Lossy DC Power Flow

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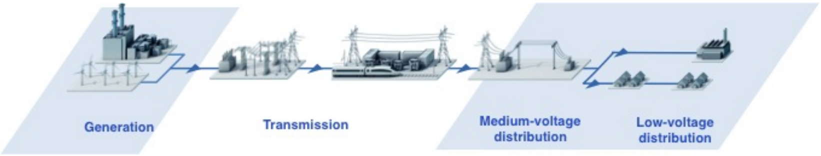
March 2, 2017

Our 20th Century Bulk Power System

A large-scale, nonlinear, hybrid, stochastic, distributed, cyber-physical . . .



Active Control → **Passive Consumption**

Today, we will focus on the network.

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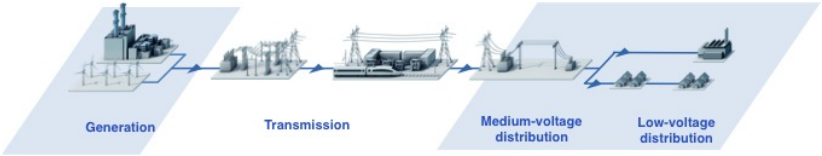

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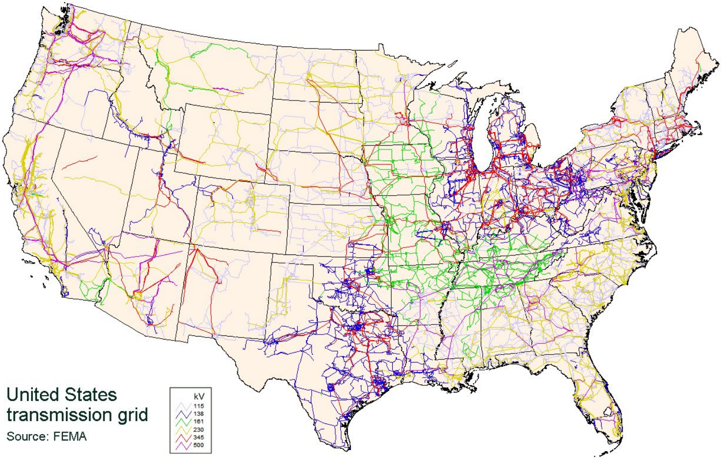
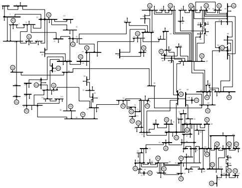
Modern Large-Scale Power Grids

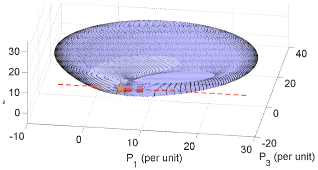


Problems in Power System Operations

Power Flow Analysis

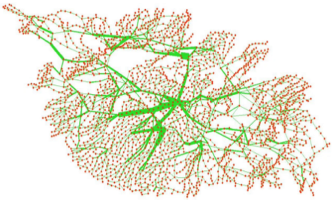


Optimal Power Flow



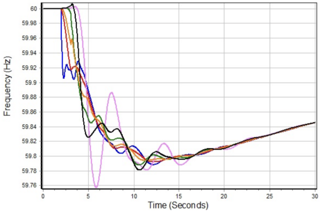
[Molzahn et al.]

Contingency Analysis



[Rezaei et al.]

Transient Stability



[Overbye et al.]

Linearized Optimal Power Flow

Idea: Optimally match supply and demand (with constraints)

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\underset{\theta, P_G}{\text{minimize}} \quad & \sum\nolimits_{i \in \mathcal{N}_G} f_i(P_i) \\
\text{subject to} \quad & P_i = \sum\nolimits_j B_{ij}(\theta_i - \theta_j) && i \in \mathcal{N}_L \cup \mathcal{N}_G \,, \\
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Big drawback: model does not have resistances

(power losses are not modelled)

AC Power Flow Equations

1. **Network:** n buses $\mathcal{N}$, m lines $\mathcal{E}$, line admittances $y_{ij} = g_{ij} + \mathbf{j} b_{ij}$

2. **Bus Variables:** voltage $V_i e^{\mathbf{j}\theta_i}$, power $S_i = P_i + \mathbf{j} Q_i$

3. **Circuit Laws:** Kirchhoff & Ohm

4. **Admittance Matrix:** $Y = G + \mathbf{j} B$

$$Y_{ij} = -y_{ij} \,, \qquad Y_{ii} = -\sum\nolimits_{j=1}^{n} y_{ij}$$

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$$P_i = \sum_{j=1}^{n} V_i V_j B_{ij} \sin(\theta_i - \theta_j) + \sum_{j=1}^{n} V_i V_j G_{ij} \cos(\theta_i - \theta_j)$$

Simplified Models of Active Power Flow

- Many applications require some simplifying assumptions

1. **constant voltage magnitudes:** $V_i \approx 1$
2. **no resistances:** $G_{ij} \approx 0$

$$P_i = \sum_{j=1}^{n} B_{ij} \sin(\theta_i - \theta_j) \qquad \text{“Decoupled PF”}$$

3. **small phase differences:** $|\theta_i - \theta_j| \ll 1$

$$P_i \approx \sum_{j=1}^{n} B_{ij}(\theta_i - \theta_j) \qquad \text{“DC PF”} \quad \Longrightarrow \quad \theta = B^{-1}P$$

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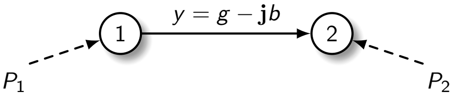
Our goal is to include resistances (remove assumption #2)

(we will not even need assumption #3)

Lossy DC Power Flow: Two-Bus Case



$$P_1 = b\sin(\theta_1 - \theta_2) + g - g\cos(\theta_1 - \theta_2)$$
$$P_2 = b\sin(\theta_1 - \theta_2) + g - g\cos(\theta_1 - \theta_2) \qquad \text{slack/ref bus}$$

- Let $\psi = \sin(\theta_1 - \theta_2)$. Then $\cos(\theta_1 - \theta_2) = \sqrt{1 - \psi^2}$

$$P_1 = b\psi + g - g\sqrt{1 - \psi^2}$$

- now isolate ψ

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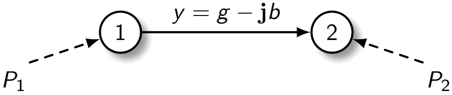

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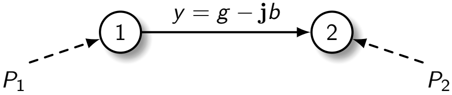

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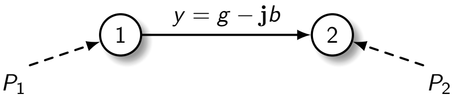

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Lossy DC Power Flow: Two-Bus Case

$$\psi = \frac{1}{b}\left(P_1 + g\sqrt{1-\psi^2} - g\right)$$

This is a **fixed-point equation** $\psi = f(\psi)$

Interpret as an **iteration**. Start with initial guess ψ_0 and update:

$$\psi_{k+1} = f(\psi_k) = \frac{1}{b}\underbrace{\left(P_1 + g\sqrt{1-\psi_k^2} - g\right)}_{:=P_{1,k}}$$

After ψ **converges**, simply compute $\theta_1 - \theta_2 = \arcsin(\psi)$

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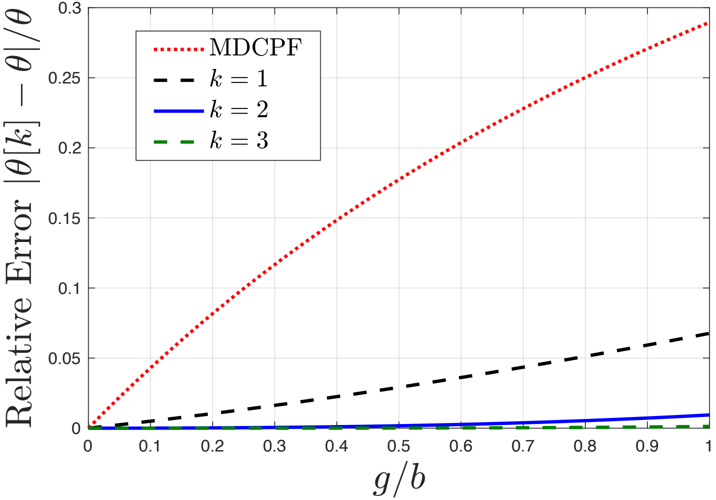
Convergence for Two-Bus Case



Can we show this converges? Yes!

Theorem: Convergence of Lossy DC Power Flow

The Lossy DC Power Flow iteration $\psi_{k+1} = f(\psi_k)$ converges exponentially to a power flow solution θ^* if

$$\left(\frac{P_1}{b}\right)^2 + 2\left(\frac{g}{b}\right)\left(\frac{P_1}{b}\right) < 1\,.$$

The power flow solution θ^* is the unique solution contained in the normal operating regime.

- P_1/b is the DC PF solution
- $\rho = g/b$ is the R/X ratio of the network

Can we extend this idea to networks?

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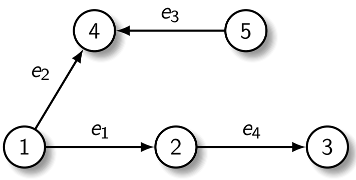
Graph and circuit matrices

- very convenient to write power flow in matrix/vector notation
- introduce **incidence matrix** $A \in \mathbb{R}^{n \times m}$

$$A_{ke} = \begin{cases} 1 & \text{if bus } k \text{ at sending-end of edge } e \\ -1 & \text{if bus } k \text{ at recieving-end of edge } e \\ 0 & \text{otherwise} \end{cases}$$



$$A = \begin{bmatrix} +1 & +1 & 0 & 0 \\ -1 & 0 & 0 & +1 \\ 0 & 0 & 0 & -1 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & +1 & 0 \end{bmatrix}$$

- $\ker(A) = \{0\}$ if graph has no cycles

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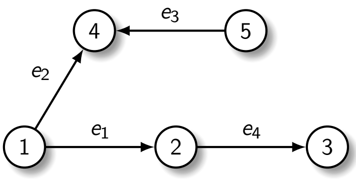

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Modified DC Power Flow

- with vector notation

$$\mathsf{D}_B = \mathrm{diag}(B_{ij})\,, \qquad P = (P_1, \ldots, P_n)^\mathsf{T}\,, \qquad \theta = (\theta_1, \ldots, \theta_n)^\mathsf{T}$$

decoupled power flow becomes

$$P_i = \sum_{j=1}^{n} B_{ij} \sin(\theta_i - \theta_j) \quad \Longleftrightarrow \quad P = A\mathsf{D}_B \,\mathbf{sin}(A^\mathsf{T}\theta)$$

- candidate solution: The "Modified" DC PF

$$\psi = \mathbf{sin}(A^\mathsf{T}\theta)\,, \qquad \psi = A^\mathsf{T} B^{-1} P$$

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Novel Insights into Lossless AC and DC Power Flow

Florian Dörfler, *Student Member, IEEE,* and Francesco Bullo, *Fellow, IEEE*

Lossy DC Power Flow for Networks

$$P_i = \sum_{j \neq i} B_{ij} \sin(\theta_{ij}) + G_{ii} + \sum_{j \neq i} G_{ij} \cos(\theta_i - \theta_j)\,,$$

$$G_{\mathrm{diag}} = (G_{11}, G_{22}, \ldots, G_{nn})^{\mathsf{T}} \qquad \mathsf{D}_G = -\mathrm{diag}(G_{ij})$$

Vectorized Lossy Power Flow

$$P = A\mathsf{D}_B \,\mathbf{sin}(A^{\mathsf{T}}\theta) + G_{\mathrm{diag}} - |A|\mathsf{D}_G \,\mathbf{cos}(A^{\mathsf{T}}\theta)$$

- substitute $\psi = \mathbf{sin}(A^{\mathsf{T}}\theta)$ like before

$$P = A\mathsf{D}_B\psi + G_{\mathrm{diag}} - |A|\mathsf{D}_G\sqrt{\mathbb{1} - \psi^2}$$

- now isolate ψ!

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Lossy DC Power Flow for Networks contd.

We obtain a **fixed-point equation**

$$\psi = f(\psi) := A^\mathsf{T} B^{-1} \left(P - G_{\mathrm{diag}} + |A| \mathrm{D}_G \sqrt{1 - \psi^2} \right)$$

which we interpret as an **iteration**

Lossy DC Power Flow Iteration

$$\psi_{k+1} = f(\psi_k) := A^\mathsf{T} B^{-1} \left(P - G_{\mathrm{diag}} + |A| \mathrm{D}_G \sqrt{1 - \psi_k^2} \right)$$

- phase angles calculated as

$$A^\mathsf{T} \theta_k = \mathbf{arcsin}(\psi_k)$$

Lossy DC Power Flow for Networks contd.

We obtain a **fixed-point equation**

$$\psi = f(\psi) := A^{\mathsf{T}} B^{-1} \left(P - G_{\text{diag}} + |A| \mathrm{D}_G \sqrt{1 - \psi^2} \right)$$

which we interpret as an **iteration**

Lossy DC Power Flow Iteration

$$\psi_{k+1} = f(\psi_k) := A^{\mathsf{T}} B^{-1} \left(P - G_{\text{diag}} + |A| \mathrm{D}_G \sqrt{1 - \psi_k^2} \right)$$

- phase angles calculated as

$$A^{\mathsf{T}} \theta_k = \mathbf{arcsin}(\psi_k)$$

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Convergence of Lossy DC Power Flow

Theorem

The Lossy DCPF iteration $\psi_{k+1} = f(\psi_k)$ converges exponentially to a power flow solution θ^* if the network has no cycles and

$$\|\psi_{\mathrm{DC}}\|_\infty^2 + 2\rho\|\psi_{\mathrm{DC}}\|_\infty < 1$$

The power flow solution θ^* is the unique solution contained in the normal operating regime

$\psi_{\mathrm{DC}} = A^{\mathsf{T}} B^{-1} P$ is the DC PF solution

$\rho = \|\mathsf{D}_B^{-1} A^{-1} |A| \mathsf{D}_G\|_\infty$ measures R/X ratio

Proof based on **contraction mapping** principle

Simulations on MATPOWER Test Cases

Table: Base Case Testing of Lossy Modified DC Power Flow

Test System	Error (deg) DCPF	Error (deg) $k = 1$	Error (deg) $k = 2$
New England 39	1.33	0.02	0.00
RTS '96 (2 area)	1.88	0.03	0.00
57 bus system	0.55	0.01	0.00
RTS '96 (3 area)	4.16	0.06	0.01
118 bus system	3.49	0.05	0.01
300 bus system	19.3	0.22	0.07
Polish 2383wp	5.32	0.31	0.02
PEGASE 2869	**21.44**	**0.61**	**0.05**
PEGASE 9241	74.05	6.02	0.37
PEGASE 13,659	242.7	111.7	5.85

Conclusions

Lossy DC Power Flow extends the DC PF

1. Iterative approximation
2. Convergence conditions
3. Successful numerical tests

Lossy DC Power Flow

John W. Simpson-Porco, *Member, IEEE*

Future work:

1. Analysis for **meshed networks**
2. Application to stability/contingency
3. Application to market LMPs

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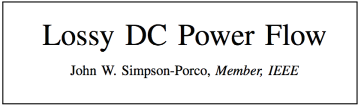
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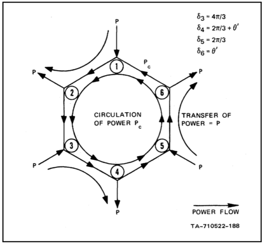

Questions

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appendix

Algorithmic Implementation

Algorithm 1: Lossy Modified/Unmodified DCPF

Inputs: grid data, approximation order $k \in \mathbb{N}$

Outputs: voltage phase angles $\theta_r[k]$

$\psi[0] = \mathbb{0}_m$

for $\ell \leftarrow 1$ **to** k **do**

  $P_r[\ell - 1] \leftarrow P_r - G_{\mathrm{diag}} + |A|_r \mathrm{D}_G \sqrt{\mathbb{1}_m - (\psi[\ell - 1])^2}$

  $\delta_r[\ell] \leftarrow \texttt{Solve}(\mathrm{L}_B \delta_r[\ell] = P_r[\ell - 1])$

  $\psi[\ell] \leftarrow A_r^{\mathsf{T}} \delta_r[\ell]$

if *Lossy Modified DCPF* **then**

  $\theta_r[k] \leftarrow \texttt{Solve}(A_r^{\mathsf{T}} \theta_r[k] = \textbf{arcsin}(\psi[k]))$

  return *L-MDCPF voltage angles* $\theta_r[k]$

else

  $\theta_r[k] \leftarrow \delta_r[k]$

  return *L-DCPF voltage angles* $\theta_r[k]$