# ECE1659H Robust and Optimal Control Project

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## **Project Description**

Each student must complete a class project related to robust and/or optimal control. As the project is intended to accommodate a wide variety of student interests and personal learning goals, there are no specific requirements on the style/nature of the investigation. Research students are strongly encouraged to apply course concepts (or related ideas in the literature) to their thesis research. Possible project styles include

- (i) a review of a technical paper, including derivation/proof of key results and reproduction of simulations,
- (ii) an application case study, applying course results or related results in the literature to a practical example, or
- (iii) an attempt at a novel research project, applying course content in a new context or to your thesis research topic.

Unless your project attempts to develop *novel* theoretical results, it *must* contain a simulation component. Broadly, the goals of the project are for you to

- (i) learn with breadth, by researching a topic outside the immediate scope of the lecture material,
- (ii) learn with depth, by digging into the details for your topic of interest, and
- (iii) develop your technical presentation skills, by conveying what you have learned to your colleagues in an oral presentation with a polished presentation deck.

#### **Project Deliverables**

There are two deliverables for the project; due dates will be posted on the course homepage.

- (i) (15% of project grade) A proposal, maximum 3/4 of a page, giving a brief description of the topic you will investigate. Your proposal must
  - state the nature of your particular project (e.g., a technical paper review, a case study, an attempt at original research, etc.)
  - explain how the proposed project broadly relates to the material covered in the course, and
  - clearly outline the what you will produce as project deliverables (e.g., review of problem formulation, review of key results, extension to some other scenario of interest, testing the method via simulations, etc.).

Your proposal mark will be based on the quality and clarity of your first submitted proposal.

You **must** receive explicit approval from the instructor before proceeding with your project, or you will receive a mark of zero. A revision to your proposal may be required before approval is granted. If you wish to change your project during the course of the work, you **must** prepare a new proposal and receive explicit approval from the instructor, or you will receive a mark of zero.

- (ii) (85% of project grade) A 25 minute presentation, plus 5 minutes for questions from the audience. The primary purpose of the presentation is for you to demonstrate that you have developed an expert-level understanding of the material you have chosen for your project, and that you have carefully considered how to best communicate your understanding and educate your colleagues on this topic. This implies that
  - you must do the individual work of developing your own understanding of the material, and
  - you must externalize this knowledge in your presentation by deciding what are the key concepts, what is important to communicate in 25 minutes and what is not, how should you organize the presentation for maximal educational value to others, and so on.

The presentation will be evaluated based on (a) the slides themselves, and (b) your presentation of the slides. Evaluation criteria may include

- Clarity of problem motivation and description
- Placement of project within literature and connection to course material
- Clarity of main technical results / takeaways
- Evidence of independent work (e.g., demonstration of developed understanding and insight, construction of simulation models, plots showing testing of controllers, etc.)
- Quality, quantity, and appropriate use of visual aids for explaining results
- Appropriateness of quantity of material presented, timing of presentation, etc.
- Typesetting, formatting, organization, finishing on time without rushing, etc.
- Is the overall organization of the material effective and did the audience learn something?

Attendance and active participation for other student presentations is required.

#### **Project Topics**

#### A Universal Decomposition for Distributed Optimization Algorithms

As a rule of thumb, if your proposed project contains any of the major course key words (robust, optimal, performance, LMI, SDP) then it is eligible. I strongly encourage you to do a bit of digging and find something you are excited to learn about or work on. I reserve the right to reject any project proposal that I feel is too close in scope to the core course material or to another project.

Below, potential papers or topics (mostly recent) for review are listed, but you are **not** restricted to these topics or references.

1. Learning/perception and robust control [1, 2, 3, 4, 5]

- 2. Data-driven control [6, 7, 8, 9, 10, 11]
- 3. Numerical aspects of LMI computation for systems and control [12]
- 4. System Level Synthesis [13, 14, 15, 16]
- 5. Linear parameter-varying (LPV) systems and gain-scheduled control [17, 18]
- 6. Design of tracking and optimizing controllers [19, 20, 21, 22]
- 7. Large-scale systems analysis [23]
- 8. Theory of integral quadratic constraints [24, 25, 26, 27, 28, 29, 30]
- 9. SDP duality theory in control [31, 32, 33]
- 10. Unfalsifiable control [34]
- 11. Fragility of optimal controllers [35]
- 12. Optimization and game theory [36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49]
- 13. Neural network control: [50, 51]
- 14. Conic-sector and SPR control synthesis: [52, 53]

Other ideas: Autonomous vehicles, Soft robotics, System identification,  $\mathcal{L}_1/\ell_1$  optimal control, estimator design, local stability analysis, robustness in biological systems (e.g., M. Khammash, J. Doyle, F. Doyle, R. Murray, ...), Sum-of-squares SOS programming (e.g., Anderson, Papachristodoulou, Peet, Parillo, ...), hybrid systems, ...

### Useful Documents and Templates

My suggestion is to use  $IAT_EX$  for producing documents and presentations which include mathematics, but I leave the decision to you for what software to use. The following may be useful to you:

- IEEE-style conference templates for MS Word and LATEX
- Template for Powerpoint presentations
- Tutorial on  $IAT_EX$
- Tutorial on Beamer for  ${\rm I\!AT}_{\rm E\!X}$  presentations
- Advice on giving presentations
- Advice on reading technical research papers

## References

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