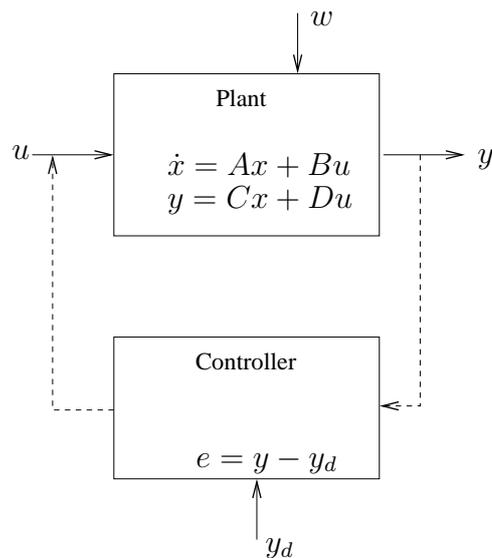


ECE410F – Control Systems
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This course concerns *multivariable* control of linear time invariant (LTI) systems. Multivariable systems are systems with multiple inputs and outputs. The motivation for studying these systems is that most industrial control problems are of this type. What is new from your previous studies of single-input single-output control systems is that we use a *state space* approach.

The basic problem we consider is the following: We are given an open loop system, called the *plant*, which is modelled by an LTI differential equation, and has the following variables: x - the (internal) state variables, y - the output, and u - the input. The control problem is to find a controller u as a function of the system output y such that the error e between the actual output y and the desired output y_d tends to zero. See the Figure below. This problem is called *output regulation*. A special instance of the problem is when $y_d = 0$. This problem is called *output stabilization*. A further special case is when $y = x$, in which case the problem is called simply *regulation* or *stabilization*. We will at times add an additional requirement that the problem be solved in some optimal sense such as minimum control effort. This is the problem of *optimal control design*. Other control requirements that we will consider are that the closed-loop system have desirable properties in its transient response such as fast response to step inputs, no oscillations, little overshoot, etc. Also, one can consider a problem in which the system has disturbances w acting on it. If the disturbance can be modelled, then we may use *disturbance rejection* design or *robust* controllers. If the disturbance is a white noise process, we will learn to design *Kalman filters*.



In mathematical terms, the state space model we consider is:

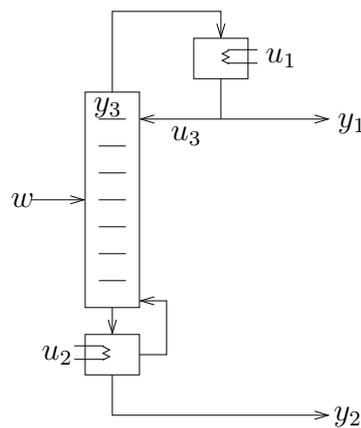
$$\dot{x} = Ax + Bu \tag{1}$$

$$y = Cx + Du, \tag{2}$$

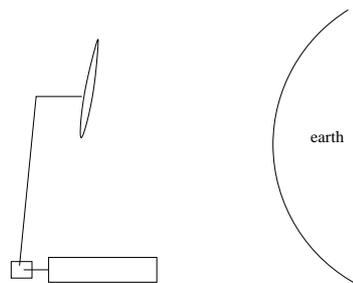
where $x \in \mathbb{R}^n$ is the *state*, $u \in \mathbb{R}^m$ is the *input*, and $y \in \mathbb{R}^p$ is the *output*.

This mathematical model captures a wide array of physical, economic, biological, and management systems. It includes nonlinear systems that are linearized about an equilibrium point. Here are some industrial examples of linear control theory.

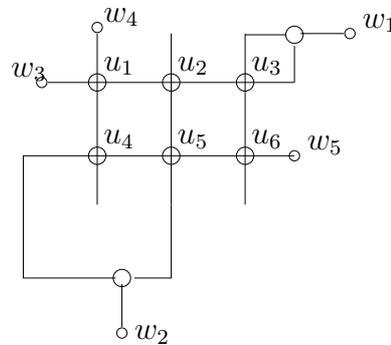
- Distillation column.** The variables are: u_1 , the boiler input, u_2 , the coolant input, u_3 , the reflux flow, y_1 , the top product composition, y_2 the bottom product composition, and y_3 , the pressure. The control problem is to regulate the temperature and pressure along the column so that the desired composition of final products is obtained.



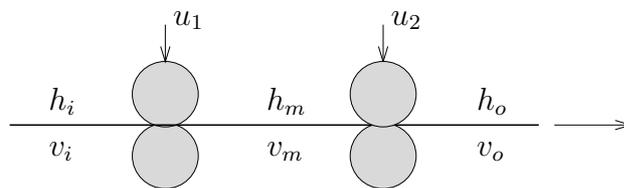
- Large flexible structure.** An example is the MSAT, the Canadian space satellite. The system is highly flexible. It has 11 outputs to be controlled and 9 control inputs with disturbances arising from particle impact, solar flux, non-symmetry of the earth, etc. The control problem is to regulate the position, attitude, and shape of the satellite.



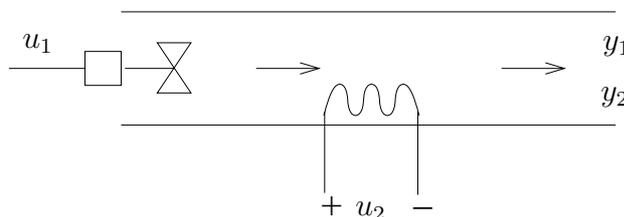
- **Traffic light control.** The objective is to control the length of queues in the N-S and E-W directions. In the figure there are 6 inputs, 12 outputs, and 5 disturbances. In the general case there can be 100's of inputs and outputs.



- **Rolling Mill.** The objective in this application is to regulate the width of the steel to a desired value. The control inputs are: u_1 , the force applied to the first roller, u_2 , the force applied to the second roller, and v_i , the speed of the incoming sheet of steel. The output is h_o , the width of the sheet. Additional states are the intermediate thickness h_m , velocity v_m , and the tensile stress, T .



- **Heat flow (building temperature control).** The objective is to regulate the temperature y_2 of the air and the flow y_1 using the control inputs u_1 the speed of the fan, and u_2 , the voltage applied to the heating element.



Course Outline

The following topics will be covered in the course.

1. **Introduction.**
Modelling, state equations.
2. **Linear Algebra.**
Change of basis, Jordan form, Cayley-Hamilton theorem.
3. **Solution of Linear ODE's.**
Solution of $\dot{x} = Ax$, state transition matrix, modal decomposition, phase portraits of 2D linear ODE's, stability.
4. **Controllability.**
Controllability matrix, invariance under change of basis, PBH test, controllable canonical form.
5. **Pole Assignment.**
Single and multi- input, stabilizability.
6. **Observability.**
Observability matrix, minimal realization, detectability.
7. **Observers.**
Design of observers and observer-based control, Kalman filter.
8. **Linear quadratic optimal control.**
9. **Tracking.**
Servomechanism problem.