University of Toronto
Department of Electrical and Computer Engineering
ECE557F Systems Control
Problem Set #5

1. In Chapter 3, we saw that controllability is unaffected by state feedback. This example shows that observability need **not** be preserved under state feedback.

   (a) Consider the system

   $$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}$$

   where

   $$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

   Show that (C, A) is observable.

   (b) Let

   $$u = Kx$$

   where $K = [0 \ \ 0 \ \ 1]$. Show that the pair $(C, A + BK)$ for the closed loop system is not observable. This shows that observability properties are not preserved under state feedback. Give also an example of an originally unobservable system which becomes observable after state feedback.

2. Construct a minimal order observer for the system

   $$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & \omega & 0 \\ -\omega & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} x \\ y &= x_1 \end{aligned}$$

   with the poles of the observer places at -1 and -2.

3. Design an output feedback compensator for the system with transfer function $\frac{1}{s^2(s+1)}$ such that all the poles of the closed loop system are located at -2. You are to use pole assignment and minimal order observer theory.