Distributed Frequency and Voltage Control of Islanded Microgrids

John W. Simpson-Porco, Florian Dorfler and Francesco Bullo

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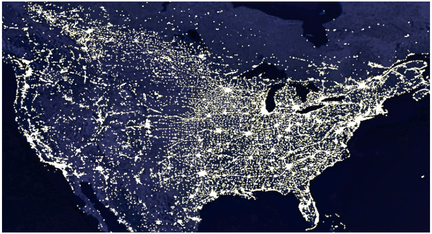
University of California, Santa Barbara

Pacific Northwest National Laboratory

March 23, 2015

Electricity & The Power Grid



- **Electricity** is the foundation of technological civilization
- **Hierarchical** grid: generate/transmit/consume
- **Challenges**: multi-scale, nonlinear, & complex

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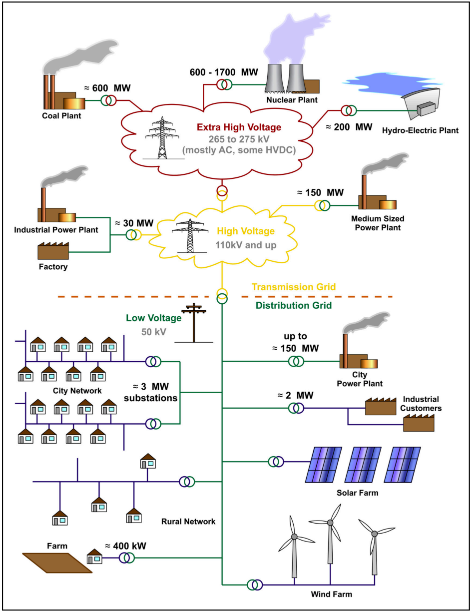

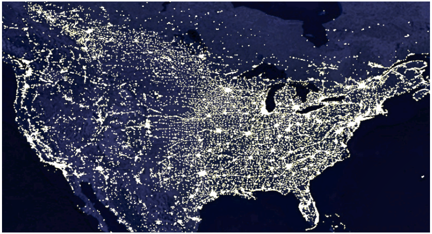

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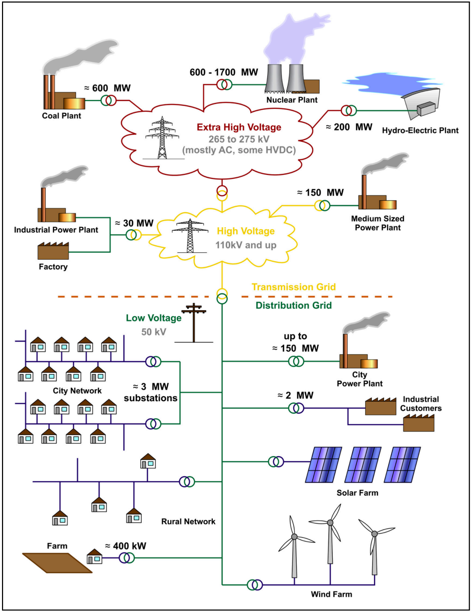

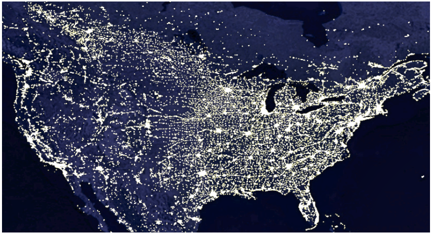

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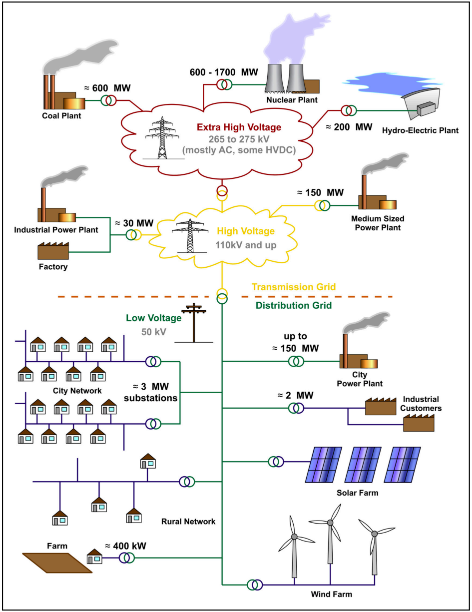

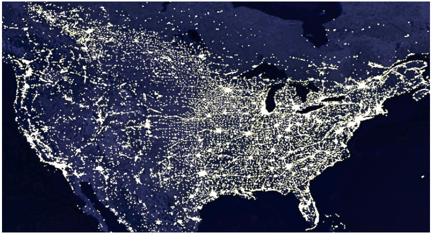

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What are the control strategies?

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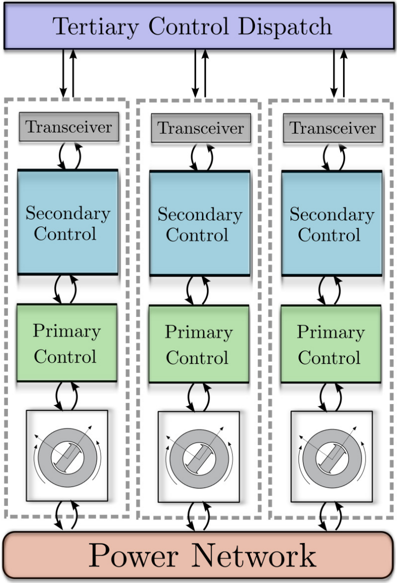
Bulk Power System Control Architecture & Objectives

Hierarchy by physics and spatial/temporal/centralization scales



3. **Tertiary control** (offline)
   - Goal: optimize operation
   - Strategy: centralized & forecast
2. **Secondary control** (minutes)
   - Goal: maintain operating point
   - Strategy: centralized
1. **Primary control** (real-time)
   - Goal: stabilization & load sharing
   - Strategy: decentralized

Q: Is this layered & hierarchical architecture still appropriate for tomorrow's power system?

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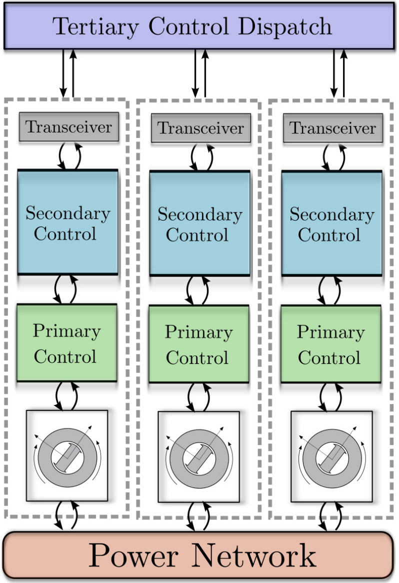

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Two Major Trends

(New York Magazine)

Trend 1: Physical Volatility

1. ~~bulk~~ distributed generation, (de)regulation
2. growing demand & old infrastructure

⇒ lowered inertia & robustness margins

Trend 2: Technological Advances

1. flexible loads, sensors & actuators (spinning reserves, PMUs, FACTS)
2. control of cyber-physical systems

⇒ cyber-coordination layer for smart grid

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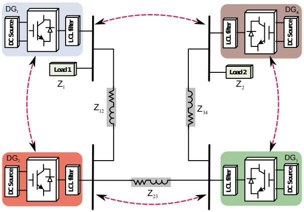
(Electronic Component News)

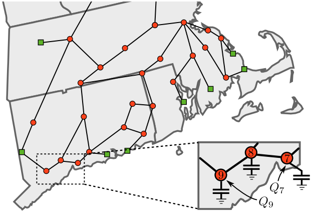
Outline

Smart Grid Project Samples

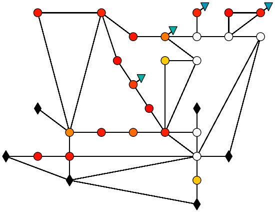
Cooperative Inverter Control



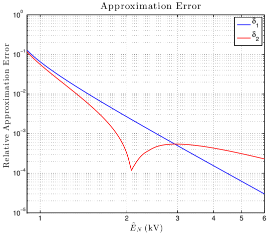
Voltage Stability/Collapse



Optimal/Sparse Voltage Support



Power Flow Approximations



Relevant Publications

J. W. Simpson-Porco, F. Dörfler, and F. Bullo. Voltage Collapse in Complex Power Grids. February 2015. Note: Submitted.

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F. Dörfler, J. W. Simpson-Porco, and F. Bullo. Breaking the Hierarchy: Distributed Control & Economic Optimality in Microgrids. *IEEE Transactions on Control of Network Systems*, January 2014. Note: Submitted.

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Microgrids

Structure

- low-voltage distribution networks
- small-footprint & *islanded*
- autonomously managed

Applications

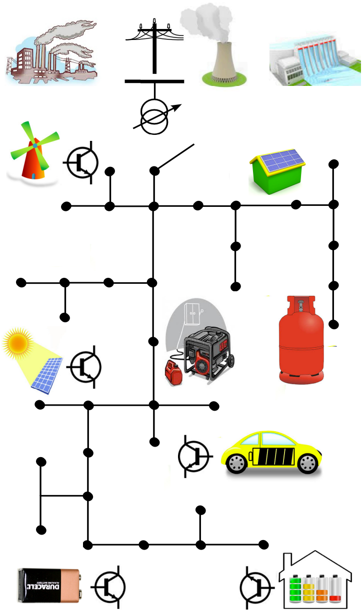
- hospitals, military, campuses, large vehicles, & isolated communities

Benefits

- naturally distributed for renewables
- scalable, efficient, & reliable

Operational challenges

- fast dynamics & low inertia
- plug'n'play & no central authority



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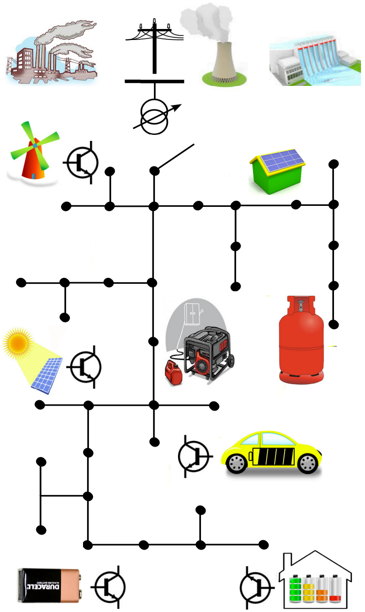
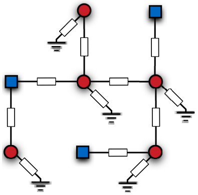


Modeling I: AC circuits



1. **Loads (●) and Inverters (■)**
2. **Quasi-Synchronous:** $\omega \simeq \omega^* \Rightarrow V_i = E_i e^{j\theta_i}$
3. **Load Model:** ZIP Loads (today, constant power)
4. **Coupling Laws:** Kirchoff and Ohm
5. **Identical Line Materials:** $R_{ij}/X_{ij} = \text{const.}$ (today, lossless $R_{ij}/X_{ij} = 0$)
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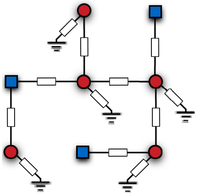

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- active power: $P_i = \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j)$
- reactive power: $Q_i = -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j)$

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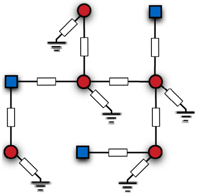

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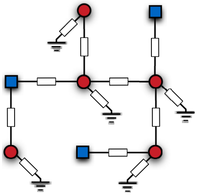

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- trigonometric active power flow: $P_i(\theta) = \sum_j B_{ij} \sin(\theta_i - \theta_j)$
- quadratic reactive power flow: $Q_i(E) = -\sum_j B_{ij} E_i E_j$

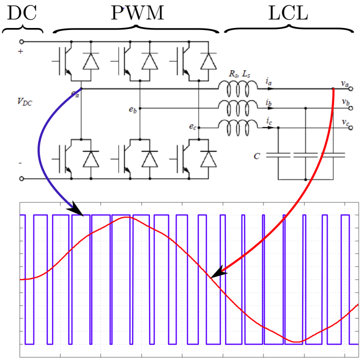
Modeling II: Inverter-interfaced distributed gen.

also applies to frequency-responsive loads

Power **inverters** are . . .

- **interface** between AC grid and DC or variable AC sources

- operated as **controllable** ideal voltage sources



Assumptions:

- Fast, stable inner-loops (voltage/current/impedance)

- Balanced 3-phase operation

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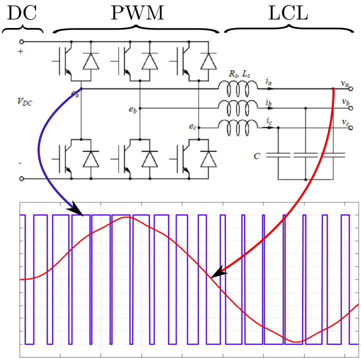
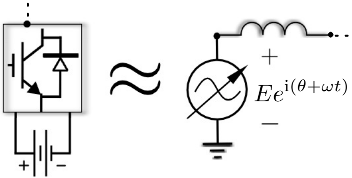
$$\omega_i = u_i^{\text{freq}}\,, \quad \tau_i \dot{E}_i = u_i^{\text{volt}}$$





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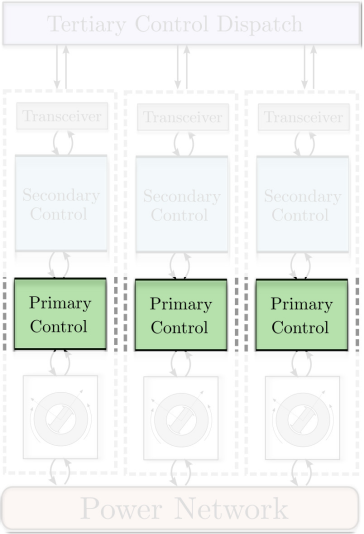

Tertiary Control Dispatch
Transceiver
Transceiver
Transceiver
Secondary Control
Secondary Control
Secondary Control
Primary Control
Primary Control
Primary Control
Power Network

Open-Loop System & Control Objectives

Frequency Open-Loop	**Voltage Open-Loop**
Inverter Dynamics:	Inverter Dynamics:
$\omega_i = \dot{\theta}_i = u_i^{\mathrm{freq}}$ $P_i(\theta) = \sum_j B_{ij} \sin(\theta_i - \theta_j)$	$\tau_i \dot{E}_i = u_i^{\mathrm{volt}}$ $Q_i(E) = \sum_j B_{ij} E_i E_j$
Load Active Power Balance:	Load Reactive Power Balance:
$0 = P_i^* - \sum_j B_{ij} \sin(\theta_i - \theta_j)$	$0 = Q_i^* - \sum_j B_{ij} E_i E_j$

Primary Control Objectives:

1. **Stabilization:** Balance system for variable loads
2. **Load Sharing:** Power injection proportional to unit capacity

Decentralized Primary Control (aka Droop Control)

A grid-forming control strategy

Key Idea: emulate self-organizing generator dynamics

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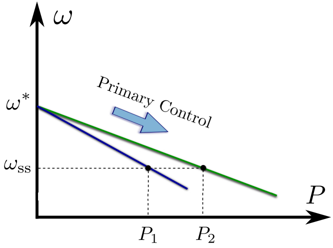
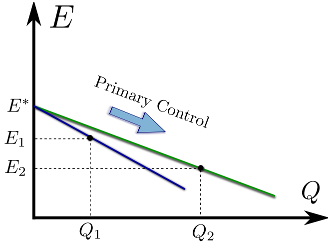
Frequency Droop Control	**Voltage Droop Control**
$\omega_i = \omega^* - m_i P_i(\theta)$	$\tau_i \dot{E}_i = -(E_i - E^*) - n_i Q_i(E)$





Spring Network Interpretations of Equilibria

Frequency Droop Control	Voltage Droop Control
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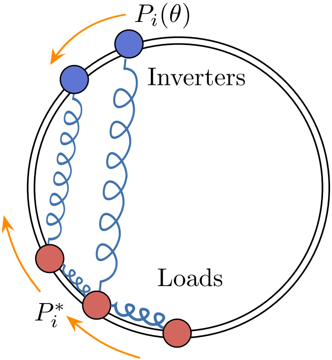

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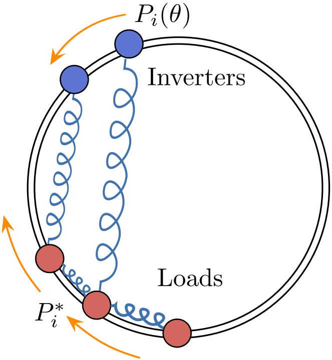

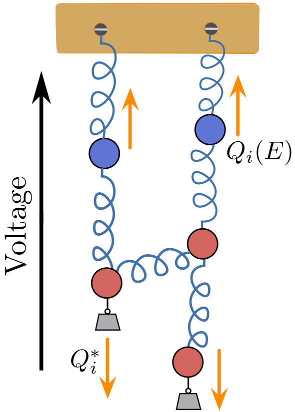

Droop Control Stability Conditions

Frequency Droop Control	**Voltage Droop Control**
$0 = P_i^* - \sum_j B_{ij} \sin(\theta_i - \theta_j)$ $\dot{\theta}_i = -m_i \sum_j B_{ij} \sin(\theta_i - \theta_j)$	$0 = Q_i^* - \sum_j B_{ij} E_i E_j$ $\tau_i \dot{E}_i = -(E_i - E_i^*) - n_i \sum_j B_{ij} E_i E_j$
Theorem: Frequency Stability (J. Simpson-Porco, F.D., & F.B., '12) ∃! loc. exp. stable angle equilibrium θ_{eq} iff $\frac{(\text{edge power flow})_{ij}}{B_{ij}} < 1$ for all branches of microgrid.	Theorem: Voltage Stability (J. Simpson-Porco, F.D., & F.B., '14) ∃! loc. exp. stable voltage equilibrium point E_{eq} if $\frac{4 \cdot \text{load} \cdot (\text{impedance})}{(\text{nominal voltage})^2} < 1$ for all load buses of microgrid.

nec. and suff. | **suff. and tight**

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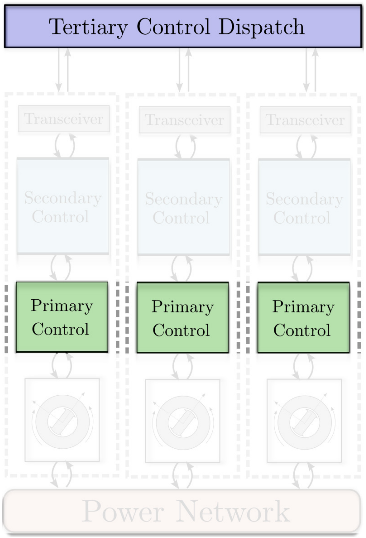

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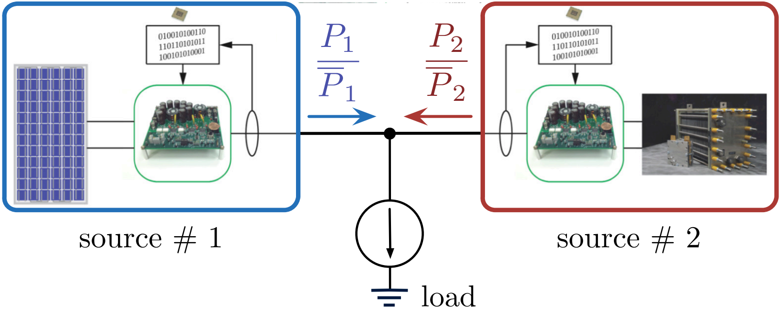
Objective I: decentralized proportional load sharing

1) Inverters have **injection constraints**: $0 \leq P_i(\theta) \leq \overline{P}_i$

2) Load must be **serviceable**: $0 \leq \left| \sum_{\text{loads}} P_j^* \right| \leq \sum_{\text{inverters}} \overline{P}_j$

3) **Fairness:** load should be shared proportionally: $P_i(\theta) \,/\, \overline{P}_i = P_j(\theta) \,/\, \overline{P}_j$



Objective I: decentralized proportional load sharing

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2) Load must be **serviceable**: $0 \leq \left| \sum_{\text{loads}} P_j^* \right| \leq \sum_{\text{inverters}} \overline{P}_j$

3) **Fairness:** load should be shared proportionally: $P_i(\theta) \,/\, \overline{P}_i = P_j(\theta) \,/\, \overline{P}_j$

Theorem (Load Sharing) [J. Simpson-Porco, FD, & F. Bullo, '12]

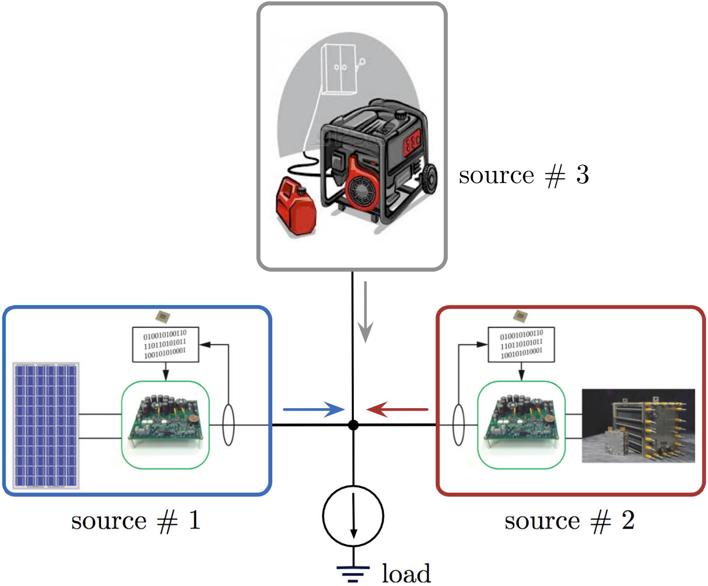
If we select the controller gains such that $\boxed{m_i \overline{P}_i = m_j \overline{P}_j}$, then

(i) Proportional load sharing: $P_i(\theta) \,/\, \overline{P}_i = P_j(\theta) \,/\, \overline{P}_j$

(ii) Constraints met: $0 \leq P_i(\theta) \leq \overline{P}_i$

What if we don't like "sharing"?

proportional load sharing is not always the right objective



Objective II: optimal economic dispatch

minimize the total accumulated generation

$$\text{minimize}_{\theta \in \mathbb{T}^n,\, u \in \mathbb{R}^{n_I}} \quad f(\theta) = \frac{1}{2} \sum_{\text{inverters}} \alpha_i [P_i(\theta)]^2$$

subject to

load power balance: $0 = P_i^* - P_i(\theta)$

branch flow constraints: $|\theta_i - \theta_j| \leq \gamma_{ij} < \pi/2$

inverter injection constraints: $P_i(\theta) \in \left[0, \overline{P}_i\right]$

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Conventional: Offline, Centralized, Model & Load Forecast

Autonomous Microgrid: On-line, decentralized, no model, no forecasts

Objective II: decentralized economic dispatch optimization

Insight: droop-controlled microgrid $=$ decentralized primal algorithm

Dispatch through droop control [H. Bouattour, FD, J. Simpson-Porco, & F. Bullo, '13]

The following statements are equivalent:

(i) econ. dispatch with cost coeffs. α_i is strictly feasible w/ global minimizer θ^*;

(ii) $\exists$ droop coefficients m_i s.t. the microgrid possesses a unique & loc. exp. stable operating point θ^* satisfying $P_i(\theta^*) \in [0, \overline{P}_i]$.

If (i) & (ii) are true, then $P_i(\theta^*) = (\omega^* - \omega_{ss})/m_i$, & $\boxed{\dfrac{\alpha_i}{m_i} = \dfrac{\alpha_j}{m_j}}$.

- similar results for **constrained** case — though not fully decentralized

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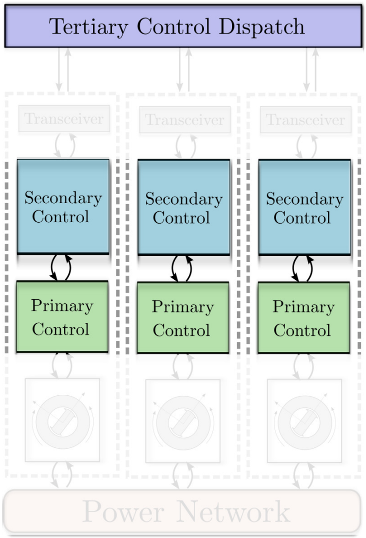
Tertiary Control Dispatch
Transceiver
Transceiver
Transceiver
Secondary Control
Secondary Control
Secondary Control
Primary Control
Primary Control
Primary Control
Power Network

Secondary frequency control in power networks

Problem: steady-state frequency deviation ($\omega_{\mathrm{ss}} \neq \omega^*$)

Solution: integral control on frequency error

Secondary frequency control in power networks

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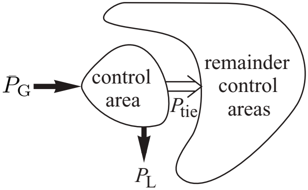
Interconnected Systems	Isolated Systems
• **Centralized** automatic generation control (AGC)	• **Decentralized** PI control



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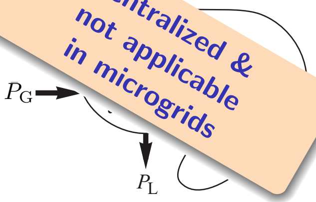

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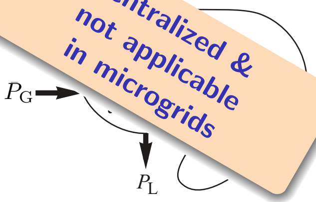

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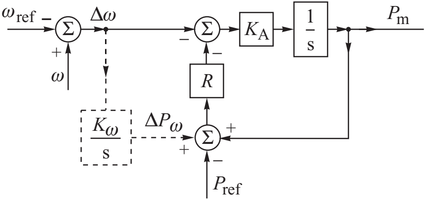

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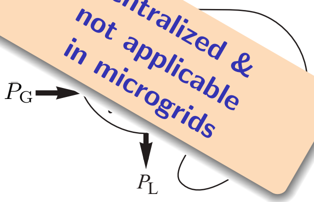

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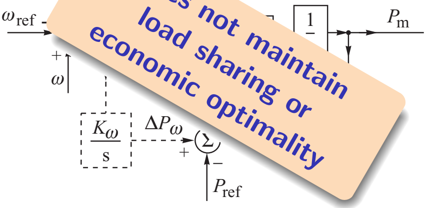

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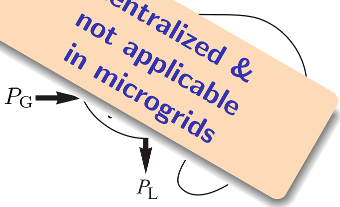

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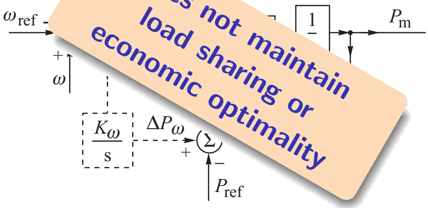

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Microgrids require **distributed** (**!**) secondary control strategies.

Distributed Averaging PI (DAPI) Frequency Control

$$\omega_i = \omega^* - m_i P_i(\theta) - \Omega_i$$

$$k_i \dot{\Omega}_i = (\omega_i - \omega^*) - \sum_{j \subseteq \text{inverters}} a_{ij} \cdot (\Omega_i - \Omega_j)$$

1. no tuning, no model dependence
2. weak comm. requirements
3. preserves optimal dispatch

Simple & Plug'n'play

Theorem: Stability of DAPI

[J. Simpson-Porco, FD, & F. Bullo, '12]

DAPI Stable

$\Updownarrow$

Primary Droop Stable

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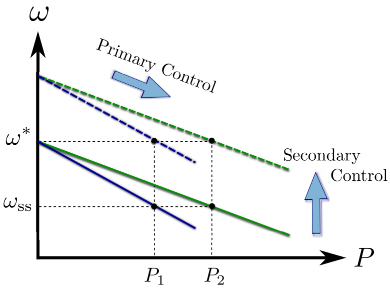

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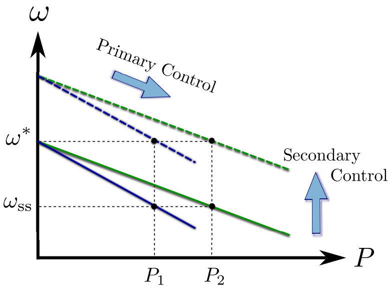

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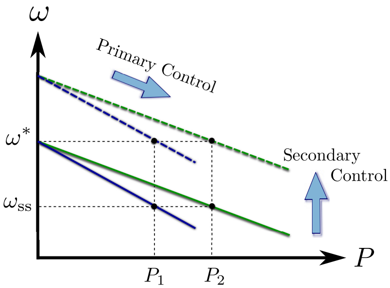

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Distributed Averaging PI (DAPI) Voltage Control

Goals: Voltage regulation $E_i \to E^*$, load sharing $Q_i/Q_i^* = Q_j/Q_j^*$

Bad News: Unlike P/ω, these goals are *fundamentally* conflicting.

Key Idea: Trade-off between voltage regulation / Q-Sharing

$$\tau_i \dot{E}_i = -(E_i - E^*) - n_i Q_i(E) - e_i$$

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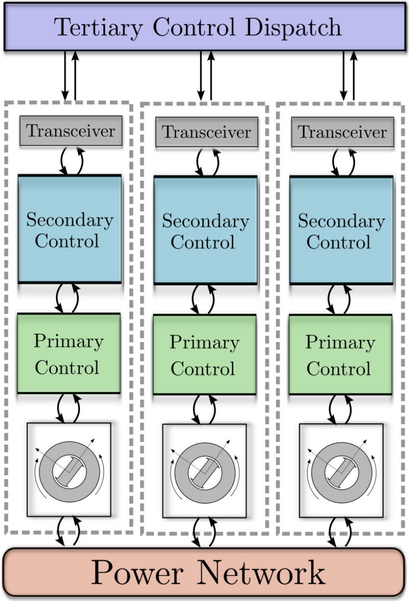
Plug'n'play architecture

flat hierarchy, distributed, no time-scale separations, & model-free



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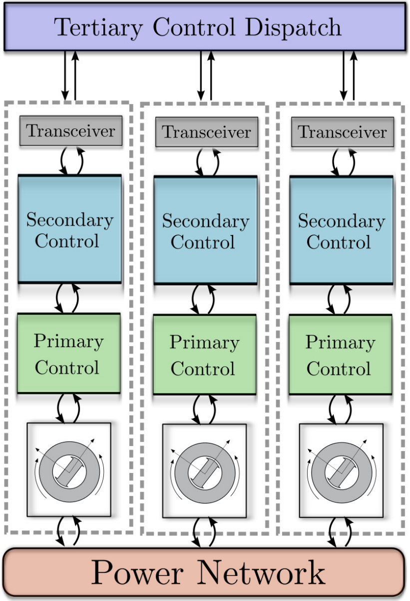
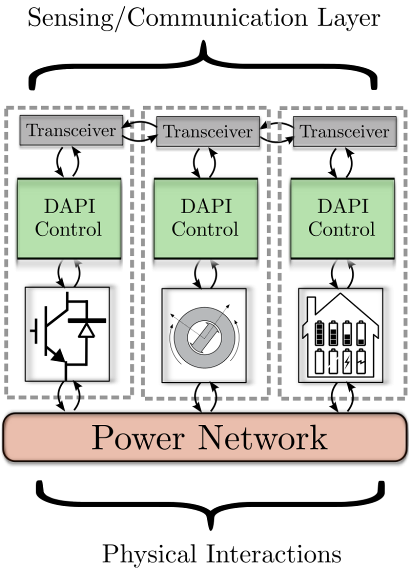
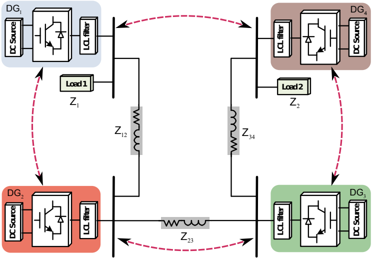




Experimental Validation of DAPI Control

Experiments @ Aalborg University with Q. Shafiee, J. C. Vasquez & J. M. Guerrero



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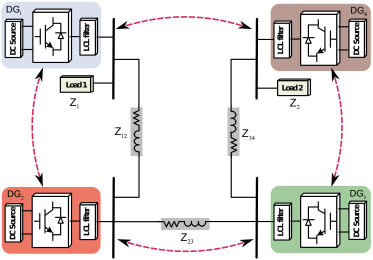
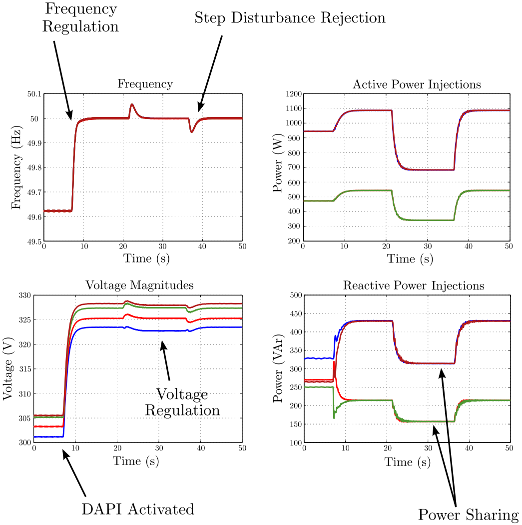
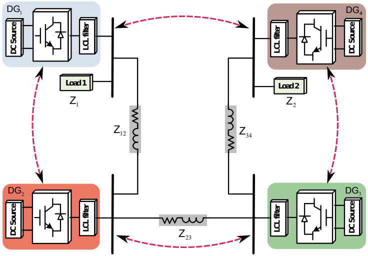




Ongoing Theoretical and Practical Challenges

1. Interaction w/ **price dynamics**
2. Cyber-security in DAPI control
3. **Performance limits** of decentralized control
4. Large-scale study w/ NS-3 comm. & more detailed load models

Summary



Distributed Inverter Control

- Primary control stability
- Distributed PI controllers
- Primary/tertiary connections
- Experiments: "It works. Really."

More Results (not shown)

- More voltage control/opt.
- Accurate approximations