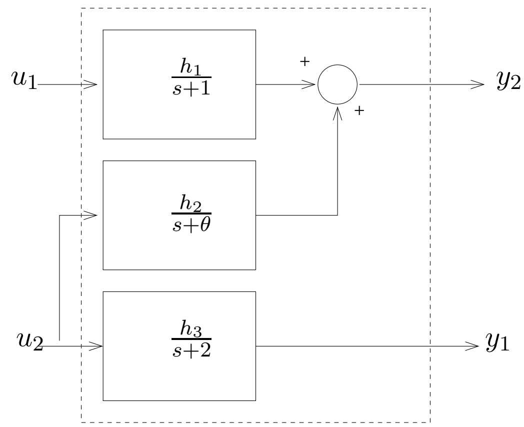
University of Toronto
Department of Electrical and Computer Engineering
ECE410F Control Systems
Problem Set #1

1. Find a state space model of the following system.



2. Find a linearized model about the equilibrium $x = 0$ of the following system:

$$\dot{x} = \begin{bmatrix} -\alpha_1 + \sin x_2 & \sin x_2 \\ \sin x_1 & -\alpha_2 + \sin x_2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u\,, \qquad y = [1\ 0]x\,.$$

where $\alpha_i > 0$ are constant parameters. Next, assume $x(0) = 0$ and let $u(t) = \mathrm{e}^{-3}\overline{\mathrm{u}}(\mathrm{t})$ be applied at $t = 0$, where $\overline{u}$ is a unit step function. Find an approximate solution to the system above.

3. Find a state space model of the system

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+2)} \\ 0 & \frac{1}{s+2} \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}$$

4. The following model approximately describes the interaction of HIV and the CD4+ T cells in the immune system:

$$\begin{aligned} \dot{x}_1 &= c_1 - \frac{c_2 x_2}{c_3 + x_2} - c_4 x_1 - c_5 x_1 x_2 + x_1 u_1 \\ \dot{x}_2 &= \frac{c_6(1-u_2)x_2}{c_7 + x_2} - c_7 x_1 x_2\,, \end{aligned}$$

where x_1 is the concentration of uninfected CD4+ T cells, x_2 is the concentration of free infectious HIV virus particles, u_1 represents drug treatment one, and u_2 represents drug treatment two. Find a linearized model of the system about the equilibrium point obtained when $u_1 = 0$ and $u_2 = 0$. Asssume $c_i = 1$, for $i = 1, \ldots, 7$. Find the eigenvalues of the resultant system. Is the system stable?

5. (a) Determine the transfer function from u to y for the linear system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & -b \\ 1 & -a \end{bmatrix} x + \begin{bmatrix} 1 \\ c \end{bmatrix} u \\ y &= \begin{bmatrix} 0 & 1 \end{bmatrix} x \end{aligned}$$

(b) Repeat for the linear system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 \\ -b & -a \end{bmatrix} x + \begin{bmatrix} c \\ 1 - ac \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} x \end{aligned}$$

and verify that the 2 transfer functions are the same.

(c) The above results show that the choice of state variables is not unique. Indeed, both state representations describe the second order scalar differential equation

$$\ddot{y} + a\dot{y} + by = u + c\dot{u}$$

Determine how the state variables from parts 1 and 2 are defined in terms of y and u.

6. Consider the linear system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 0 & 1 \end{bmatrix} x \end{aligned}$$

(a) Determine its transfer function from u to y, and verify that it is stable.

(b) Show that in general, the total response, which includes the initial condition response, results in the output y increasing exponentially without bound even when the input u is bounded. This shows one must be careful in interpreting results based on input-output transfer functions.

7. Find the Jordan canonical form of

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & -2 & 0 \\ 5 & 0 & -1 \end{bmatrix}$$

8. Consider the autonomous system $\dot{x} = Ax$.

(a) Suppose that $eigs(A) = \{-1, -3, -3, -1 + j2, -1 - j2\}$. Also, suppose the rank of $(A - \lambda I)_{\lambda=-3}$ is 4. Determine Λ, the (complex) Jordan form of A.

(b) Suppose that $eigs(A) = \{-1, -2, -2, -2\}$. Also, suppose the rank of $(A - \lambda I)_{\lambda=-2}$ is 3. Determine Λ, the Jordan form of A.

(c) Suppose that $eigs(A) = \{-1, -2, -2, -2, -3\}$. Also, suppose the rank of $(A - \lambda I)_{\lambda=-2}$ is 3. Determine Λ, the Jordan form of A.