UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATION, April 26, 2010
ECE356 - LINEAR SYSTEMS AND CONTROL
Exam Type: C
Calculator Type: 4
Examiner: M. Broucke

FAMILY NAME ______________________________

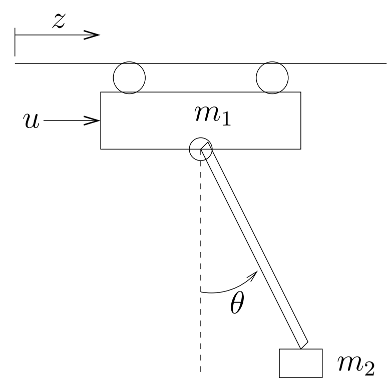
GIVEN NAME(S) ______________________________

STUDENT NUMBER ______________________________

Instructions
No Calculators. Write your solutions clearly in the spaces provided below the problem statements. Use the back sides of the pages for rough work.

Problem	Mark
1	/40
2	/20
3	/20
4	/10
5	/20
Total	/110

1. Consider a hanging crane system depicted below.



The equations of motion, with normalized parameters, are

$$\begin{aligned} \ddot{\theta} + \sin\theta &= -\ddot{z}\cos\theta \\ 2\ddot{z} + \dot{z} + \ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta &= u\,. \end{aligned}$$

Variable z is the position of the top of the crane, u is the force input, and θ is the angle of the load m_2. The control objective is to maintain the load in the downward position $\overline{\theta} = 0$ without swinging.

(a) Assume the state vector is $x = (z, \dot{z}, \theta, \dot{\theta})$ and the output is $y = \theta$. Find a nonlinear state model $\dot{x} = f(x, u)$, $y = h(x, u)$.

(b) Find all equilibrium pairs $(\overline{x}, \overline{u})$ appropriate for the control objective.

(c) Assume $\overline{u} = 0$. Linearize the system in part (a) about the equilibrium $\overline{x}$ in part (b).

(d) Suppose the crane is carrying a heavy load so that the dynamics of the mass m_1 are negligible and $u = \ddot{z}$ is the new input to the system. The equations of motion of the load are:

$$\ddot{\theta} + \sin\theta = -u\cos\theta\,.$$

Assume the state vector is $x = (\theta, \dot{\theta})$. Find a nonlinear state model $\dot{x} = f(x, u)$, $y = h(x, u)$ for the reduced system.

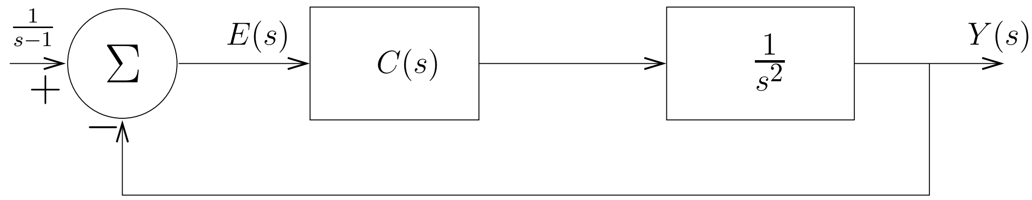
(e) Using the model of part (d), find all equilibrium pairs $(\overline{x}, \overline{u})$ appropriate for the control objective.

(f) Linearize the system in part (d) about the equilibrium in part (e).

(g) Using the model in part (f), design a linear state feedback $\tilde{u} = K\tilde{x}$ to stabilize the equilibrium pair $(\overline{x}, \overline{u})$ such that errors decay according to poles at $\{-1, -1\}$.

(h) Sketch a block diagram of the closed-loop system, assuming the nonlinear model in part (d). Identify in your sketch: the plant, the controller, the sensor, the actuator, and the feedback loop. Also indicate the boundary between processes in the physical world and those on a computer.

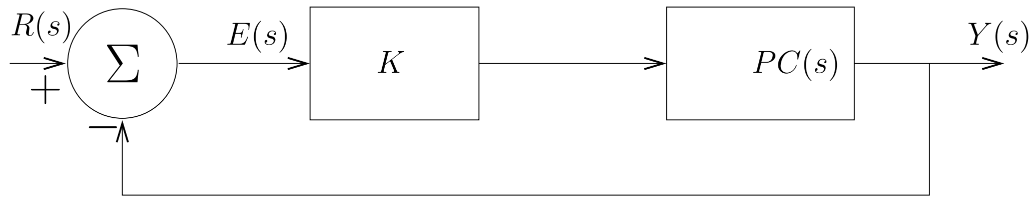
2. Consider the feedback system



(a) Show that if $C(s) = \widehat{C}(s)\frac{1}{s-1}$ and $\widehat{C}(s)$ is selected so that the CLS is stable, then $e_{ss} = \lim_{t\to\infty} e(t) = 0$.

(b) Show that if $C(s)$ is selected so that the CLS is stable and $C(s)$ has no poles at $s = 1$, then $e_{ss} \neq 0$.

3. Consider the unity feedback system



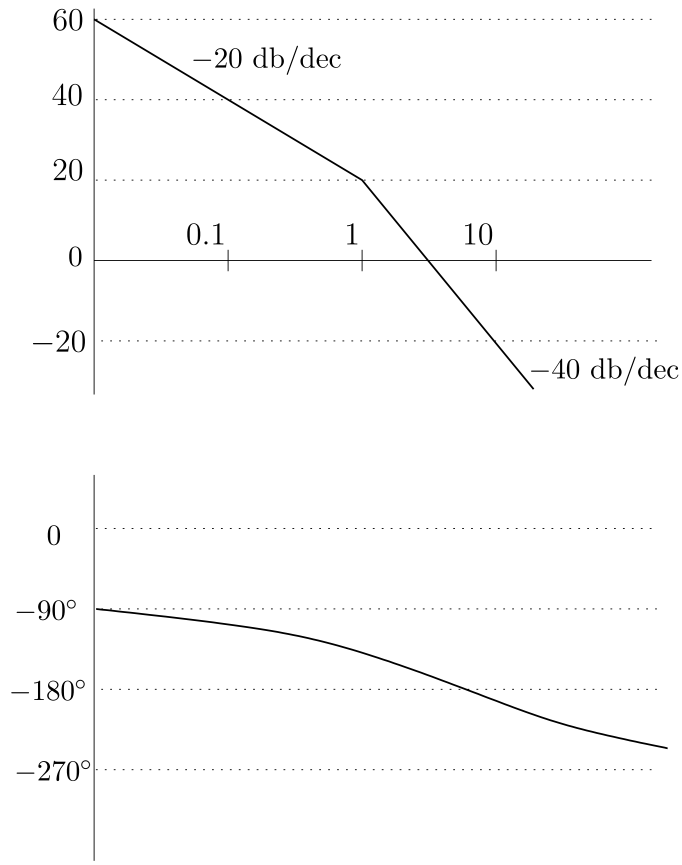
(a) Draw the Nyquist plot for

$$PC(s) = \frac{1}{s}.$$

(b) Use the Nyquist criterion to find the range of K such that the CLS is stable.

(c) Verify your result in part (b) using the Routh Criterion.

4. A stable system $G(s)$ has the following Bode plot



Find a controller $C(s)$ such that:

(i) The closed loop system is asymptotically stable.

(ii) The phase margin of the closed-loop system satisfies $PM > 45°$.

5. Consider the speed dynamics of a motor

$$\dot{x} = -3x + u\,,$$

where x is the state and u is the torque input. The motor speed y is subject to a constant bias whose amplitude is unknown. That is,

$$y = x + c\,,$$

where c represents the constant bias. The control objective is for the motor to track a desired reference speed $y_d(t) = 0$.

(a) Propose an exosystem $\dot{w} = Sw$, $y_d = C_d w$ which models both the disturbance in the output y and the desired reference y_d.

(b) Design an open-loop exact tracking controller $u(t)$ and give plant initial conditions $x(0)$ such that $e(t) = 0$, $t \geq 0$.

(c) Design an asymptotic tracking controller such that $e(t) \to 0$ and error transients decay according to a pole at -10.

(d) Design a robust regulator such that $e(t) \to 0$ and estimator error transients decay according to poles at $\{-30, -30\}$. Your final answer should be in the form of a state equation for the regulator.