University of Toronto
Department of Electrical and Computer Engineering
ECE410F Control Systems
Problem Set #4

1. You are given the MIMO system

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u \,.$$

Design a state-feedback controller so that the closed-loop poles are $\{-10, -10, -10\}$.

2. Given

$$A = \begin{bmatrix} 5 & 2 & 1 \\ -2 & 1 & -1 \\ -4 & 0 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} -2 & 2 \\ 2 & -1 \\ 2 & -2 \end{bmatrix}$$

Find a feedback gain K such that the closed-loop poles are assigned to $\{-10, -2 \pm i\}$.

3. Suppose we are given two systems S_1 and S_2 where S_1 is given by

$$\begin{aligned} \dot{x} &= A_1 x + B_1 u \\ z &= C_1 x + D_1 u \end{aligned}$$

and S_2 is given by

$$\begin{aligned} \dot{\xi} &= A_2 \xi + B_2 u \\ w &= C_2 \xi + D_2 u \,. \end{aligned}$$

Let $y = z + w$. Assume the eigenvalues of A_1 and A_2 are disjoint. Show that the overall system S is controllable and observable iff the subsystems are controllable and observable. Next, suppose the spectra of the A_i's are not disjoint. Does the result still hold?

4. Suppose we are given two systems S_1 and S_2 where S_1 is given by

$$\begin{aligned} \dot{x} &= A_1 x + B_1 u_1 \\ y_1 &= C_1 x + D_1 u_1 \end{aligned}$$

and S_2 is given by

$$\begin{aligned} \dot{\xi} &= A_2 \xi + B_2 u_2 \\ y_2 &= C_2 \xi + D_2 u_2 \,. \end{aligned}$$

Suppose they are connected in series so that $u_2 = y_1$, $u = u_1$ and $y = y_2$. When is the overall system controllable and observable?

5. A first order differential equation is given by

$$\dot{x} = x + u\,.$$

A feedback controller is to be designed such that $u(t) = kx$ such that $x = 0$ is a stable equilibrium. The cost function is

$$J = \int_0^\infty x^2 dt\,.$$

Assume that the initial condition is $x(0) = \sqrt{2}$. Obtain the value of k in order to minimize J. Is this k physically realizable?

To account for expenditure of energy and resources, the control input is often included in the cost function. A suitable cost function that includes the effect of the magnitude of the control is

$$J = \int_0^\infty (x^2(t) + u^2(t))dt\,.$$

6. (Matlab) Consider the following model of a 2-degree of freedom vehicle:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -4.4 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ -4.2E^{-3} \\ 0 \\ -2.8e^{-2} \end{bmatrix} u\,, \qquad y = [1\ 0\ 0\ 0]x\,.$$

Design a controller, using an observer, to stabilize the unstable system and to minimize the performance index

$$J = \int_0^\infty \left[y^2(\tau) + \epsilon u^2(\tau) \right] d\tau\,,$$

where $\epsilon = 1e^{-5}$. Simulate the response of the resultant closed-loop system for the initial condition $x0 = (1, 1, 1, 1)$.