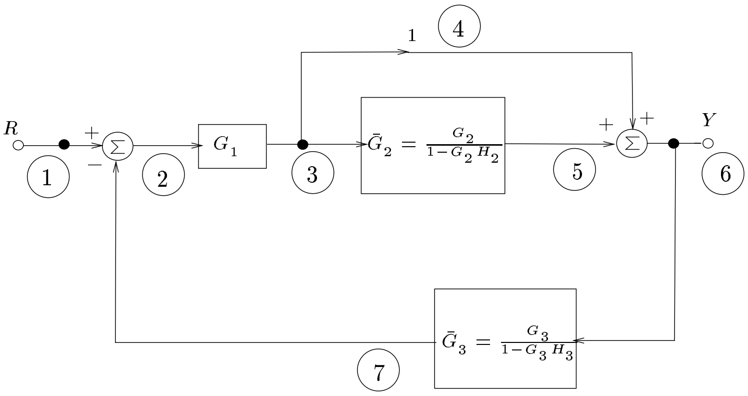
Therefore, the system transfer function is

$$\frac{Y}{R} = \frac{G_1[1 + \frac{G_2}{1-G_2 H_2}]}{1 + G_1[1 + \frac{G_2}{1-G_2 H_2}][\frac{G_3}{1-G_3 H_3}]}.$$

Or

$$\frac{Y}{R} = \frac{G_1(1 - G_2H_2)(1 - G_3H_3) + G_1G_2(1 - G_3H_3)}{(1 - G_2H_2)(1 - G_3H_3) + G_1G_3(1 - G_2H_2) + G_1G_2G_3}.$$

b) Using Mason's rule, we have



Forward path

$$\begin{array}{lrcl} 1\ 2\ 3\ 5\ 6 & p_1 & = & G_1\bar{G}_2 \\ 1\ 2\ 3\ 4\ 6 & p_2 & = & G_1 \end{array}$$

Loops

$$\begin{array}{lrcl} 2\ 3\ 4\ 7 & L_1 & = & G_1\bar{G}_3 \\ 2\ 3\ 5\ 7 & L_2 & = & G_1\bar{G}_2\bar{G}_3 \end{array}$$

Therefore,

$$\frac{Y}{R} = \frac{p_1 + p_2}{1 + L_1 + L_2} = \frac{G_1(1 + \bar{G}_2)}{1 + G_1\bar{G}_3(1 + \bar{G}_2)}$$

which is the same as derived in part (a) after substituting for $\bar{G}_2$ and $\bar{G}_3$.

□