Problem Set 9 Solutions

Problem 1

The Bode plot of $\frac{500}{s(s+10)(s+100)}$ with phase and gain margins is depicted in Figure 1.

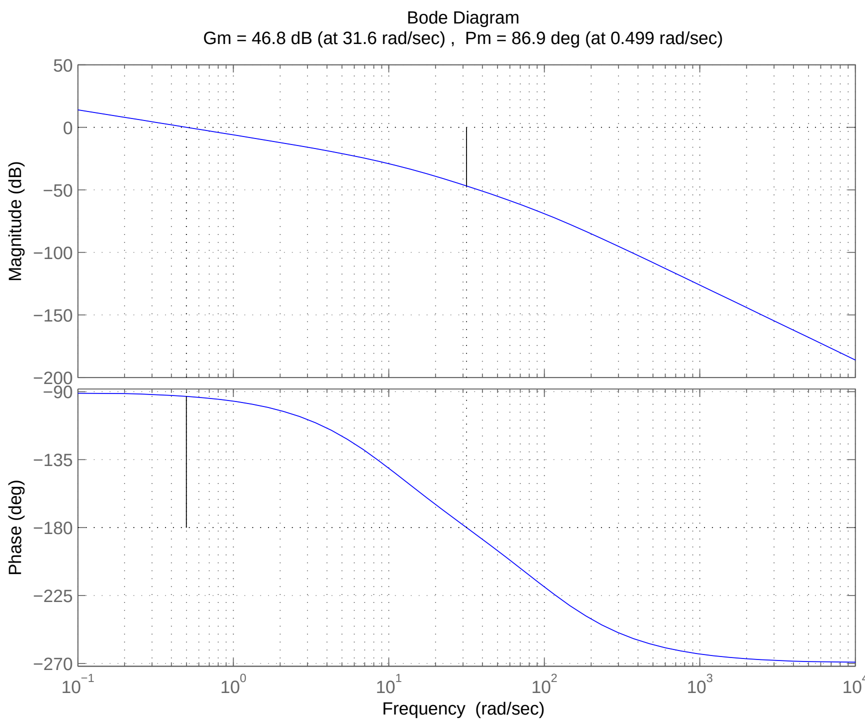


Figure 1: Problem 1, part 1.

The Bode plot of $\frac{100(s+1)}{s(s^2+2s+100)}$ with phase and gain margins is depicted in Figure 2.

The Bode plot of $\frac{100(s+1)(s+10)}{s(s^2+2s+100)}$ with phase and gain margins is depicted in Figure 3. Note that the GM is ∞.

Problem 2

The phase margin of the system is calculated by finding the crossover frequency w_c, i.e. the frequency at which the log magnitude crosses 0dB, and then calculating $\sphericalangle G(jw_c) - (-180)$. By inspection we find that $w_c \cong 30\ rad/sec$ and $PM \cong 80°$.

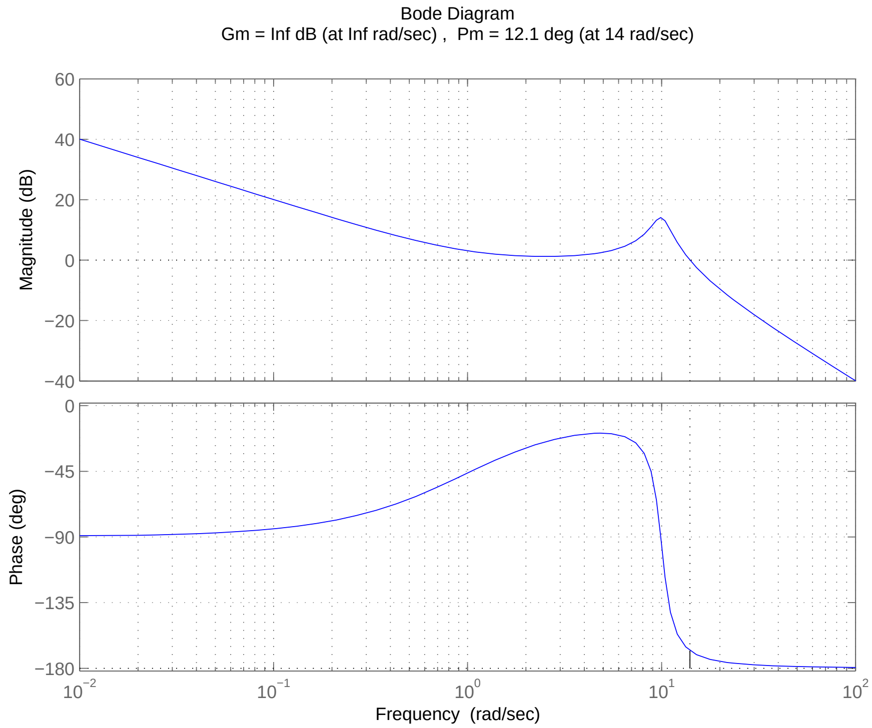


Figure 2: Problem 1, part 2.

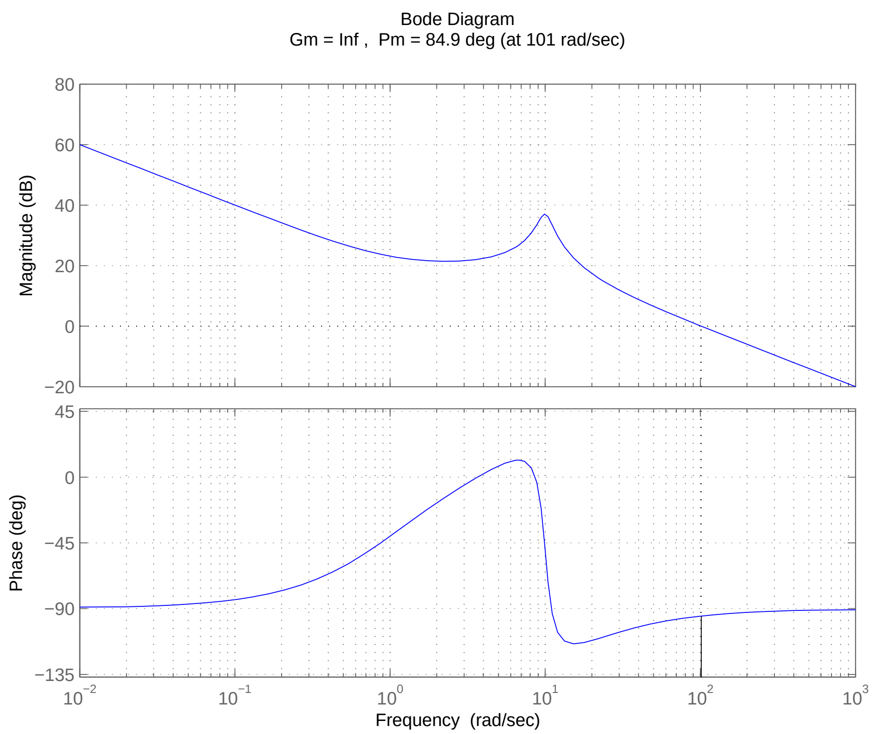
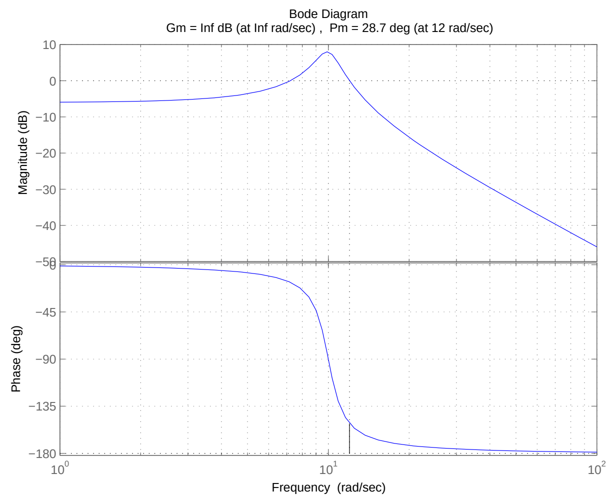


Figure 3: Problem 1, part 3.

Problem 3

We plot the Bode diagrams of the system *before compensation*



and we notice that

$$PM = 28.7^\circ \quad w_c = 12 rad/sec.$$

Note: When the low frequency gain of the frequency response is less than one (i.e., when the magnitude plot at low frequency "starts" from below 10dB), the magnitude plot may have *two* crossover frequencies. This is the case, for instance, in the figure above. In such cases, one takes the phase margin to be the *minimum* between the two phase margins. In this example, the two phase margins corresponding to the crossover frequencies at 7 and 12 rad/sec are 160° and 28.7°, respectively. We thus pick the phase margin to be $\min\{160, 28.7\} = 28.7^\circ$ at 12 rad/sec.