ECE311 Homework solutions # 8

6.3a

$$G(s) = \frac{1}{(s+1)^2(s^2+s+4)}$$

We can rewrite this into Bode form.

$$\begin{aligned} G(s) &= \tfrac{1/4}{(s+1)^2((s/2)^2+2\zeta(s/2)+1)} \\ where\ \zeta &= 0.25 \end{aligned}$$

Sketch the magnitude plot of G(s) in dB

$$\begin{aligned} |G(s)| &= 20log_{10}(1/4) - 2 \times 20log_{10}|s+1| \\ &\quad -20log_{10}|(s/2)^2 + 2\zeta(s/2)+1| \end{aligned}$$

$$-2 \times 20log_{10}|s+1| \cong \left\{ \begin{array}{l} 0dB for\ \omega < 1rad/s \\ 2 \times -20dB/decade for\ \omega \geq 1rad/s \end{array} \right\}$$

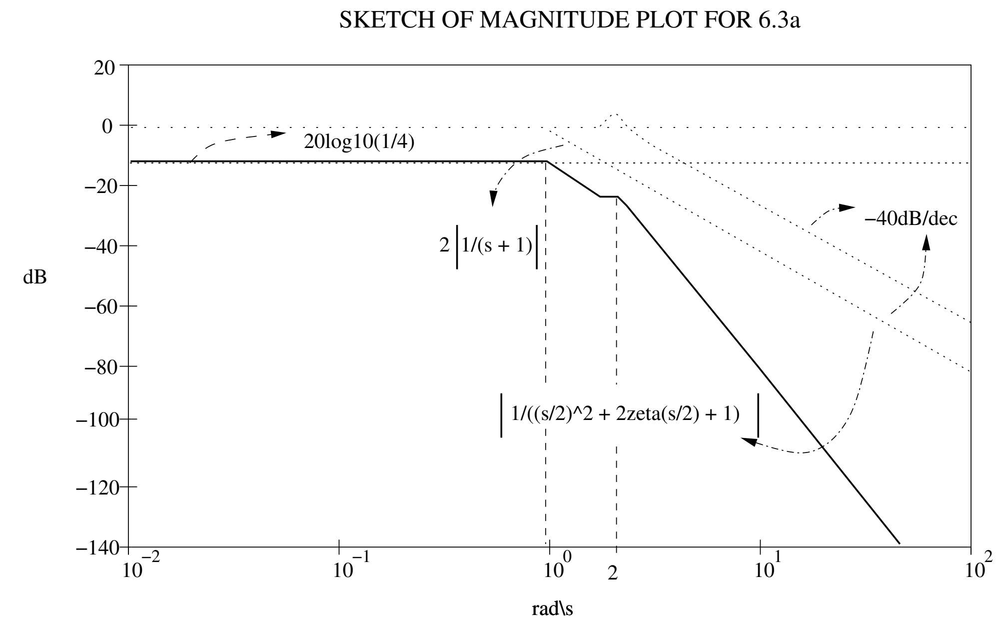
$$-20log_{10}|(s/2)^2 + 2\zeta(s/2)+1| \cong \left\{ \begin{array}{l} 0dB for\ \omega < 2rad/s \\ 20log_{10}(1/2\zeta)dB for\ \omega = 2rad/s \\ -40dB/decade for\ \omega > 2rad/s \end{array} \right\}$$

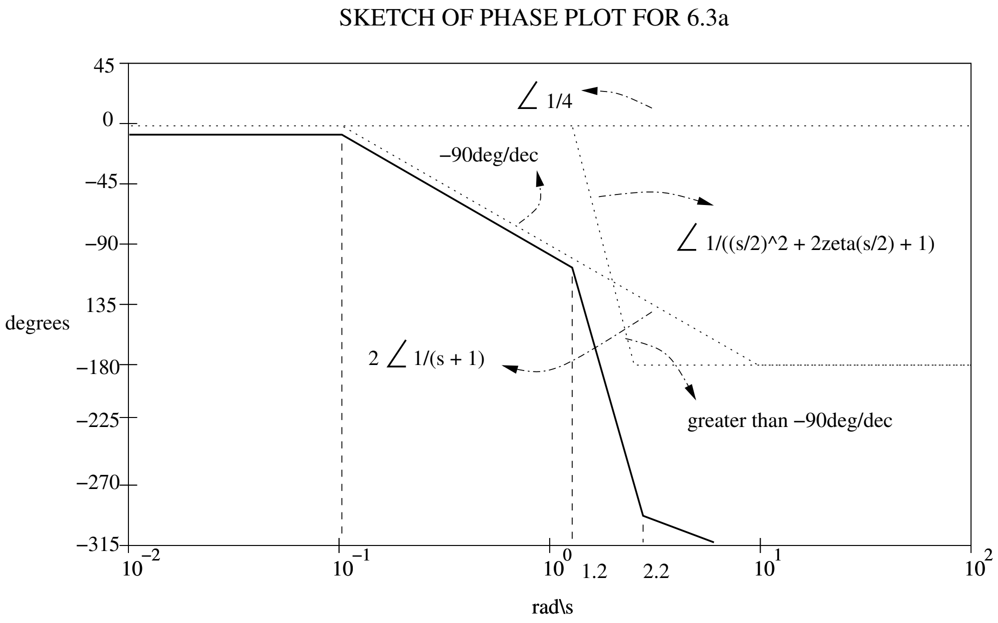
Sketch the phase plot of G(s). Figure 6.2b on p.342 of the textbook was used to approximate phase change of second order term.

$$\angle G(s) = \angle(1/4) - 2 \times \angle(s+1) - \angle\left((s/2)^2 + 2\zeta(s/2)+1\right)$$

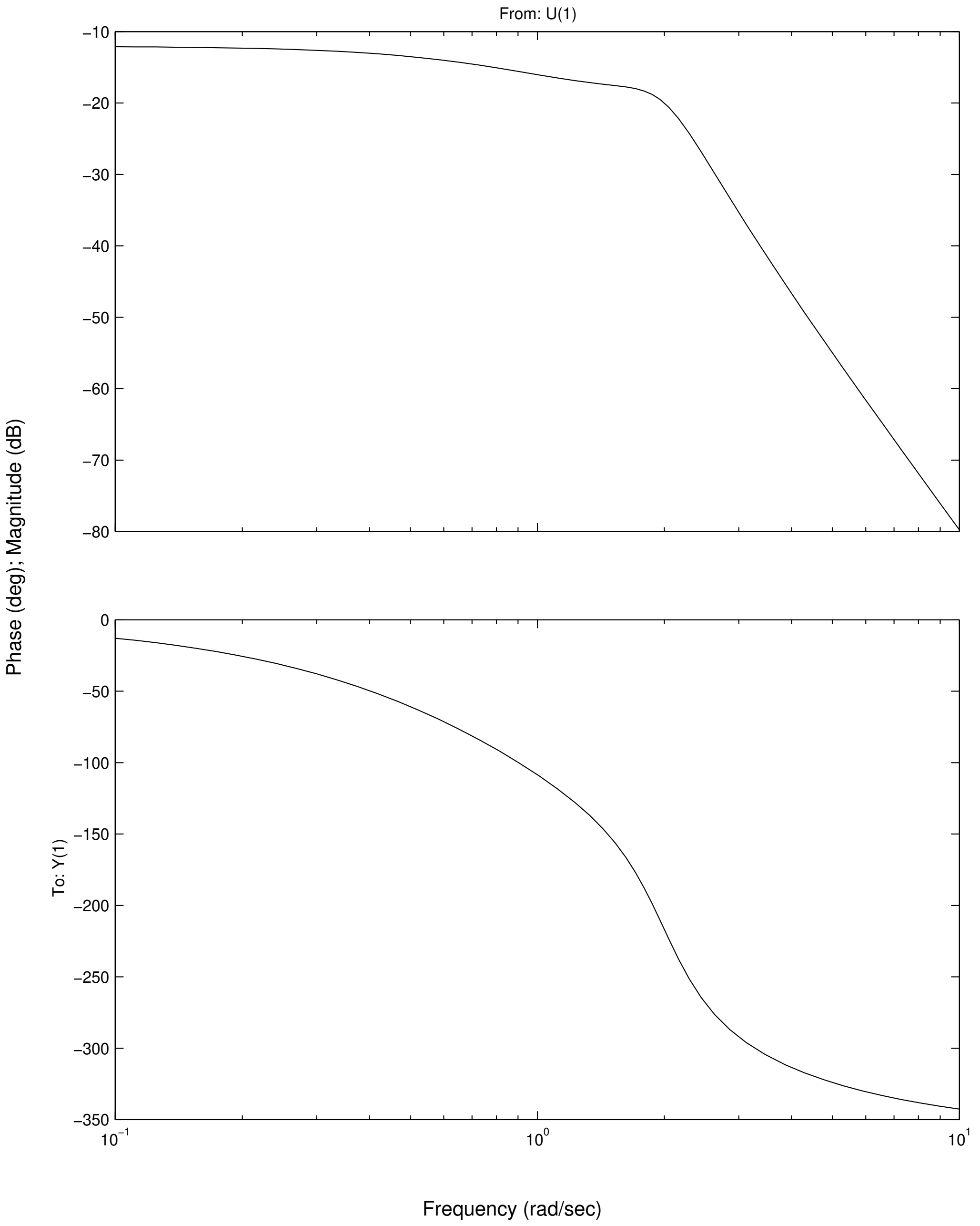
$$-2 \times \angle(s+1) \cong \left\{ \begin{array}{l} 0^o\ for\ \omega < 0.1rad/s \\ 2 \times -45^o/decade for\ 0.1rad/s < \ \omega \ < 10rad/s \\ -180^o for\ \omega > 10rad/s \end{array} \right\}$$

$$-\angle((s/2)^2 + 2\zeta(s/2)+1) \cong \left\{ \begin{array}{l} 0^o for\ \omega < 1.2rad/s \\ -180^o change for\ \omega_n \times 0.6 = 1.2rad/s < \ \omega \ < \omega_n \times 1.1 = 2.2rad/s \\ -180^o for\ \omega > 2.2rad/s \end{array} \right\}$$


SKETCH OF MAGNITUDE PLOT FOR 6.3a
20
0
−20
−40
−60
−80
−100
−120
−140
dB
20log10(1/4)
2 |1/(s + 1)|
−40dB/dec
| 1/((s/2)^2 + 2zeta(s/2) + 1) |
10^−2
10^−1
10^0
2
10^1
10^2
rad\s


SKETCH OF PHASE PLOT FOR 6.3a
45
0
−45
−90
135
−180
−225
−270
−315
degrees
∠ 1/4
−90deg/dec
∠ 1/((s/2)^2 + 2zeta(s/2) + 1)
2 ∠ 1/(s + 1)
greater than −90deg/dec
10^−2
10^−1
10^0
1.2
2.2
10^1
10^2
rad\s

Bode Diagrams in Matlab for 6.3a



6.3e

$$G(s) = \frac{1000(s+1)}{s(s+1)(s^2+8s+64)}$$

We can rewrite this into Bode form.

$$\begin{aligned} G(s) &= \tfrac{1000/128(s+1)}{s((s/2)+1)((s/8)^2+2\zeta(s/8)+1)} \\ where\ \zeta &= 0.5 \end{aligned}$$

Sketch the magnitude plot of G(s) in dB

$$\begin{aligned} |G(s)| &= 20log_{10}(1000/128) - 20log_{10}\,|s| - 20log_{10}\,|(s/2)+1| \\ &+20log_{10}\,|s+1| - 20log_{10}\,|(s/8)^2 + 2\zeta(s/8)+1| \end{aligned}$$

$$-20log_{10}\,|s| \cong \left\{ \begin{array}{l} -20dB/decade for all\ \omega \\ intersecting\ 0dB\ at\ 1rad/s \end{array} \right\}$$

$$+20log_{10}\,|s+1| \cong \left\{ \begin{array}{l} 0dB for\ \omega < 2rad/s \\ 20dB/decade for\ \omega \geq 1rad/s \end{array} \right\}$$

$$-20log_{10}\,|(s/2)+1| \cong \left\{ \begin{array}{l} 0dB for\ \omega < 2rad/s \\ -20dB/decade for\ \omega \geq 2rad/s \end{array} \right\}$$

$$-20log_{10}\,|(s/8)^2 + 2\zeta(s/8)+1| \cong \left\{ \begin{array}{l} 0dB for\ \omega < 8rad/s \\ 20log_{10}(1/2\zeta)dB for\ \omega = 8rad/s \\ -40dB/decade for\ \omega > 8rad/s \end{array} \right\}$$

Sketch the phase plot of G(s). Figure 6.2b on p.342 of the textbook was used to approximate phase change of second order term.

$$\begin{aligned}
\angle G(s) &= \angle(1000/128) - \angle(s) - \angle((s/2)+1) \\
&\quad +\angle(s+1) - \angle((s/8)^2 + 2\zeta(s/8)+1)
\end{aligned}$$

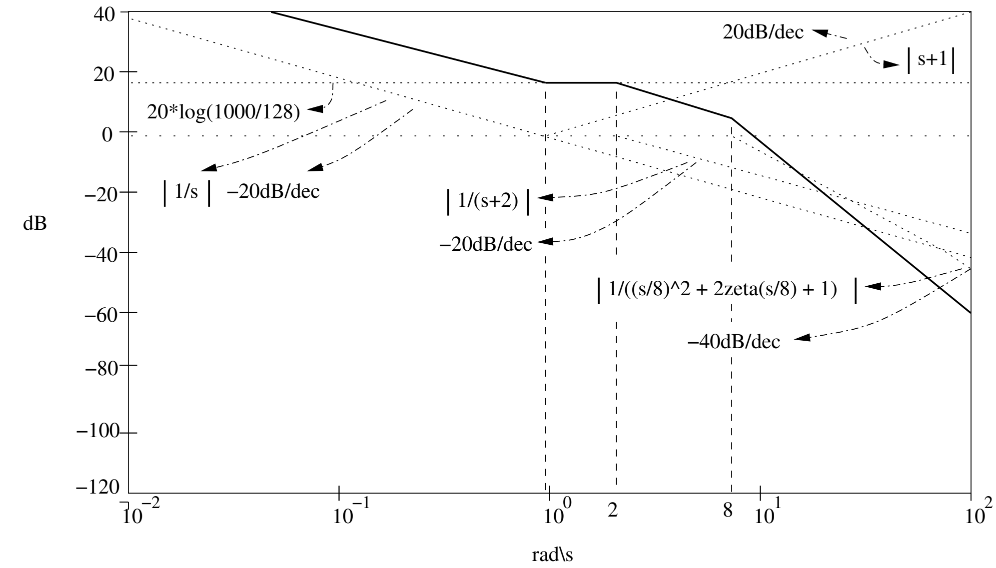
$$-\angle(s) = \left\{ \; -90^o forall\ \omega \; \right\}$$

$$+\angle(s+1) \cong \left\{ \begin{array}{l} 0^o\ for\ \omega < 0.1rad/s \\ 45^o/decade for\ 0.1rad/s < \ \omega \ < 10rad/s \\ 90^o for\ \omega > 10rad/s \end{array} \right\}$$

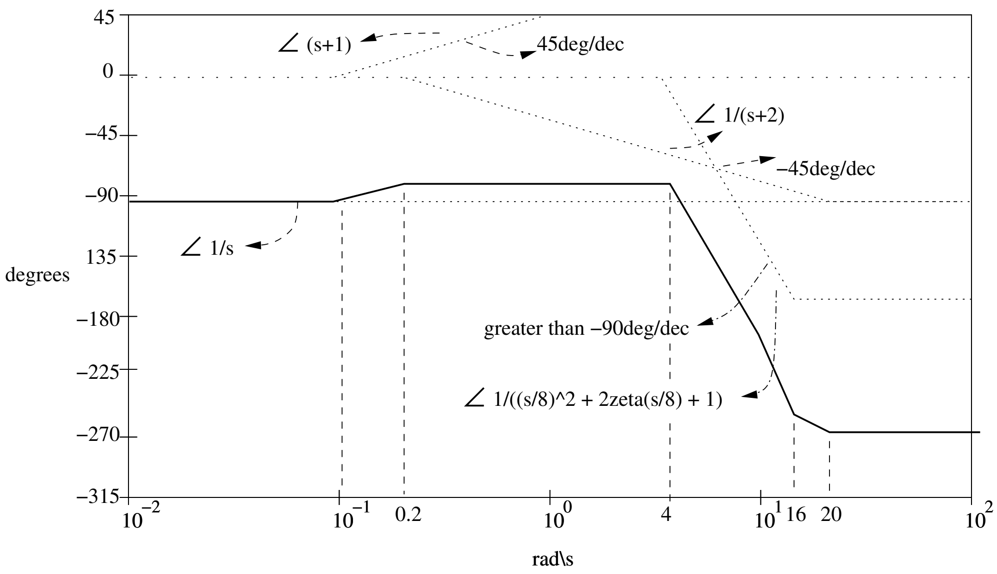
$$-\angle((s/2)+1) \cong \left\{ \begin{array}{l} 0^o\ for\ \omega < 0.2rad/s \\ -45^o/decade for\ 0.2rad/s < \ \omega \ < 20rad/s \\ -90^o for\ \omega > 20rad/s \end{array} \right\}$$

$$-\angle((s/8)^2 + 2\zeta(s/8)+1) \cong \left\{ \begin{array}{l} 0^o for\ \omega < 4rad/s \\ -180^o change for\ \omega_n \times 0.5 = 4rad/s < \ \omega \ < \omega_n \times 2 = 16rad/s \\ -180^o for\ \omega > 16rad/s \end{array} \right\}$$

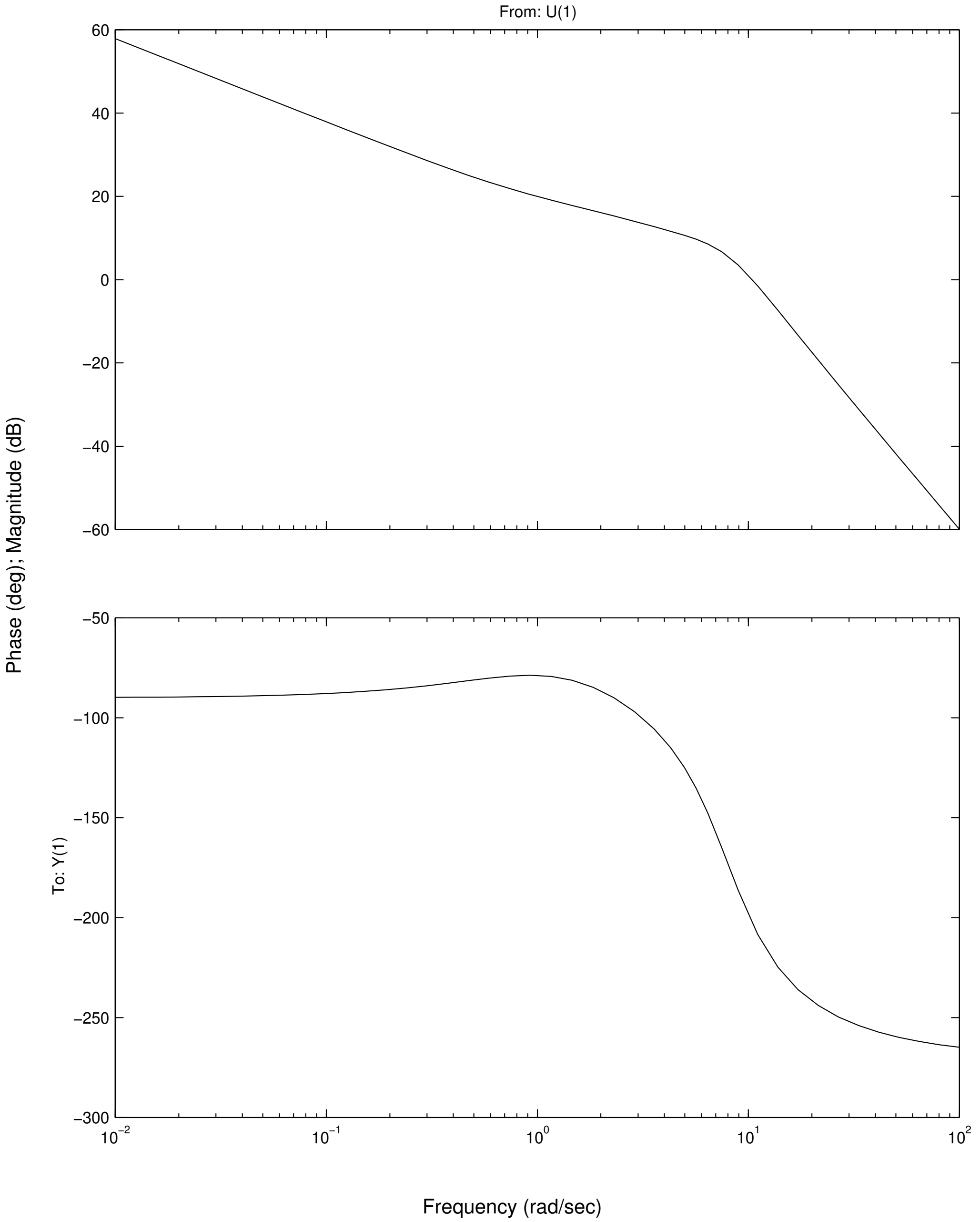
SKETCH OF MAGNITUDE PLOT FOR 6.3e



SKETCH OF PHASE PLOT FOR 6.3e



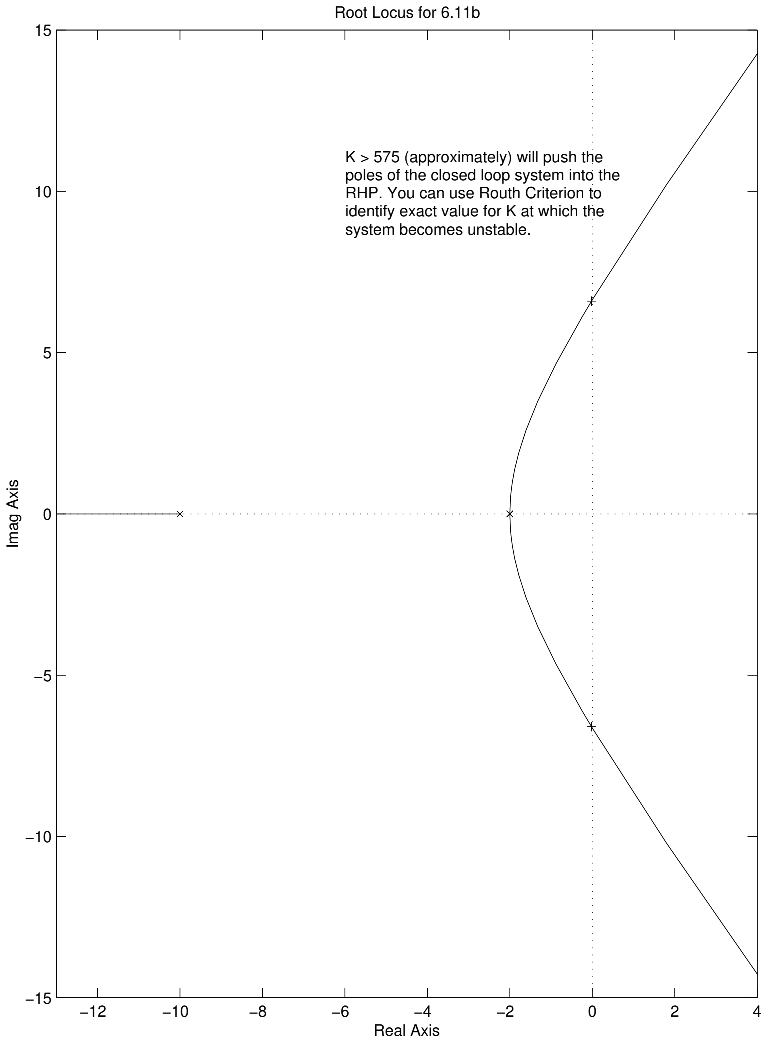
Bode Diagrams in Matlab for 6.3e

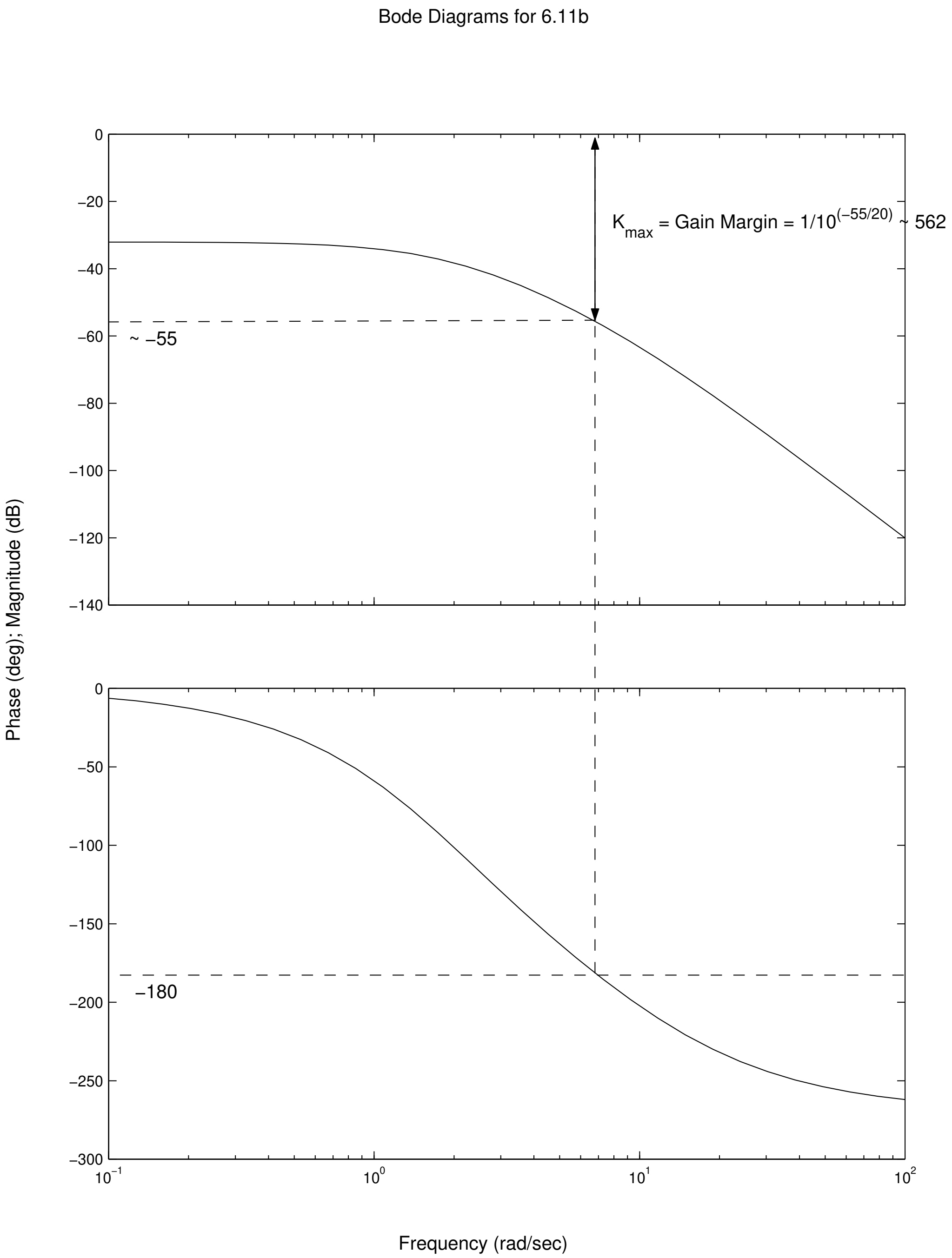


6.11b

$$KG(s) = \frac{K}{(s+10)(s+2)^2}$$

From the root locus we can obtain a range for K for which the C.L. system is stable. The maximum value of K which the system is stable corresponds to the Gain Margin. Using the bode plot (see next page) the maximum value of K for which the system is stable could be obtained by finding the gain of the open loop system when its phase is -180^o. The Gain Margin = range of K for which the system is stable = $1/(10^{(GAIN AT\ -180^o\ OF PHASE/20)}$.



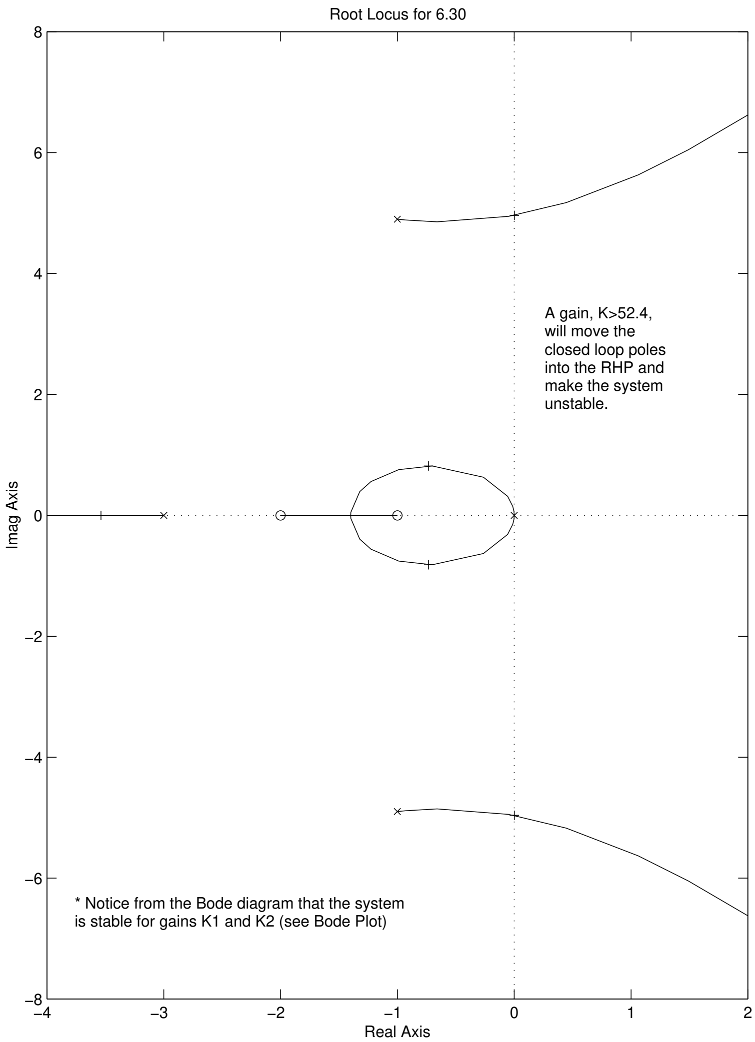


Bode Diagrams for 6.11b
0
−20
−40
−60
−80
−100
−120
−140
K_max = Gain Margin = 1/10^(−55/20) ~ 562
~ −55
Phase (deg); Magnitude (dB)
0
−50
−100
−150
−200
−250
−300
−180
10^−1
10^0
10^1
10^2
Frequency (rad/sec)

6.30

$$KG(s) = \frac{K(s+1)(s+2)}{s^2(s+3)(s^2+2s+25)}$$

To obtain the gains that provide a Phase Margin of 20^o (graphically) identify at what frequencies the phase plot is 20^o away from the -180^o mark. At those frequencies find out what gains are required to have the magnitude plot intersect the 0dB mark. To identify whether the system is stable for all such gains find the range of K (graphically) on the root locus plot for which the system is stable. If all such gains lie within this range of K, then the system is stable for all such gains.



Bode Diagrams for 6.30