

University of Toronto
Department of Electrical and Computer Engineering
ECE410F Control Systems
Problem Set #3

1. Determine if the following system is observable.

$$\dot{x} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} x, \quad y = [5 \ 6]x.$$

2. Consider the inverted pendulum problem in which the base of the pendulum is movable. Let l be the length of the pendulum, g the gravitational acceleration, and m the mass. An approximate linearized model of the system about the upright equilibrium is

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ \frac{mg}{l} & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{-mg}{l} \end{bmatrix} u.$$

where $x = [y \ \dot{y}]^T$. Determine if this system is controllable and observable.

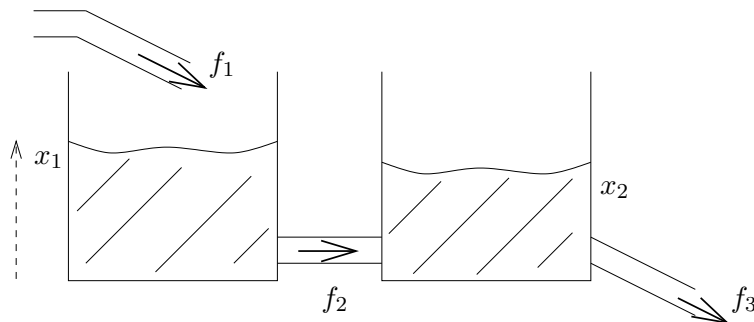
3. Determine if the following system is stabilizable and detectable.

$$\dot{x} = \begin{bmatrix} 100 & 0 & 0 \\ -1000 & -700 & 1000 \\ -700 & -500 & 800 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} u.$$

$$y = \begin{bmatrix} 4 & 2 & -2 \\ -4 & -2 & 4 \end{bmatrix} x.$$

Also, find the minimal realization of the system above.

4. Design an observer for the minimal realization obtained in the previous problem so that the observer is asymptotically stable with poles all equal to -100 .
5. Consider the two tank system shown in the figure below.



Let f_1 be the in-flow rate to the first tank, f_2 , the flow from tank 1 to tank 2, and f_3 the out-flow rate from tank 2. Valves at the in-flow pipe of tank 1 and out-flow pipe of tank 2

allow one to control f_1 and f_3 , respectively. A is the cross-sectional area of each tank. Let x_1 and x_2 be the height of liquid in tank 1 and tank 2, respectively. Write a state space model of the system, assuming that the flow rate between the two tanks is given by:

$$f_2 = \sqrt{2g(x_1 - x_2)}.$$

where g is the gravitation acceleration. Assume that

$$k := \frac{\sqrt{2g}}{A} = 0.26$$

Next, linearize this model about an equilibrium $x_1 - x_2 := H$. Check that the resulting linear system is controllable. Next, design a feedback controller to stabilize (in a neighborhood of the equilibrium point) the height of the tanks at $x_1 - x_2 = 1$. What model will you use for this task?

Returning to the original linear system you obtained, suppose that only x_2 can be measured. Check that the system is observable. Then design an observer to estimate x_1 using only x_2 , f_1 and f_3 . Set the poles of the observer at $s = -2, -2$. Write the final equation of the observer in terms of all the variables.

6. Find a minimal realization of the system

$$\dot{x} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix} x + \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ -1 & 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 \end{bmatrix} x.$$

7. Consider a satellite in a circular equatorial orbit at an altitude of 250 nautical miles above Earth. The satellite motion is described by the normalized state equations

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 3\omega^2 & 0 & 0 & 2\omega \\ 0 & 0 & 0 & 1 \\ 0 & -2\omega & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} u_r + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u_t.$$

The state x is the normalized perturbations about a circular orbit, u_r is the input from a radial thruster, u_t is the input from a tangential thruster, and $\omega = 0.0011 \text{ rad/s}$ is the orbital rate. Is the satellite controllable from the radial thruster alone? Is the satellite controllable from the tangential thruster alone? Is the satellite is controllable using both thrusters?

8. (Matlab) The following model has been proposed to describe the motion of a constant-velocity guided missile:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -0.1 & -0.5 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 \\ 0.5 & 1 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = [0 \ 0 \ 0 \ 1 \ 0]x.$$

- (a) Using the `Matlab` command `eig`, determine if this model is stable.
- (b) Verify that the system is not controllable by using the `Matlab` command `ctrb`.
- (c) Develop a controllable model by decomposing the model into controllable and uncontrollable parts. First do this in the state space. Second do it using a transfer function approach. For the transfer function approach, compute the transfer function from u to y , cancel any common factors in numerator and denominator, and then convert the modified transfer function back to the state space using the `Matlab` command `ss`. Compare the results of your two approaches.
- (d) Verify that the modified state space model obtained in the previous step is controllable.