**University of Toronto**
**Department of Electrical and Computer Engineering**
**ECE557F Systems Control**
**Problem Set #7**

1. Consider the linear system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x.$$

   (a) Suppose the control objective is closed loop stability and that the output $y$ asymptotically tracks a constant reference set point $r$. Design a full information output regulator which places the closed loop poles at $\{-1, -2\}$ and achieves asymptotic output tracking. Express your controller in terms of $x$ and $r$.

   (b) Now verify that the augmented system formed by plant and exosystem is observable from the output $e = y - r$, and design an error feedback output regulator which ensures that the design objectives are satisfied. In doing so, place the observer poles at $\{-10, -20, -30\}$.

   (c) Determine the closed loop transfer function of the compensator and verify that the internal model principle is satisfied.

2. The Canadian Transportation Agency has contracted you to design a longitudinal controller for an automated snowplow. Let $x_1$ be position, $x_2$ velocity, $u$ force input, $m$ mass, and $k$ viscous friction. A simplified model of the longitudinal dynamics of the snowplow is

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= \frac{1}{m}(-kx_2 + u)\,. \end{aligned}$$

   Suppose $m = k = 1$. The control objective is, starting from rest, bring the snowplow to a velocity of $20m/s$ by tracking a reference position of $p(t) = t^2$, $0 \le t \le 10$.

   Design a tracking controller (exact matching and asymptotic parts) to track $p(t)$, $0 \le t \le 10$, assuming full state information, so that the error between the plant state and exosystem state decays as $\mathrm{e}^{-\mathrm{t}}$ and $\mathrm{e}^{-2\mathrm{t}}$.

3. Suppose that after the snowplow reaches a speed of $20m/s$ it must move at a constant speed thereafter. Design a tracking controller so that the snowplow tracks a constant reference speed of $20m/s$ starting from $t = 10$ sec, with exponential convergence of the tracking error of $\mathrm{e}^{-5t}$.

4. Consider the problem or controlling a platoon of two automated cars moving in a straight line. The leader of the platoon moves at a constant speed and the second car is to follow the leader at a fixed distance $d$. You are to design a controller for the second car. Let $x \in \Re$ denote the position of the second car, and suppose that its control input is its velocity,

$$\dot{x} = u$$
$$y = x.$$

Let $r(t)$ be the position of the leader. The control objective is to achieve $y(t)-r(t)=d$ while the leader moves at a constant speed (i.e., $\ddot{r}(t)=0$). Letting $e(t)=y(t)-r(t)-d$, we want $e(t)\to 0$.

(a) Define an exosystem $\dot{w}=Sw$ generating the class of reference signals we want to track. Hint: the exosystem will have three states.

(b) Find a full-information controller $u=K_1x+K_2w$ making the tracking error converge to zero at a rate $\exp(-5t)$.

(c) Consider the augmented system with state $(x,w)$, and output $e$. Show that this system is unobservable and, in fact, undetectable. Note that the car model *is* observable.

(d) Eliminating the redundancy in the exosystem, design an error feedback output regulator solving the problem. Place the poles of the observer at $-10$.

(e) Find the transfer function of the controller, from $e$ to $u$.

5. Consider the control system

$$\dot{x}=\begin{bmatrix}0&1\\0&0\end{bmatrix}+\begin{bmatrix}0\\1\end{bmatrix}u$$

with state $x\in\Re^2$. The control objective is to make $x$ approach a circle in the plane parametrized as $(\cos t,\sin t)$. In other words, we want

$$(x_1(t),x_2(t))\to(\cos t,-\sin t).$$

(a) Is the problem solvable? If so, find a full information output regulator solving it.

(b) Now assume that the system is affected by a constant disturbance $d$ as follows

$$\dot{x}=\begin{bmatrix}0&1\\0&0\end{bmatrix}+\begin{bmatrix}0\\1\end{bmatrix}u+\begin{bmatrix}1\\0\end{bmatrix}d.$$

Is the problem solvable?