# Distributed Control of Inverter-Based Power Grids

*Workshop on Communications, Computation and Control for Resilient Smart Energy Systems*

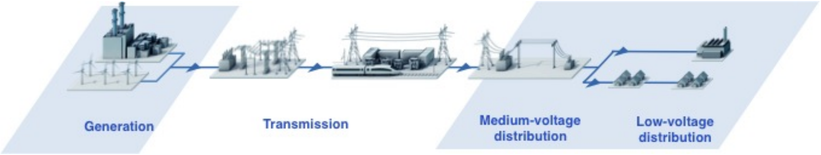
*ACM e-Energy 2016*

John W. Simpson-Porco

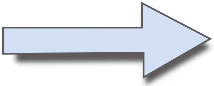
UNIVERSITY OF WATERLOO

# Our 20th Century Bulk Power System

A large-scale, nonlinear, hybrid, stochastic, distributed, cyber-physical . . .




**Active Control**

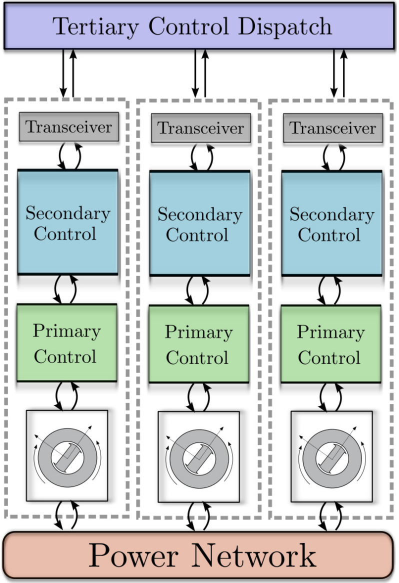


**Passive Consumption**

What kind of control is used?

# Bulk Power System Control Architecture & Objectives

## Hierarchy by spatial/temporal scales and physics




3. **Tertiary control** (offline)
   - Goal: optimize operation
   - Strategy: centralized & forecast
2. **Secondary control** (minutes)
   - Goal: restore frequency
   - Strategy: centralized
1. **Primary control** (real-time)
   - Goal: stabilize freq. and volt.
   - Strategy: decentralized

**Q:** Is this hierarchical architecture still appropriate for new applications?

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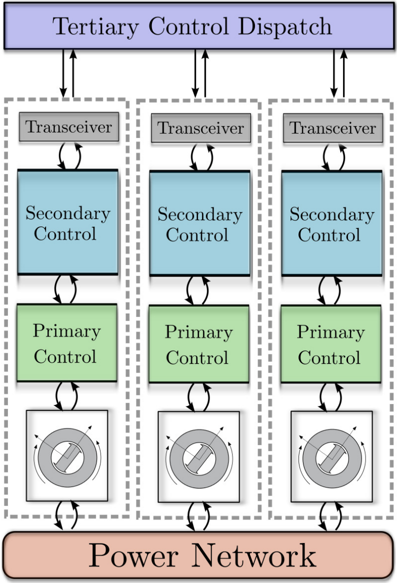


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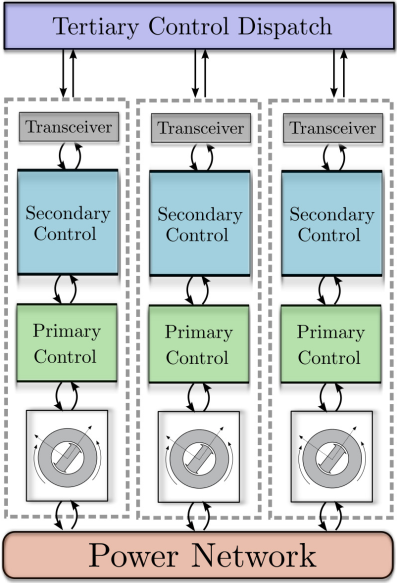


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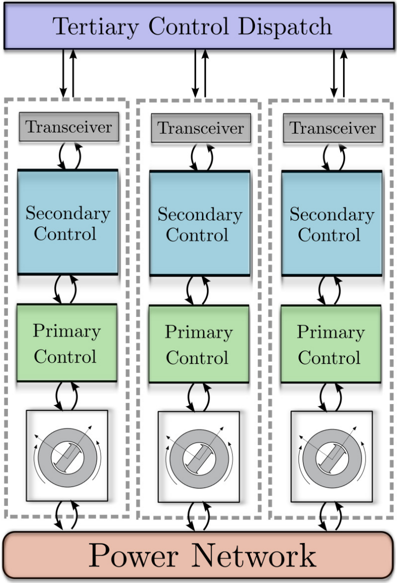


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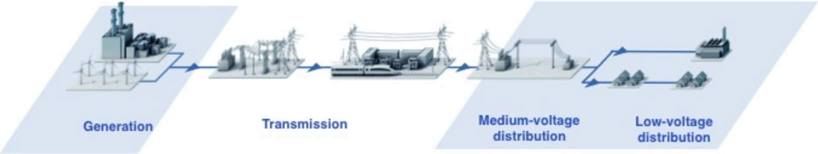

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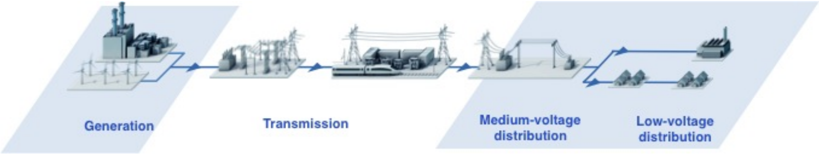

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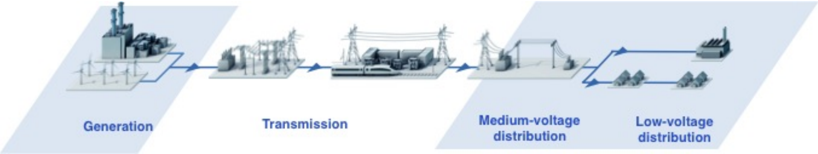
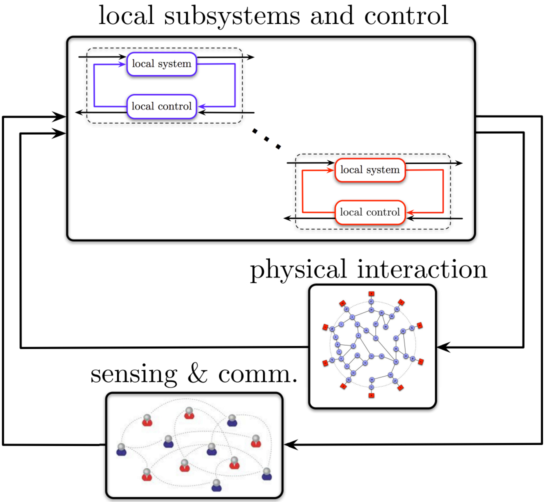



If central control authority fades . . . how to coordinate new actuators?

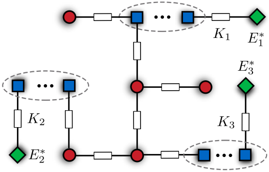
# My Perspective: Distributed Control Systems



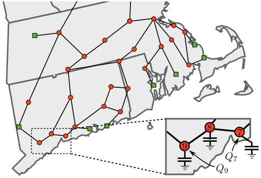

$\{$Simple Models$\} \cup \{$Analysis$\} \cup \{$Optimization$\} \cup \{$Control$\}$

# Smart Grid Project Samples

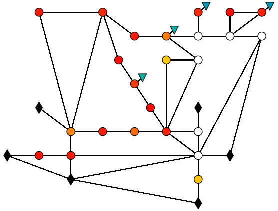
## Quadratic Droop Control (TAC)



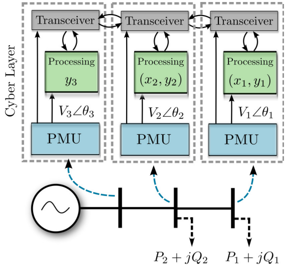

## Power Flow Solns. (Nat. Comms.)




## Optimal Distrib. Volt/Var (CDC)



## Wide-Area Monitoring (TSG)



# Microgrids

## Structure

- low-voltage, small footprint
- grid-connected or islanded
- autonomously managed

## Applications

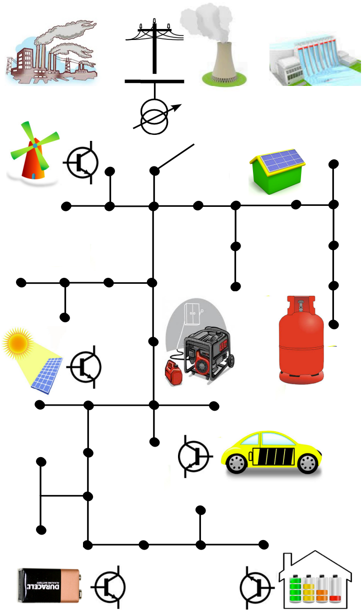
- hospitals, military, campuses, large vehicles, & remote locations

## Benefits

- naturally distributed for renewables
- scalable, efficient & redundant

## Operational challenges

- low inertia & uncertainty
- plug'n'play & no central authority



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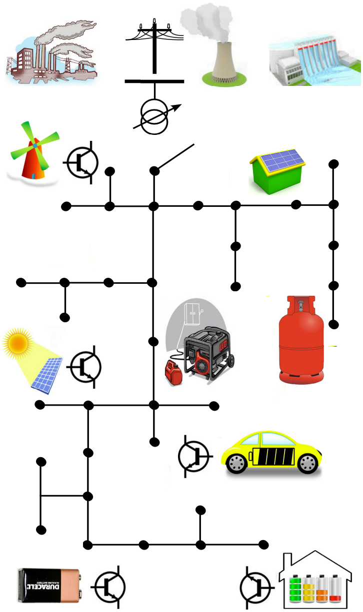
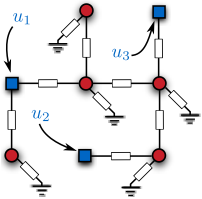


# Modeling I: AC circuits

1. **Loads (●) and Inverters (■)**
2. **Quasi-Synchronous:** $\omega \simeq \omega^* \ \Rightarrow V_i = E_i e^{j\theta_i}$
3. **Load Model:** Constant powers $P_i^*$, $Q_i^*$
4. **Coupling Laws:** Kirchoff and Ohm: $Y_{ij} = G_{ij} + jB_{ij}$




5. **Line Characteristics:** $G_{ij}/B_{ij} =$ const. (today, lossless $G_{ij} = 0$)
6. **Decoupling:** $P_i \approx P_i(\theta)$ & $Q_i \approx Q_i(E)$ (normal operating conditions)

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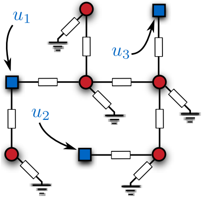


- active power: $P_i \;=\; \sum_j B_{ij}E_iE_j \sin(\theta_i - \theta_j) + G_{ij}E_iE_j\cos(\theta_i - \theta_j)$
- reactive power: $Q_i \;=\; -\sum_j B_{ij}E_iE_j \cos(\theta_i - \theta_j) + G_{ij}E_iE_j\sin(\theta_i - \theta_j)$

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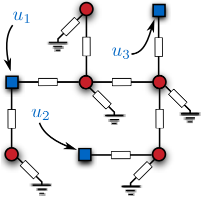


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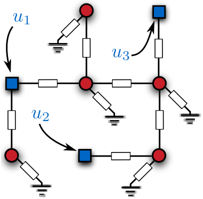


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- trigonometric active power flow: $P_i(\theta) \quad = \quad \sum_j B_{ij} \sin(\theta_i - \theta_j)$
- quadratic reactive power flow: $Q_i(E) \quad = \ -\sum_j B_{ij} E_i E_j$

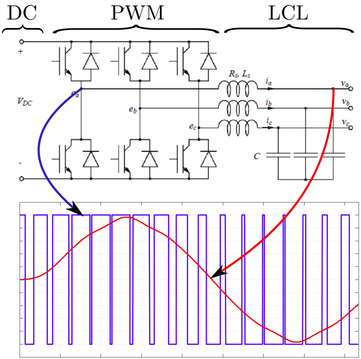
# Modeling II: Inverter-interfaced sources

also applies to frequency-responsive loads

Power **inverters** are ...

- **interface** between AC grid
  and DC or variable AC sources

- operated as **controllable** ideal
  voltage sources




Assumptions:

- Fast, stable inner/outer loops
  (voltage/current/impedance)
- Good harmonic filtering
- Balanced 3-phase operation

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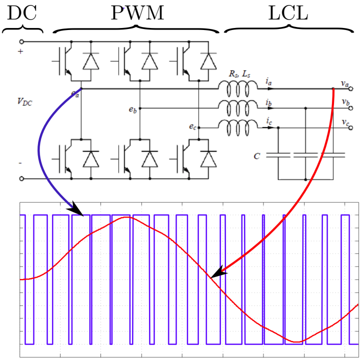
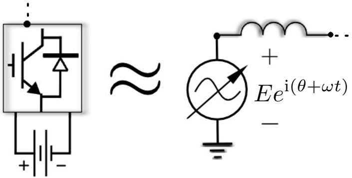
$$\tau_i \dot{\omega}_i = u_i^{\mathrm{freq}}, \quad \tau_i \dot{E}_i = u_i^{\mathrm{volt}}$$

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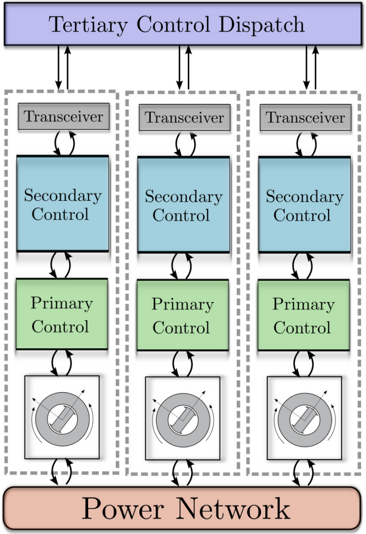
Tertiary Control Dispatch
Transceiver
Transceiver
Transceiver
Secondary Control
Secondary Control
Secondary Control
Primary Control
Primary Control
Primary Control
Power Network

# Open-Loop System & Control Objectives

| **Frequency Open-Loop** | **Voltage Open-Loop** |
|---|---|
| Inverter Dynamics ($i \in \mathcal{I}$): | Inverter Dynamics ($i \in \mathcal{I}$): |
| $\omega_i = \dot{\theta}_i = u_i^{\mathrm{freq}}$ <br> $P_i(\theta) = \sum_j B_{ij} \sin(\theta_i - \theta_j)$ | $\tau_i \dot{E}_i = u_i^{\mathrm{volt}}$ <br> $Q_i(E) = -\sum_j B_{ij} E_i E_j$ |
| Power Balance ($i \in \mathcal{L}$): | Power Balance: ($i \in \mathcal{L}$) |
| $0 = P_i^* - \sum_j B_{ij} \sin(\theta_i - \theta_j)$ | $0 = Q_i^* + \sum_j B_{ij} E_i E_j$ |

**Primary Control Objectives:**

1. **Stabilization:** Ensure stable frequency/voltage dynamics
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| **Frequency Droop Control** | **Voltage Droop Control** |
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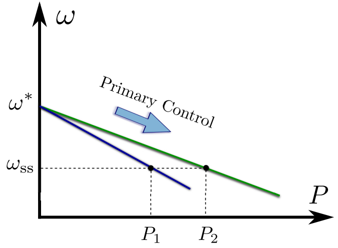
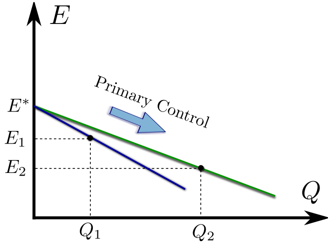
| $\omega_i = \omega^* - m_i P_i(\theta)$ | $\tau_i \dot{E}_i = -(E_i - E^*) - n_i Q_i(E)$ |



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"Grid-forming" decentralized control

**Key Idea:** emulate generator speed & AVR control

| **Frequency Droop Control** | **Quad. Voltage Droop Control** |
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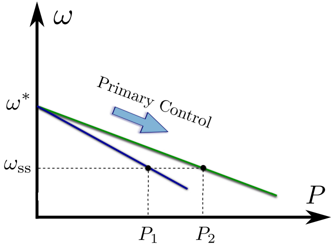


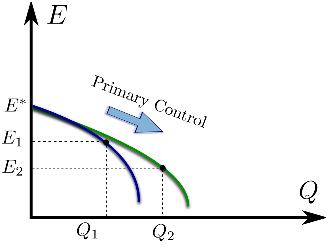

# Spring Network Interpretations of Equilibria

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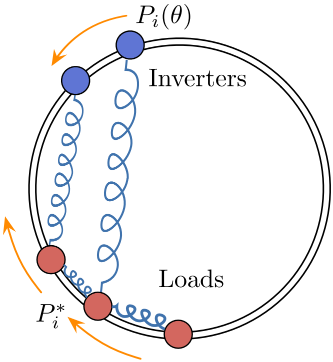

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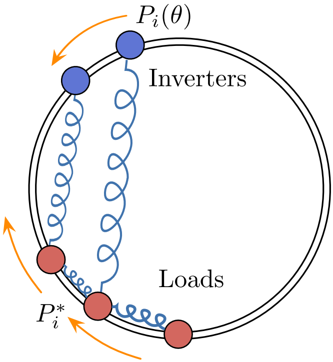

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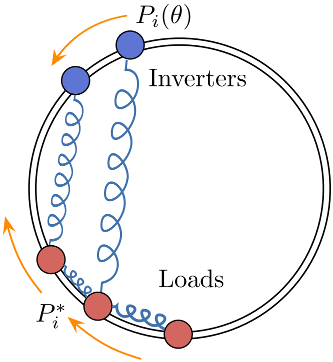


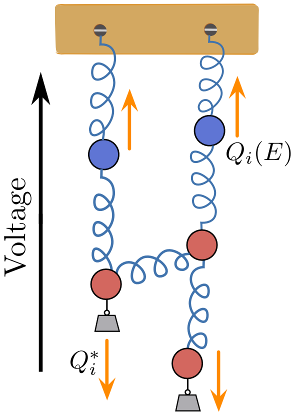

# Droop Control Stability Conditions

| **Frequency Droop Control** | Voltage Droop Control |
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| $0 = P_i^* - \sum_j B_{ij} \sin(\theta_i - \theta_j)$<br>$\dot{\theta}_i = -m_i \sum_j B_{ij} \sin(\theta_i - \theta_j)$ | $0 = Q_i^* + \sum_j B_{ij} E_i E_j$<br>$\tau_i \dot{E}_i = -E_i(E_i - E^*) + n_i \sum_j B_{ij} E_i E_j$ |
| Theorem: Frequency Stability (JWSP, FD, & FB '12)<br>∃! loc. exp. stable angle equilibrium $\theta_{\rm eq}$ iff<br>$\boxed{\dfrac{(A^\dagger P)_{ij}}{B_{ij}} < 1}$<br>for all lines $(i, j)$ of microgrid. | Theorem: Voltage Stability (JWSP, FD, & FB '15)<br>∃! loc. exp. stable voltage equilibrium point $E_{\rm eq}$ if<br>$\boxed{\frac{4}{(E^*)^2}(B_{\rm red}^{-1} Q_L)_i < 1}$<br>for all load buses $i$ of microgrid. |
| Necessary and Sufficient | Tight and Sufficient |

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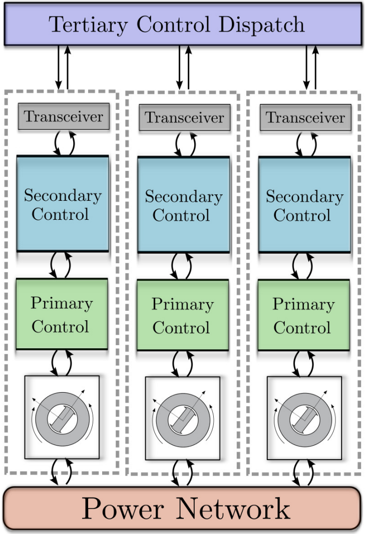
# Open Problems in Primary Control Stability

1. Analysis of *standard* voltage droop controller
2. Coupled droop equilibrium and unbal 3-phase analysis
3. Design or adaptation for non-uniform $R/X$ ratios
4. Limits of decentralized control based on $(\omega_i, V_i, P_i, Q_i)$

1. Stability of inverters + synchronverters
2. Interaction between droop and DR/battery management
3. Multi-harmonic extensions
4. Regulation vs. mechanism design


Tertiary Control Dispatch
Transceiver
Transceiver
Transceiver
Secondary Control
Secondary Control
Secondary Control
Primary Control
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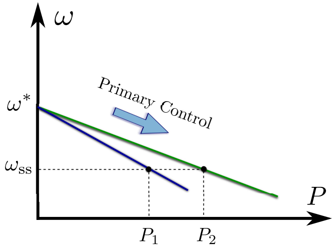
Primary Control
Power Network

# Secondary Control for Optimal Frequency Regulation

**Problem:** Usually $\omega_{\mathrm{ss}} \neq \omega^*$, and system subject to many disturbances




**Droop Control + Sec. Input**

$$\omega_i = \omega^* - m_i(P_i(\theta) - P_i^* - p_i)$$

**Active Power Flow**

$$P_i(\theta) = \sum\nolimits_{j=1}^{n} B_{ij} \sin(\theta_i - \theta_j),$$

**Problem:** Update $p_i$ online, in a model-free manner s.t.

(i) $\omega_{\mathrm{ss}} = 0$

(ii) "optimality" or power sharing

(iii) reject unknown disturbances

**Opt. Freq. Reg. Problem**

$$\begin{aligned} \underset{p}{\text{minimize}} \quad & \sum\nolimits_{i \in \mathcal{I}} J_i(p_i) \\ \text{subject to} \quad & \sum\nolimits_{i=1}^{n} (P_i^* + p_i) = 0 \\ & \underline{p}_i \leq p_i \leq \overline{p}_i \end{aligned}$$

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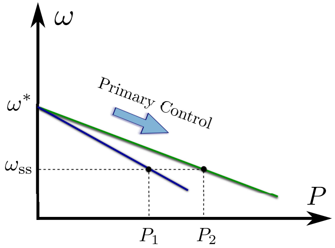


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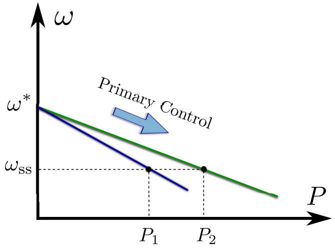


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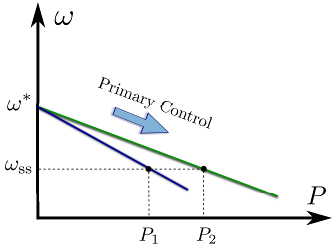


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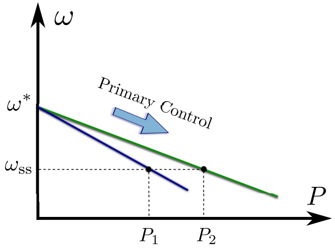


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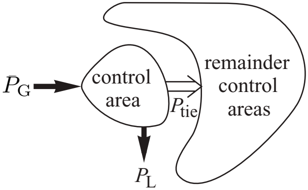

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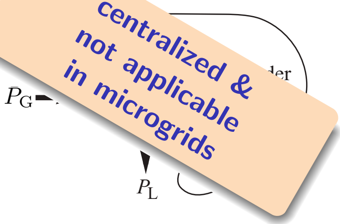


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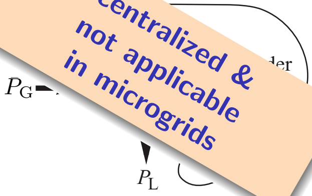


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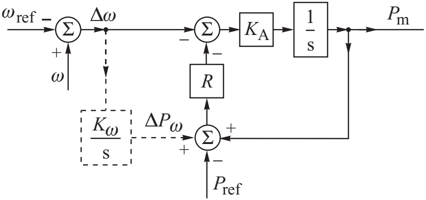

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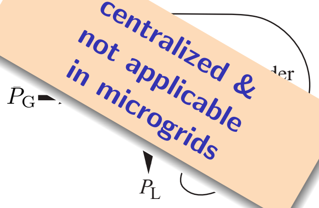
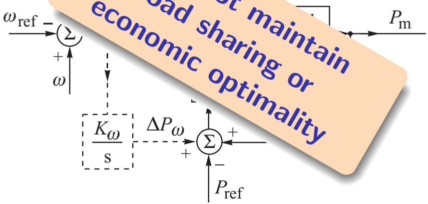



## Isolated Systems

- **Decentralized** PI control (isochronous mode)

**does not maintain load sharing or economic optimality**



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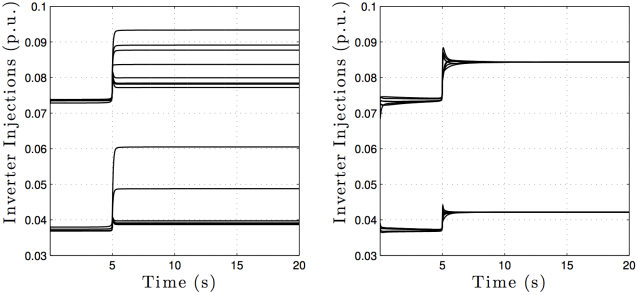


(a) Decentralized control (b) Centralized control

$P_m$

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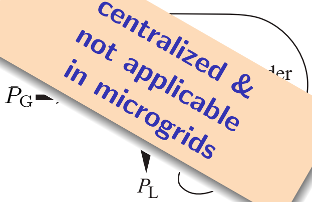


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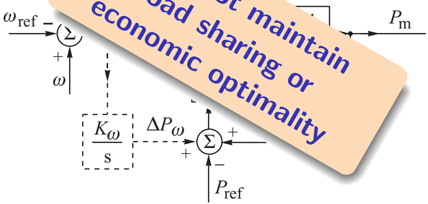


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Can we strike a middle ground between these two approaches?

# Distributed Averaging PI (DAPI) Frequency Control

$$\omega_i = \omega^* - m_i(P_i(\theta) - P_i^* - p_i)$$

$$k_i \dot{p}_i = (\omega_i - \omega^*) - \sum_{j \subseteq \text{inverters}} a_{ij} \cdot (m_i p_i - m_j p_j)$$

1. Power sharing: $\nabla J_i(p_i) = m_i p_i$
2. no tuning, no model dependence
3. weak comm. requirements
4. enforces equal marginal costs
   (share burden of sec. control)

Keep It Simple

**Theorem: Stability of DAPI**
[JWSP, FD, & FB, '13]

DAPI-Controlled System Stable
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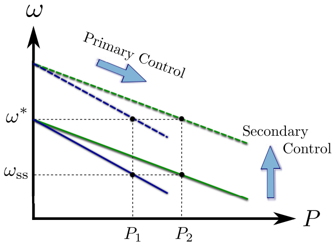


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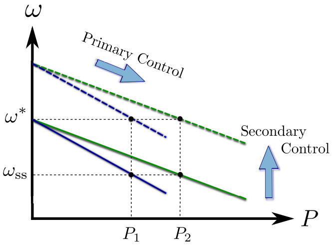


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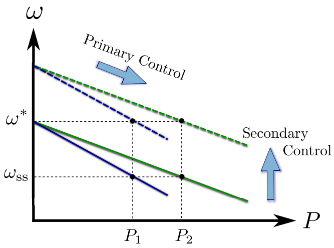


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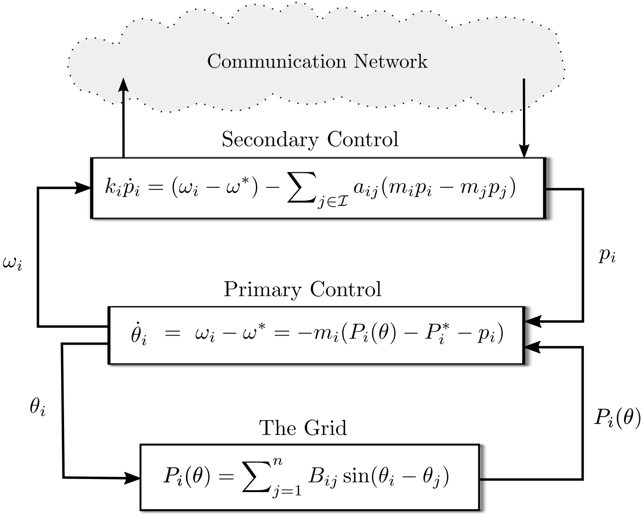

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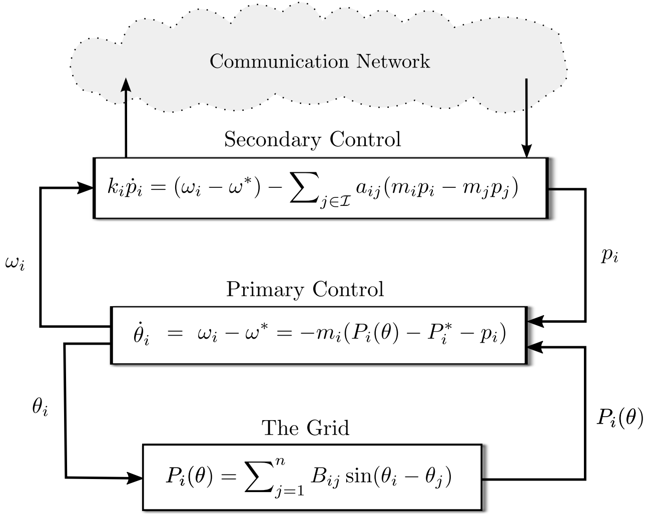


Build grid controls via ~~time-scale separation~~ **passivity**.

# DAPI Control From The Utility Side

**Question**

With this distributed controller, what does the microgrid look like from "the outside"?

It looks like there is no microgrid.

- Different units respond uniformly to disturbances and commands
- Microgrid acts as a *single entity*, a rigid formation of devices
- Coordination complexity hidden "behind the transformer"

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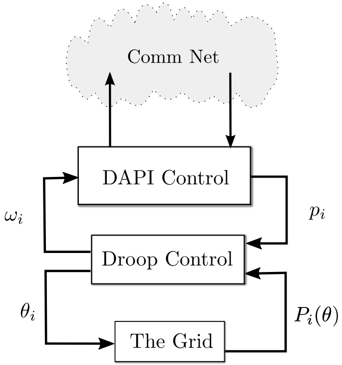
For real-time implementation . . . what about **I/O performance**?

**System norms** quantify amplification from **disturbances** (sensor noise, faults, uncertainty, etc.) to **controlled outputs**



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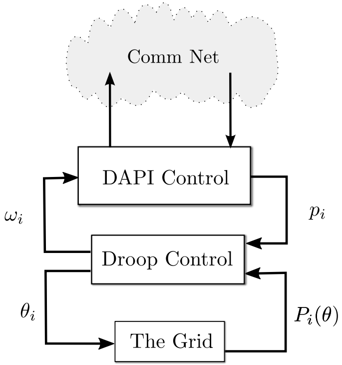

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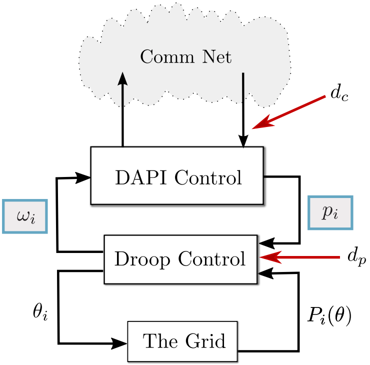


# Quick Review: The $\mathcal{H}_2$ System Norm

**Exp. Stable Linear System**

$$\begin{aligned} \dot{x} &= Ax + Bd \\ z &= Cx \end{aligned} \quad \Longleftrightarrow \quad G(s) = C(sI - A)^{-1}B$$

$$\|G\|_{\mathcal{H}_2}^2 \triangleq \frac{1}{2\pi} \int_{\mathbb{R}} \mathrm{Tr}\left[ G^{\mathsf{T}}(-j\omega) G(j\omega) \right] \, \mathrm{d}\omega$$

Useful Interpretations:

(i) **Steady-state output variance** $\lim_{t\to\infty} \mathbb{E}[z(t)^{\mathsf{T}} z(t)]$ when $d$ noise

(ii) "Average" gain over all frequencies from $d(\cdot)$ to $z(\cdot)$

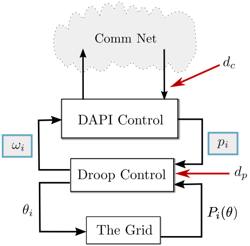
**If $(A, C)$ observable**

$$A^{\mathsf{T}} Y + YA + C^{\mathsf{T}} C = 0 \quad \Longrightarrow \quad \|G\|_{\mathcal{H}_2}^2 = \mathrm{Tr}(B^{\mathsf{T}} Y B)$$

# $\mathcal{H}_2$-Performance of Distributed Frequency Control

## Linearized, Network-Reduced, Simplified

$$G: \begin{cases} \dot{\theta}_i = \omega_i \\ \tau\dot{\omega}_i = -\omega - m_i(P_i(\theta) - p_i) + m_i d_{p,i} \\ k\dot{p}_i = -\omega_i - \gamma \sum_{j=1}^{n} a_{ij}\,(m_i p_i - m_j p_j - d_{c,j}) \\ z_{\omega,i} = \omega_i \\ z_{2,i} = \sum_{j=1}^{n} a_{ij}(m_i p_i - m_j p_j) \end{cases}$$




**Theorem: High-Gain Performance**

In the high-gain limit $\gamma \to \infty$, we have

$$\frac{1}{n}\|G\|^2_{d_p \to \omega} = \frac{m^2}{\tau} \qquad \frac{1}{n}\|G\|^2_{d_c \to \omega} = \frac{m^2}{2\tau} \cdot \frac{n-1}{n}$$

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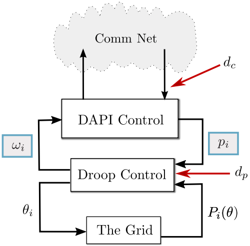


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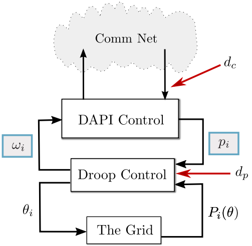


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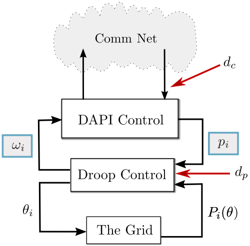


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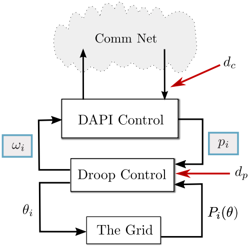


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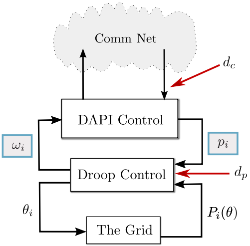


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# Distributed Averaging PI (DAPI) Voltage Control [TIE '15]

**Problem:** steady-state voltage deviations ($E_i \neq E_i^*$)

**Goals:** Voltage regulation $E_i \to E_i^*$, "load" sharing $Q_i/Q_i^* = Q_j/Q_j^*$

**Bad News:** These goals are *fundamentally* conflicting.

We propose a **heuristic compromise**.

$$\tau_i \dot{E}_i = -(E_i - E_i^*) - n_i Q_i(E) - e_i$$

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**Tuning Intuition:**

1. $\beta_i \gg \sum_j b_{ij} \implies$ voltage regulation
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# DAPI Voltage Control – Performance [TIE '15]

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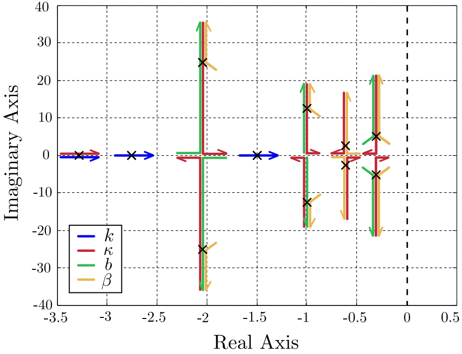
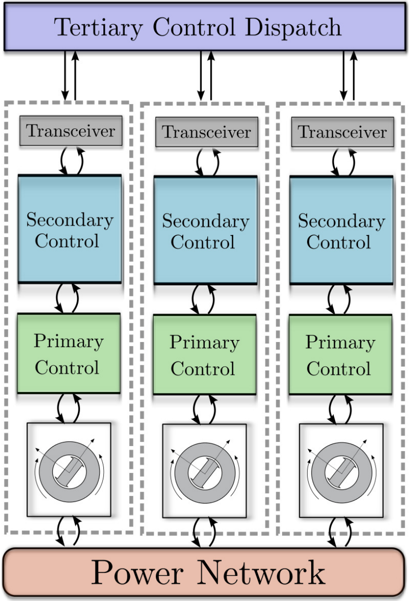


# From Hierarchical Control to DAPI Control

flat hierarchy, distributed, no time-scale separations, & model-free



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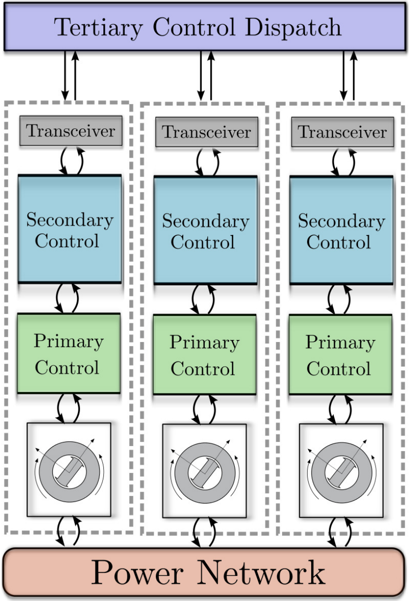
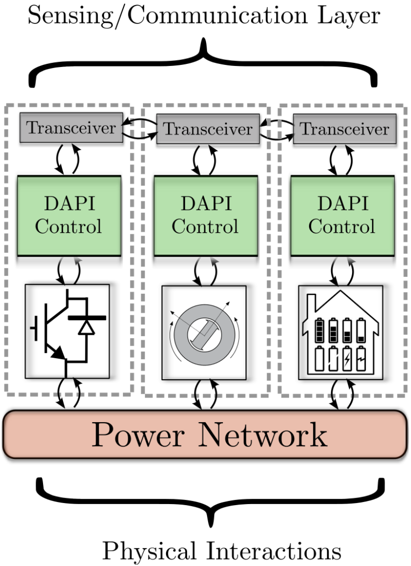
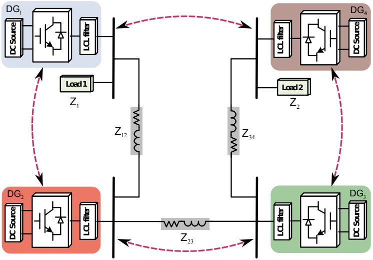


# Experimental Validation of DAPI Control

## Experiments @ Aalborg University Intelligent Microgrid Laboratory



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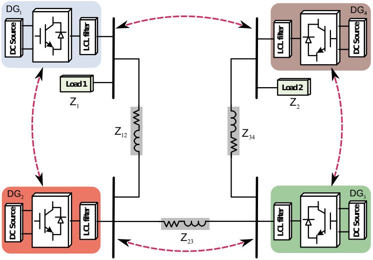


1. $t < 7$: Droop Control
2. $t = 7$: DAPI Control
3. $t = 22$: Remove Load 2
4. $t = 36$: Attach Load 2

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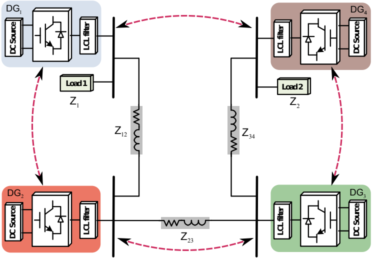


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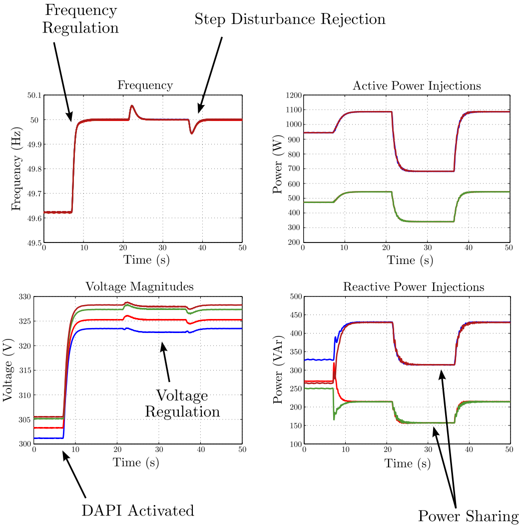
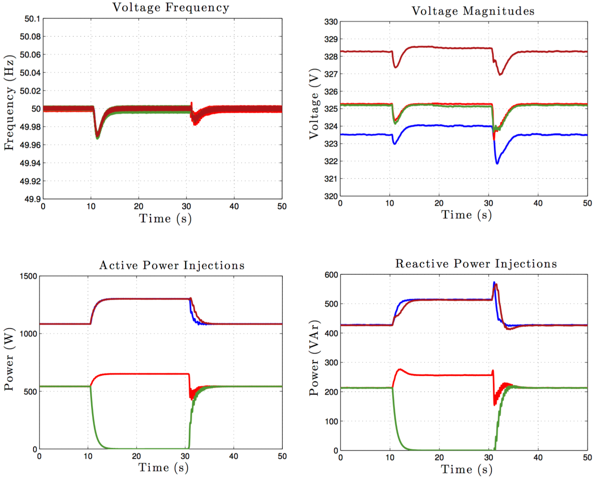


# Experiments – Plug-and-Play Operation

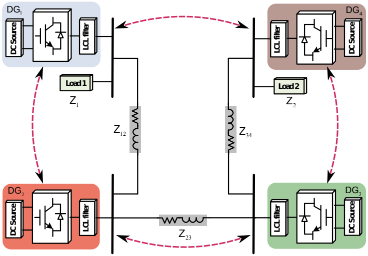
## Unit 3 (green) disconnected then reconnected



# Summary

## Distributed Inverter Control

- Primary control stability
- Distributed controllers
- Controller performance
- Extensive validation




## Future Work

- More detailed models
- More systematic designs
- More optimal control
- Monitoring $\Longleftrightarrow$ Feedback
- Distributed control security
- LV markets for control

# Acknowledgements

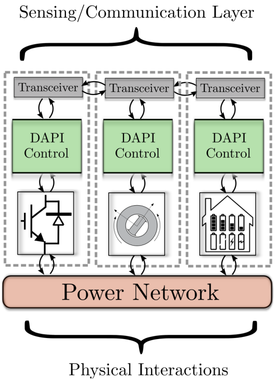
Florian Dörfler

Francesco Bullo

Qobad Shafiee

Josep Guerrero

# Questions




https://ece.uwaterloo.ca/~jwsimpso/
jwsimpson@uwaterloo.ca

# supplementary slides

# An incomplete literature review of a busy field

ntwk with unknown disturbances $\cup$ integral control $\cup$ distributed averaging

- all-to-all source frequency & injection averaging [Q. Shafiee, J. Vasquez, & J. Guerrero, '13] & [H. Liang, B. Choi, W. Zhuang, & X. Shen, '13] & [M. Andreasson, D. V. Dimarogonas, K. H. Johansson, & H. Sandberg, '12]
- optimality w.r.t. economic dispatch [E. Mallada & S. Low, '13] & [M. Andreasson, D. V. Dimarogonas, K. H. Johansson, & H. Sandberg, '13] & [X. Zhang and A. Papachristodoulou, '13] & [N. Li, L. Chen, C. Zhao & S. Low '13]
- ratio consensus & dispatch [S.T. Cady, A. Garcıa-Domınguez, & C.N. Hadjicostis, '13]
- load balancing in Port-Hamiltonian networks [J. Wei & A. Van der Schaft, '13]
- passivity-based network cooperation and flow optimization [M. Bürger, D. Zelazo, & F. Allgöwer, '13, M. Bürger & C. de Persis '13, He Bai & S.Y. Shafi '13]
- distributed PI avg optimization [G. Droge, H. Kawashima, & M. Egerstedt, '13]
- PI avg consensus [R. Freeman, P. Yang, & K. Lynch '06] & [M. Zhu & S. Martinez '10]
- decentralized "practical" integral control [N. Ainsworth & S. Grijalva, '13]