On Area Control Errors, Area Injection Errors, and Textbook Automatic Generation Control

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***Abstract*—We reexamine and critique the classical area control error concept used in automatic generation control, and propose a new error signal termed the area injection error. Unlike the ACE, the AIE uses direct measurement of generator power levels, yielding improved AGC performance in the presence of bias uncertainty and nonlinearity in generator turbine-governor responses. As a by-product of our analysis, we conclude that the effective frequency biasing found in a common textbook implementation of AGC is larger than intended, resulting in unintended interaction between control areas. Our theory is validated via simulations on a detailed two-area test system.**

***Index Terms*—Area control error, automatic generation control, load-frequency control, secondary frequency control**

I. Introduction

Automatic generation control (AGC) is a decentralized control layer in modern interconnected power systems, which adjusts local generation to meet local load within each balancing authority area (control) area. This re-balancing is deliberately slow, and is achieved after several minutes, i.e., after primary control dynamics have settled, but before economic re-dispatch occurs [1]; see [2], [3] for textbook treatments, [1], [4], [5] for surveys, and [6]–[15] for recent contributions.

The key idea enabling the decentralized operation of AGC is the frequency-biased net-interchange concept [16]. Balancing authority k collects local measurements of frequency deviation Δf_k and net interchange deviation $\Delta\mathsf{NI}_k$, and combines them into the *area control error* (ACE)

$$\mathsf{ACE}_k \triangleq \Delta\mathsf{NI}_k + b_k\Delta f_k, \tag{1}$$

where $b_k > 0$ (units MW/Hz) is the *frequency bias* for area k, and is a tuneable gain. Through a low-gain integral controller, generation levels are then slowly adjusted to drive ACE_k to zero. This remarkable control scheme requires no coordination of information between adjacent control areas.

One major design requirement for an AGC system is that it should respect the *non-interaction principle* [4], [17], meaning that the AGC system in area k should not respond to a disturbance occurring in any other area j. Note that primary control response will (and indeed, must) still occur in all areas; non-interaction refers only to the AGC response. The requirement of non-interaction is conventionally targeted through careful tuning of the frequency bias constant b_k in (1). The tuning procedure is based on the following two assumptions:

(A1) there exists a constant of proportionality $\beta_k > 0$ [MW/Hz] (the *frequency response characteristic* (FRC)) between the quasi steady-state frequency deviation in area k and the quasi steady-state active power imbalance in area k;

(A2) β_k can be accurately estimated.

Under (A1)–(A2), textbook analysis [2], [3] then asserts that the tuning $b_k = \beta_k$ leads to non-interactive behaviour.[1] We make two observations concerning (A1)–(A2). First, implicit in (A1) is that primary control systems respond linearly. Put differently, the effects of turbine-governor nonlinearity are neglected. Second, reliable estimates of β_k are difficult to obtain, as the FRC varies with load composition, disturbance size, seasonality, and more [1]; see [18] for more information. As such, some balancing authorities eschew explicit estimation altogether, and simply set b_k at 10% of peak load [18, Page 22]. In the US Eastern Interconnection, it is claimed that this leads to roughly a 100% over-biasing of ACE [18, Page 29] relative to the ideal tuning $b_k = \beta_k$. In sum, there is considerable practical motivation for exploring alternatives to the frequency-biasing concept, which (i) do not implicitly assume turbine-governor linearity, and (ii) rely less heavily on accurate FRC estimation to ensure non-interactive response.

In [15] the author has presented a rigorous dynamic stability and performance analysis of AGC; this letter contains an offshoot of those technical results. The present letter contains two specific contributions. First, we introduce a technical distinction between *exact* and *proxy* error signals for use in AGC, and critically reexamine the classical ACE concept through this lens. In particular, we show that ACE is generally a proxy error signal for the true area imbalance, and that even this conclusion holds only under the assumption of linear turbine-governor droop response. This leads us to propose a measurement-based error signal for use in AGC systems, which we term the *area injection error (AIE)*. Unlike the ACE, which captures generator response though the linear frequency bias term, the AIE is based on direct measurement of generator power injections, and largely eliminates the need for FRC estimation. Second, we leverage our analysis to show that a common textbook AGC implementation [2], [3] is unintentionally overbiased with respect to frequency compensation, causing unintended interaction between areas. We illustrate our results with a simple case study on a full-order test system.

II. Quasi Steady-State Model for AGC

Consider an interconnected power system with N balancing/control areas, labeled as $\mathcal{A} = \{1,\ldots,N\}$. Area $k \in \mathcal{A}$ has

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[1] As we will point out in Section IV however, this conclusion depends on the particular AGC implementation under consideration, and is in fact invalid for at least one textbook implementation.

a set of generators $\mathcal{G}_k$, and the subset $\mathcal{G}_k^{\rm AGC} \subseteq \mathcal{G}_k$ participate in AGC. Each generator $i \in \mathcal{G}_k$ has an electrical power output $P_{k,i}$ and its governor accepts a load reference command $u_{k,i}$, with $u_{k,i}^\star$ denoting the base reference determined by dispatch. For generators not participating in AGC, we of course always have $u_{k,i} = u_{k,i}^\star$, and for notational convenience, we let

$$\Delta u_k \triangleq \sum\nolimits_{j\in\mathcal{G}_k}(u_{k,j} - u_{k,j}^\star) = \sum\nolimits_{j\in\mathcal{G}_k^{\rm AGC}}(u_{k,j} - u_{k,j}^\star)$$

denote the total change made in set-points for generation in area k relative to the economic dispatch point. We let $\Delta f_k = f_k - f_k^\star$ and $\Delta\mathsf{NI}_k = \mathsf{NI}_k - \mathsf{NI}_k^\star$ be measurements of frequency and net interchange deviation from their scheduled values; by definition, NI_k is the net power leaving area k [19].

We will focus on the quasi steady-state which is obtained in the interconnected system *after* the action of primary frequency control [15]. Let $\Delta P_k^{\rm L}$ denote the net load deviation (including losses) in area k. In quasi steady-state, the system is synchronized $\Delta f_1 = \Delta f_2 = \cdots = \Delta f_N = \Delta f_{\rm ss}$ with common deviation $\Delta f_{\rm ss}$. Following NERC, each tie line is metered at a common point [19], which implies that the net interchange deviations $\Delta\mathsf{NI}_k$ satisfy the balance relationship

$$0 = \sum\nolimits_{k\in\mathcal{A}} \Delta\mathsf{NI}_k. \tag{2}$$

The change in power in any control area k is balanced by the primary control action, which we express as

$$0 = \sum\nolimits_{i\in\mathcal{G}_k}(P_{k,i} - u_{k,i}^\star) - D_k\Delta f_k - \Delta P_k^{\rm L} - \Delta\mathsf{NI}_k \tag{3}$$

where $D_k > 0$ is the load damping factor[2] and where $P_{k,i}$ is the electrical power of unit $i \in \mathcal{G}_k$. To incorporate turbine-governor saturation and deadband effects, we allow $P_{k,i}$ to be a generic nonlinear function of $u_{k,i}$ and Δf_k, written as

$$P_{k,i} = h_{k,i}(u_{k,i}, \Delta f_k) \tag{4}$$

for some function $h_{k,i} : \mathbb{R} \to \mathbb{R}$. The form most commonly used in (4) when discussing AGC is the *linear* model

$$h_{k,i}(u_{k,i}, \Delta f_k) = u_{k,i} - \Delta f_k/R_{k,i}, \tag{5}$$

where $R_{k,i} > 0$ is the governor droop constant. Given the linear model (5), the FRC β_k of area $k \in \mathcal{A}$ is given by $\beta_k \triangleq D_k + \sum_{i\in\mathcal{G}_k} R_{k,i}^{-1}$. Finally, the AGC system in area k will operate by integrating some error signal e_k and allocating the resulting control signal to the participating units via

$$\tau_k\dot{\eta}_k = -e_k, \qquad u_{k,i} = u_{k,i}^\star + \alpha_{k,i}\eta_k, \tag{6}$$

where $\tau_k > 0$ is the integral time constant and $\{\alpha_{k,i}\}_{i\in\mathcal{G}_k^{\rm AGC}}$ are nonnegative participation factors which sum to one.

III. Area Control and Injection Errors

The purpose of AGC is to asymptotically match the local net load change $\Delta P_k^{\rm L}$ with an equal change in local commanded generation Δu_k. Unfortunately, $\Delta u_k - \Delta P_k^{\rm L}$ cannot be used as the error signal in (6), since $\Delta P_k^{\rm L}$ is unmeasurable. We now carefully define the ideas of *exact* and *proxy* error signals. We say that a measurable signal e_k is an *exact error signal* for the imbalance $\Delta u_k - \Delta P_k^{\rm L}$ if $e_k = \Delta u_k - \Delta P_k^{\rm L}$, and is a *proxy error signal* if instead we have that $e_k = 0$ if and only if $\Delta u_k - \Delta P_k^{\rm L} = 0$; the latter is a weaker requirement. Note that at any *non-zero* value, a proxy error signal need not equal the true area imbalance.

[2]Typical estimated values for D_k are 1%–2.5% of load [18].

A. The Area Control Error

To begin our analysis in earnest, we ask a fundamental question: is the classical ACE (1) a proxy or exact error signal? Assuming a linear turbine-governor model (5), we may substitute (5) into (3) and rearrange to obtain

$$\Delta u_k - \Delta P_k^{\rm L} = \Delta\mathsf{NI}_k + \beta_k\Delta f_k. \tag{7}$$

Comparing (1) and (7), one can quickly conclude that ACE_k is an exact error signal if and only if b_k can be set equal to β_k. While we omit the details here, it is not difficult to further show that if $b_k \neq \beta_k$, the ACE is a proxy error signal [15, Lemma 2.3]. We draw two important conclusions from this simple calculation. First, note that we used linearity of the turbine-governor response to arrive at (7); the ACE concept crucially relies on linearity. Second — even assuming such a linear turbine-governor droop response — the numerical value of the ACE is *only* physically meaningful if $b_k = \beta_k$. As discussed below (A2), successfully achieving this tuning is extremely challenging in practice.

B. The Area Injection Error

The ACE (1) incorporates both load damping and generator primary control action through the linear response term $b_k\Delta f_k$. If direct measurements of generator output powers are available, then an alternative error signal can be simply constructed which does not assume linearity of the turbine-governor response and does not require FRC estimation. Adding Δu_k to both sides of (3) and simplifying, we obtain

$$\Delta u_k - \Delta P_k^{\rm L} = \Delta\mathsf{NI}_k + D_k\Delta f_k + \sum\nolimits_{i\in\mathcal{G}_k}(u_{k,i} - P_{k,i}). \tag{8}$$

Motivated by (8), we define the *area injection error*

$$\mathsf{AIE}_k \triangleq \Delta\mathsf{NI}_k + d_k\Delta f_k + \sum\nolimits_{i\in\mathcal{G}_k}(u_{k,i} - P_{k,i}). \tag{9}$$

where $d_k > 0$ is a small frequency-bias tuning term.[3] Compared to the ACE (1), the AIE (9) removes the linear turbine-governor droop model embedded in the coefficient b_k, and replaces it with direct measurement of generator output powers $P_{k,i}$; this allows the AIE to account for nonlinear turbine-governor response. The remaining tuning parameter d_k accounts for the (comparatively, small) effect of load damping, and should ideally be set equal to D_k to obtain an exact error signal. Even if load damping D_k is deemed small enough to neglect, we recommend using a small positive value for d_k in practice (e.g., 1% of peak load), as this will improve the robustness of secondary frequency regulation to errors in power measurements. We conclude that, in practice, the

[3]Intermediate ideas between the ACE and AIE are also possible, where the response of some generation units is captured through additional frequency biasing and the response of other units is captured via power measurements; we omit the details.

numerical value of the AIE is likely to be much closer to the true imbalance than the ACE, since the importance of setting an accurate bias factor has been greatly reduced.

IV. An Inconsistency in Textbook AGC

Our general approach can be leveraged to analyze the tuning of a textbook implementation of AGC found in [2, Pg. 354], [3, Pg. 620] (see also [20, Fig. 2(b)], [21, Eq. (8)] [22, Eq. (7)]). This implementation uses the composite error signal

$$\begin{aligned} e_k^{\mathrm{Text}} &= \mathsf{ACE}_k + \sum\nolimits_{i \in \mathcal{G}_k} (u_{k,i} - P_{k,i}) \\ &= \Delta \mathsf{NI}_k + b_k \Delta f_k + \sum\nolimits_{i \in \mathcal{G}_k} (u_{k,i} - P_{k,i}). \end{aligned} \tag{10}$$

which directly sums the ACE with the direct power feedback term. Comparing (10) with (8), we see that (10) is an exact error signal if and only if $b_k = D_k$, which is a substantially lower bias setting than the conventionally suggested tuning $b_k = \beta_k$. Put differently, with the conventional tuning $b_k = \beta_k$, the error signal e_k^{Text} is *unintentionally overbiased* with respect to frequency. This overbiasing induces unintended interaction between the AGC systems of different areas, as illustrated in Scenario #1 of the next section.

V. Case Study on Kundur Test System

We illustrate our results with simulations on the Kundur two-area four-machine test system (Figure 1), as implemented in MATLAB's Simscape Electrical. The model is three-phase and includes full-order machine, turbine-governor, excitation, and PSS models. All four generators have 5% primary governor droop; only generators G1 and G3 participate in AGC. Load damping in the system is negligible.

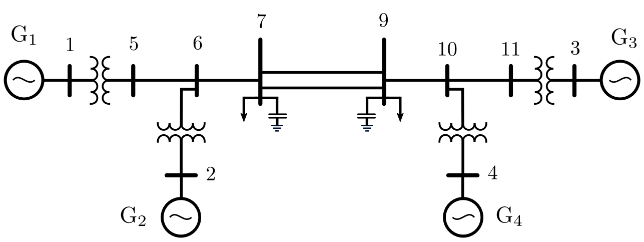
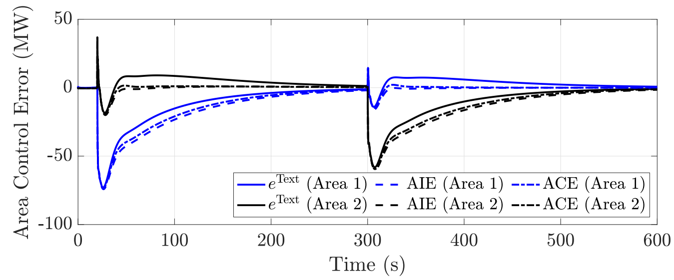


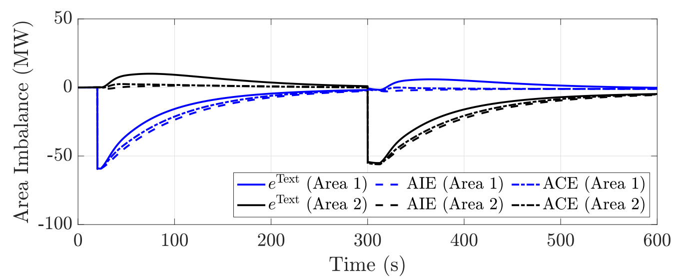
Fig. 1: Two-area four-machine test system.

We will compare the action of the AGC controller (6) for different choices of error signals; in all cases the integral time constants are $\tau_k = 80$s. The disturbance considered in all tests is a 60MW load increase in Area 1 at bus 7 at $t = 10$s, followed by a 60MW load increase in Area 2 at bus 9 at $t = 300$s.

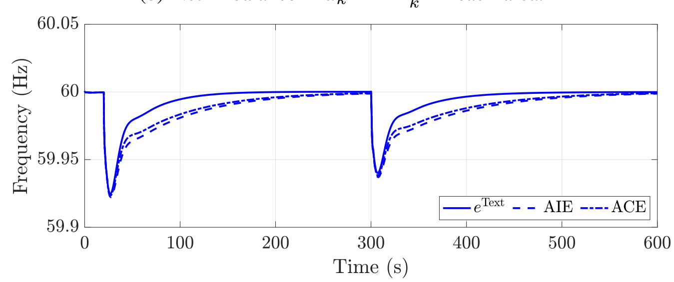
Scenario #1 — Baseline Comparison of control based on e^{Text}, ACE, *and* AIE: We consider three cases, by using each of e^{Text}, ACE, and AIE as the error signal in (6), and simulating the disturbance described above. The bias terms b_k in (1) and (10) are set equal to the exact FRC's $\beta_k = 40$ p.u./p.u. Figure 2 shows the response of the ACE, the area imbalance $\Delta u_k - \Delta P_k^{\mathrm{L}}$, and the system frequency for each case. Note that when the textbook error signal e^{Text} is used, both the ACE (Figure 2(a)) and the commanded power (Figure 2(b)) in Area 2 show significant response to the disturbance in Area 1, and vice-versa, and that this response is sustained even after the action of primary control. Conversely, when one uses the AIE or the "pure" ACE as the error signal, one observes excellent decoupling between area responses. As shown in Figure 2(c), and in agreement with the predictions of Section IV, the textbook controller (10) produces a more aggressive frequency recovery compared to the ACE or AIE-based designs.



(a) Area control errors $\mathsf{ACE}_k = \Delta\mathsf{NI}_k + b_k \Delta f_k$.



(b) Net imbalance $\Delta u_k - \Delta P_k^{\mathrm{L}}$ in each area.

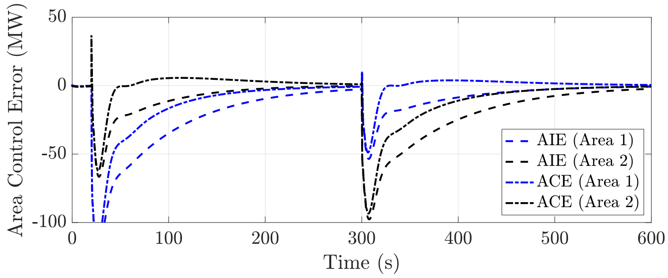


(c) Frequency of generator G1 in Area 1.

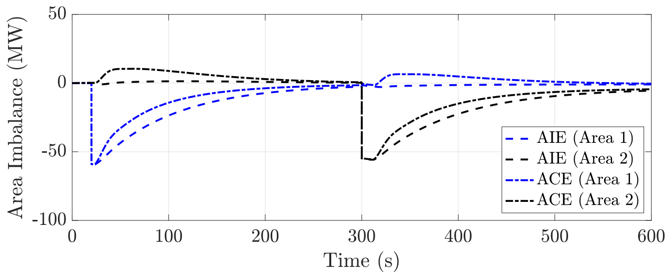
Fig. 2: Response of two-area four-machine Kundur system.

Scenario #2 — Comparing ACE *vs.* AIE *with 100% over-biasing of ACE:* To illustrate the benefit of the AIE (9) being a measurement-based error signal which does not rely on FRC estimation, we compare the response of the two control schemes when the bias values used in the ACE have been over-estimated by 100%, and set $b_1 = b_2 = 80$ p.u./p.u; in this case. The results are plotted in Figure 3; Note that the ACE is a very poor quantitative measure of the true imbalance. In Figure 3(b), the ACE controller displays significant interaction between the two control areas, while the AIE-based controller retains the desired non-interactive behaviour.

Scenario #3 — Comparing ACE *vs.* AIE *with turbine-governor nonlinearity:* As discussed in Section III, the ACE implicitly assumes a linear turbine-governor response, while the AIE does not. To clearly illustrate the effect of this, we use the ideal bias settings $b_k = \beta_k$ for the ACE-based controller, but we incorporate a frequency deadband of 36mHz in all turbine-governor systems. For clarity, we apply only the first



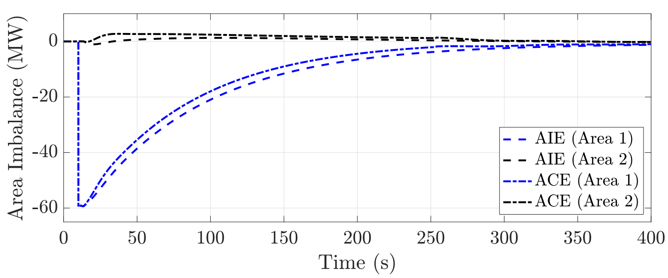
(a) Area control errors $\mathsf{ACE}_k = \Delta\mathsf{NI}_k + b_k \Delta f_k$.



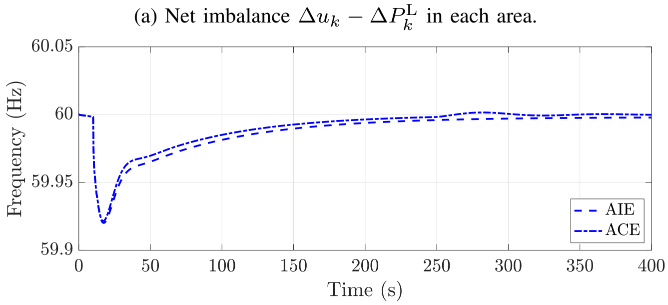
(b) Net imbalance $\Delta u_k - \Delta P_k^{\mathrm{L}}$ in each area.

Fig. 3: Response of Kundur system with 100% overbiasing.

disturbance to Area 1, and the results are plotted in Figure 4. Since the measurement-based AIE scheme accounts for the new nonlinearity, the corresponding response displays a slight improvement in non-interaction compared to the ACE scheme.



(a) Net imbalance $\Delta u_k - \Delta P_k^{\mathrm{L}}$ in each area.



(b) Frequency of generator G1 in Area 1.

Fig. 4: Response of Kundur system with governor deadband.

VI. Conclusions

This letter has analyzed and critiqued the area control error concept and proposed a new measurement-based error signal called the area injection error for use in AGC systems. The AIE eliminates much of the need for difficult bias tuning, and accounts for turbine-governor nonlinearity. As a by-product, our analysis shows that an implementation of AGC found in standard textbook references is overbiased with respect to frequency compensation, and all theoretical results are confirmed via simulation. Future work will rigorously quantify the dynamic performance of AIE vs. conventional ACE-based control, and will seek to incorporate voltage-frequency coupling into the analysis and design of AGC systems.

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